Optimal Discriminatory Disclosure*

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Abstract

A seller of an indivisible good designs a selling mechanism for a buyer who knows privately the distribution of his value for the good (his type) but not the realization of his value. The seller also controls how much information about the realized value the buyer is allowed to learn privately. In a model of two types with an increasing likelihood ratio, we show that under some regularity conditions the disclosure policy in an optimal mechanism has a nested interval structure: the high type is allowed to learn whether his value is above the seller's cost, while the low type is allowed to learn whether his value is in an interval above the cost. Information discrimination is in general necessary in an optimal mechanism.

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1 Introduction

In many bilateral trade environments with one-sided incomplete information, the informed party (say the buyer) is endowed with some private information about the underlying state of the potential trade, but his initial private information is often incomplete and he can learn additional information over time. In these environments, however, the buyer's access to the additional information may be controlled by the uninformed party (say the seller). Examples include the seller designing product trials, restricting the nature and the number of tests that the buyer can carry out, and managing the buyer's access to data. How should the seller design the information policy together with the selling mechanism?

Recent advancement in information technologies has made it easier to compute and refine personalized prices, and at the same time has also enhanced dissemination of personalized information to potential buyers. Interactions between price discrimination and information discrimination in mechanism design are a new theoretical issue that we study in this paper. In particular, do optimal mechanisms generally require information discrimination?

Answering these questions is potentially important for businesses who can provide customized data access to their customers, especially in environments where discriminatory disclosure may face regulatory or legal challenges.[1] Moreover, a better understanding the interaction between price discrimination and information discrimination can help regulatory authorities enact better policies to protect consumers.

We adapt the framework of sequential screening (Courty and Li (2000)) to study the issue of information disclosure. The buyer's type represents his initial private information regarding from which distribution his value for the seller's product is drawn. There are two types, and we assume that the value distribution of the "high type" dominates in likelihood ratio order that of the "low type." The seller also controls an additional signal about the buyer's value that she can disclose without observing its realization. The seller chooses a menu of experiments, one for each type, and a menu of option contracts, each consisting of an advance payment and a strike price. The advance payment here can be interpreted as the price for both the call option and the access to the additional information controlled by the seller.

[1] For example, in financial markets, the *2000 Regulation Fair Disclosure* (Reg FD) prohibits the disclosure of nonpublic, material information to selected parties to ensure investors have a fair access to information. In practice, however, security brokers often help their premium customers arrange one-to-one meeting with firm managers by hosting private "non-deal roadshows (NDRs)" (see for example, Bradley, Jame, and Williams (2021)). NDRs are costly to arrange and the information shared during private meetings may be material, in violation of Reg FD.

We assume that all information about the underlying state except the buyer's ex ante type is under the seller's control. Hence, the additional signal controlled by the seller is correlated with the buyer's type. Our main characterization is that, under some regularity conditions, the optimal discriminatory disclosure policy is a pair of partitions of the value support with a nested interval structure. More precisely, each buyer type is recommended to buy if the realized value lies inside some interval, without knowing the exact realization, and is otherwise recommended not to buy, again without knowing the exact realization. Furthermore, the "buy intervals" for the two types are nested: the low type's buy interval is a subset of the high type's buy interval. The partitioning for the high type is efficient in the sense that the buy interval includes all realized values higher than the seller's cost (reservation value), and is therefore monotone. The partitioning for the low type is inefficient in that the buy interval lies above the the seller's cost, and more interestingly, can be non-monotone. Depending on the level of likelihood ratio at the top, the buy interval of the low type may exclude an interval of highest realized values. Intuitively, if the likelihood ratio of the two distributions is sufficiently high for an interval of highest values, excluding these realizations from the low type's buy interval may significantly reduce the high type's information rent with little sacrifice on the trading surplus with the low type. This is because the deviating high type would have gained most from buying at these realizations which occur with high probability for the high type but with low probability for the low type.

In the original sequential screening model (Courty and Li (2000)), there is only price discrimination because the buyer privately learns his value after the seller commits to a mechanism. Some features of the optimal mechanism from Courty and Li (2000) remain in the present model with both price and information discrimination.[2] In particular, there is no allocation distortion at the top, meaning that the high type buys the good whenever his realized value is above the seller's cost, and only downward distortion at the bottom, in that the low type never buys when his value is below a cutoff that is strictly higher than the cost. With information discrimination in this paper, the high type is allowed to have the necessary information for the efficient allocation, but the information disclosed to the low type leads to a different form of downward distortion when the partition is non-monotone: the low type is also prevented from buying when his value is above the buy interval. This is impossible in Courty and Li (2000), where incentive compatibility after the buyer learns his realized value requires allocation

[2] Courty and Li (2000) studies both first order and second order stochastic dominance ranking of value distributions by type. In this paper, we assume likelihood ratio dominance ranking, which implies first order stochastic dominance ranking.

monotonicity, but in the present paper this is used to reduce the information rent for the high type.

The interaction between information and price discriminations makes it necessary to adapt the standard approach to mechanism design, and one contribution of this paper is to show how to do it. In contrast to Courty and Li (2000), here with endogenous information structure the individual rationality condition for the high type in general does not follow from the incentive compatibility constraint of the high type and the individual rationality constraint of the low type. We provide "regularity conditions" on the primitives of the mechanism design problem for any optimal information disclosure to allow the high type to buy the good with a greater probability after a deviation than the truthful low type. This ensures that the individual rationality constraint of the high type can be dropped in a suitably relaxed problem.

To answer the question of whether optimal mechanisms require information discrimination, we study whether the same profit achieved by the optimal discriminatory disclosure policy can be replicated by a non-discriminatory disclosure policy, generated by the coarsest common refinement of the pair of partitions. The buyer is now given the same information regardless of his type report, and the critical question is whether the high type's incentives would remain the same as under discriminatory disclosure, especially after lying about his type (off-path). Due to the nested-interval structure, the answer is "yes" when the partition for the low type is monotone. However, when the partition is non-monotone, the answer is "no" because the high type may profit from disobeying recommendation after misreporting as the low type. We provide an analytical example that this is indeed the case, and information discrimination is necessary.

The rest of the paper is organized as follows. We conclude the Introduction by discussing related papers in the existing literature. Section 2 sets up the model. Section 3 characterizes the optimal mechanism including the disclosure policy. Section 4 discusses how our analysis may be extended to settings where the information controlled by the seller is independent of the buyer's type and to settings where price discrimination is not possible. Section 5 offers some suggestions about future research directions in mechanism design with endogenous information structures.

1.1 Related literature

Our paper belongs to the literature of private information disclosure where the realization of the seller's signal is privately observable to the buyer but not to the seller.

The idea of private disclosure was first introduced by Lewis and Sappington (1994) to the mechanism design literature.[3]

The joint design problem of information policy and pricing scheme has been previously investigated by a number of papers. Bergemann and Pesendorfer (2007) consider an auction setting without ex ante private information and show that, if the seller cannot charge fee for information, the optimal disclosure in an optimal auction must assign asymmetric partitions to ex ante homogeneous buyers. If buyers have ex ante private information and the seller can charge fee for information, Eső and Szentes (2007) show that full disclosure is optimal when the seller is restricted to disclosing only the orthogonal component of the seller's information, that is, the part of seller's information that is independent of the buyers' private information.[4]

Li and Shi (2017) consider a bilateral trade setting similar to the one in Eső and Szentes (2007), but allow the seller to directly garble the information under her control. Their goal is to show that full disclosure is then generally suboptimal.[5] In particular, they show that monotone binary partitions of the true value dominate full disclosure in terms of the seller's revenue, by limiting the buyer's additional private information to only whether his true value is above or below some partition threshold, instead of allowing him to learn the exact value as under full disclosure. They do not solve the joint design problem of information policy and pricing scheme in their setup. Moreover, even though the disclosure policy with monotone binary partitions they use to establish the sub-optimality of full disclosure is discriminatory, this does not imply that information discrimination is necessary for profit maximization. Therefore, our research questions of optimal disclosure policy and of the necessity of information discrimination remain open.

This paper aims to address these two open questions in the same setting of Li and Shi (2017), except that the buyer's type is assumed to be binary. We show that the optimal disclosure policy consists of a pair of intervals, which nests as a special case the monotone binary partitions that Li and Shi (2017) use to show the sub-optimality of full disclosure. Although effective in both creating trade surplus and extracting

[3]See also an earlier contribution by Kamien, Tauman, and Zamir (1990). Subsequent literature on static private disclosure includes Che (1996), Ganuza (2004), Johnson and Myatt (2006), and Ganuza and Penalva (2010).

[4]Hoffmann and Inderst (2011) and Bergemann and Wambach (2015) also consider information disclosure in the sequential screening setting, but they focus on the case where the information released by the seller is independent of the buyer's private information. See also Lu, Ye, and Feng (2021) for a related analysis of how a seller can use a two-stage mechanism to induce bidders to acquire additional information.

[5]Krähmer and Strausz (2015) show that the irrelevance theorem in Eső and Szentes (2007) fails if the buyer's type is discrete. They present an example in which full disclosure is not optimal.

information rent, a monotone partition for the low type can be too informative for the deviating high type, generating a large information rent. Therefore, non-monotone partitioning in the form of intervals may be needed for profit maximizing when the likelihood ratios are large for the highest values.[6]

This paper also extends the profit equivalence between discriminatory and non-discriminatory disclosure with independent information in Eső and Szentes (2007) to a setting with binary types (see Section 4). But this equivalence breaks down if the buyer's participation constraint is posterior rather than interim. Wei and Green (2020) consider a similar design problem with independent information, but assume that the buyer's payoff must be non-negative for every type and every signal realization. The non-negative payoff restriction rules out advance payment and hence the optimal selling mechanism takes the form of type-dependent posted prices. They show that information discrimination and price discrimination are complements and optimal mechanism must feature both. In other words, the profit attained by optimal mechanism with discriminatory disclosure cannot be replicated by non-discriminatory disclosure.[7]

The issue of (non-)equivalence between discriminatory and non-discriminatory disclosure has been investigated in the literature of Bayesian persuasion. If the receiver's type is independent of the sender's information, Kolotilin, Mylovanov, Zapechelnyuk, and Li (2017) show that, for any incentive compatible discriminatory disclosure policy, there is a non-discriminatory disclosure policy that yields the same interim payoff for both parties. In other words, incentive compatibility alone implies equivalence. If the receiver's type is correlated with the sender's information, however, Guo and Shmaya (2019) show that equivalence does not follow from incentive compatibility but optimality does imply equivalence. That is, the sender-optimal discriminatory disclosure can be implemented as a non-discriminatory disclosure.

As in Guo and Shmaya (2019), our seller's signal is correlated with the buyer's type. Different from Guo and Shmaya (2019), our seller can use prices, in additional to information, to discriminate against different buyer types, and moreover, her goal is

[6]Krähmer (2020) considers a design setting similar to ours and allows the seller to secretly randomize information structures via a secret randomization device. He shows that, if the contract can be made contingent on the seller's randomization outcome, then the seller can use a scheme similar to Crémer and McLean (1988) to extract the full surplus. Zhu (2017) studies a similar problem in a multi-agent setting and shows that an individually uninformative but aggregately revealing disclosure policy can extract full surplus. Transfers are not needed in his construction. Such randomization of information structures and contracting technology are not allowed in our paper.

[7]Smolin (2020) considers a model where the buyer's value is a weighted average of the value of several product attributes. He shows that it is without loss to consider linear disclosure which reveals whether a weighted value of attributes is above some threshold, and that the optimal menu may be non-discriminatory.

to maximize profit rather than the expected purchase probability. We have shown that equivalence fails in general if price discrimination is possible. If price discrimination is impossible – different buyer types must receive the same pricing scheme, we show that optimality implies equivalence as in Guo and Shmaya (2019) if the gain from trade is certain. If the gain from the trade is uncertain, we show in Section 4 through an example that optimal disclosure policy may not be a pair of nested intervals and that information discrimination is necessary to obtain the maximal profit.

2 The Model

A risk-neutral seller (she) has a product for sale to a risk-neutral buyer (he). The buyer's value for the product is ω, which is drawn from $\Omega = [\underline{\omega}, \overline{\omega}]$ and is initially unknown to both players. The buyer has private information about his value, which we refer to as his type. Let θ denote the buyer's type and assume a binary type space $\theta \in \{H, L\}$, with ϕ_H and $\phi_L = 1 - \phi_H$ being the probabilities of type H and type L respectively. Let $F_\theta(\cdot)$ be the cumulative distribution function of the buyer's value ω conditional on θ. We assume that $F_\theta(\cdot)$ has a positive and finite density $f_\theta(\cdot)$, and that H stochastically dominates L in likelihood ratio order, i.e., $f_H(\omega)/f_L(\omega)$ is weakly increasing for all ω. We denote by μ_θ the mean of $F_\theta(\cdot)$, i.e., $\mu_\theta = \int_{\underline{\omega}}^{\overline{\omega}} \omega \; \mathrm{d}F_\theta(\omega)$. If the buyer chooses not to participate in the seller's mechanism, he gets a payoff of zero regardless of his type. The seller's production cost (reservation value for the product) is known to be c, with $c < \overline{\omega}$.

For simplicity, we assume that all information of the buyer about ω except his ex ante type θ is under the seller's control. That is, the buyer may not acquire any additional private information about ω on his own.

The seller can commit to a menu of two contracts, one for each type. Each contract consists of a *pricing scheme* and an *information policy*. A pricing scheme $(a^\theta, p^\theta) \in \mathbf{R}_+ \times \mathbf{R}$ consists of an advance payment a^θ and a strike price p^θ.[8] A type-θ buyer transfers the advance payment a^θ to the seller before he is allowed to receive additional private information about ω, and has the option to buy the product at the strike price p^θ after he receives the additional information. Instead of an advancement payment and a strike price, we can alternatively model a pricing scheme as a deposit and a refund. The only restriction we have imposed is that pricing schemes are deterministic.[9]

[8]We use superscripts for reported types and subscripts for true types.

[9]See Section 5 for a brief discussion on this restriction.

An information policy for type θ is an experiment on Ω. Each experiment is a mapping from Ω to a set of outcomes. Since the pricing scheme for type θ is deterministic, it is without loss to model the experiment for θ as $\sigma^{\theta} : \Omega \to \Delta(\{\text{buy}, \text{don't-buy}\})$ where the outcome space $\{\text{buy}, \text{don't-buy}\}$ is binary.[10] Further, without loss we can restrict to contracts that are *obedient* in the sense that for a buyer who reports his type truthfully, he purchases the product when learning that the outcome is buy and does not purchase after learning that the outcome is don't-buy. Given that the outcome space is binary, with a slight abuse of notation, we denote the experiment for type θ as $\sigma^{\theta} : \Omega \to [0, 1]$, with $\sigma^{\theta}(\omega)$ representing the probability of the buy outcome for a truthful buyer type θ conditional on signal realization ω. A seller's *disclosure policy* is a pair of information policies $\left(\sigma^{L}, \sigma^{H}\right)$, and a disclosure policy is *discriminatory* if information policies differ cross buyer types: $\sigma^{L} \neq \sigma^{H}$.

An experiment may be thought of as a product trial or a pilot program for type θ. The seller designs the trial length and chooses which aspects of the product are available for trial to control how much type θ privately learns about ω. There are only two possible trial outcomes, "buy" and "don't-buy." Type θ does not learn the realization of the signal z, only whether or not the seller recommends a truthful type θ to purchase the product. The advance payment a^{θ} can be interpreted as the price for both the trial *and* the option to purchase the product at the strike price p^{θ}.

The timing of the game is as follows. The seller first commits to a menu of contracts $\left(a^{\theta}, p^{\theta}, \sigma^{\theta}\right)$, one for each type. The buyer then chooses one contract $\left(a^{\theta}, p^{\theta}, \sigma^{\theta}\right)$ from the menu and pays the advance payment a^{θ}. Finally, the buyer learns additional information about ω through experiment σ^{θ}, and decides whether to buy at the given strike price p^{θ}.

We conclude the model description by introducing several terminologies on the information policy. An information policy $\sigma^{\theta} : \Omega \to [0, 1]$ is *partitional* if $\sigma^{\theta}(\omega)$ is either 0 or 1. A partition σ^{θ} has an *interval structure* if there is an interval $[\underline{k}, \overline{k}] \subseteq [\underline{\omega}, \overline{\omega}]$ such that $\sigma^{\theta}(\omega) = \mathbb{1}\{\omega \in [\underline{k}, \overline{k}]\}$, where $\mathbb{1}\{\cdot\}$ is the indicator function, and we refer to $[\underline{k}, \overline{k}]$ as the *buy interval* for type θ. A partition $\sigma^{\theta}(\cdot)$ with an interval structure $[\underline{k}, \overline{k}]$ is *monotone* if $\overline{k} = \overline{\omega}$, and is *non-monotone* if $\overline{k} < \overline{\omega}$.

An information policy σ^{θ} is a *monotone (binary) partition* if there is a threshold k^{θ}

[10] To see this, suppose the experiment for type θ has more than two outcomes. Under a deterministic pricing scheme, for every experiment outcome, a type-θ buyer can choose either to buy or not to buy the product. If we pool all the outcomes after which type θ buys, and pool the outcomes after which he does not buy, neither the payoff of type θ nor the seller's profit is affected. Pooling however makes it less attractive for the other type $\tilde{\theta}$ to mimic type θ since type θ's experiment becomes less informative. Thus, we can restrict to experiments with the outcome space $\{\text{buy}, \text{don't-buy}\}$.

such that $\sigma^\theta(\omega) = \mathbb{1}\{\omega \geq k^\theta\}$. It is clear that, for any menu of option contracts with full disclosure, if we replace full disclosure by monotone partitions with $k^\theta = p^\theta$ while keeping the option contracts unchanged, then equilibrium outcome and hence profit remain the same. Li and Shi (2017) have shown that one can strictly increase profit by further raising p^θ and adjusting a^θ accordingly to bind the relevant participation and incentive constraints. We will show in this paper that non-monotone partitions may do even better than monotone binary partitions.

3 Optimal Disclosure

The seller's problem is to choose a pricing scheme (a^θ, p^θ) and an information policy $\sigma^\theta : \Omega \to [0, 1]$ for each reported θ, to maximize her profit:

$$\sum_{\theta=H,L} \phi_\theta \left(a^\theta + \left(p^\theta - c\right) \int_{\underline{\omega}}^{\overline{\omega}} \sigma^\theta(\omega) f_\theta(\omega) d\omega \right), \tag{1}$$

subject to: (i) two ex ante participation constraints,

$$-a^\theta + \int_{\underline{\omega}}^{\overline{\omega}} (\omega - p^\theta)\sigma^\theta(\omega) f_\theta(\omega) d\omega \geqslant 0, \ \forall \theta; \tag{IR$_\theta$}$$

(ii) two interim participation constraints, so each type is willing to buy after the "buy" outcome and is willing to pass after the "don't-buy" outcome:

$$\int_{\underline{\omega}}^{\overline{\omega}} (\omega - p^\theta)\sigma^\theta(\omega) f_\theta(\omega) d\omega \geqslant 0 \geqslant \int_{\underline{\omega}}^{\overline{\omega}} (\omega - p^\theta)(1 - \sigma^\theta(\omega)) f_\theta(\omega) d\omega; \ \forall \theta, \tag{PB$_\theta$}$$

and (iii) two incentive compatibility constraints:

$$\begin{aligned} & -a^H + \int_{\underline{\omega}}^{\overline{\omega}} (\omega - p^H)\sigma^H(\omega) f_H(\omega) d\omega \\ & \quad \geqslant -a^L + \max\left\{ \int_{\underline{\omega}}^{\overline{\omega}} (\omega - p^L)\sigma^L(\omega) f_H(\omega) d\omega, \int_{\underline{\omega}}^{\overline{\omega}} (\omega - p^L) f_H(\omega) d\omega \right\}, \end{aligned} \tag{IC$_H$}$$

$$\begin{aligned} & -a^L + \int_{\underline{\omega}}^{\overline{\omega}} (\omega - p^L)\sigma^L(\omega) f_L(\omega) d\omega \\ & \quad \geqslant -a^H + \max\left\{ \int_{\underline{\omega}}^{\overline{\omega}} (\omega - p^H)\sigma^H(\omega) f_L(\omega) d\omega, 0 \right\}. \end{aligned} \tag{IC$_L$}$$

In the statement of IC$_H$ constraint, we use the fact that, if the high type reports low, the most profitable deviation is either to buy after the "buy" outcome, or to buy all the time. For IC_L constraint, we use the fact that, if the low type reports high, the most profitable deviation is either to buy after the "buy" outcome, or not to buy at all. Here, it is easy to see that assuming $a^\theta \geqslant 0$ is without loss, since the buyer is offered an option to buy at the price p^θ. The value of this option is weakly positive.

To ease exposition, we introduce two more notations. For all $\theta, \tilde{\theta} = H, L$, denote the posterior estimate of a type θ buyer who reports $\tilde{\theta}$ and then observes the "buy" outcome as

$$v_\theta^{\tilde{\theta}} = \frac{\int_{\underline{\omega}}^{\overline{\omega}} \omega \sigma^{\tilde{\theta}}(\omega) f_\theta(\omega) d\omega}{\int_{\underline{\omega}}^{\overline{\omega}} \sigma^{\tilde{\theta}}(\omega) f_\theta(\omega) d\omega}.$$

Similarly, denote the posterior estimate of a type θ buyer who reports $\tilde{\theta}$ and then observes the "don't-buy" outcome as

$$u_\theta^{\tilde{\theta}} = \frac{\int_{\underline{\omega}}^{\overline{\omega}} \omega \left(1 - \sigma^{\tilde{\theta}}(\omega)\right) f_\theta(\omega) d\omega}{\int_{\underline{\omega}}^{\overline{\omega}} \left(1 - \sigma^{\tilde{\theta}}(\omega)\right) f_\theta(\omega) d\omega}.$$

Under likelihood ratio dominance, we have $v_H^\theta \geq v_L^\theta$ and $u_H^\theta \geq u_L^\theta$, for each $\theta = H, L$.[11] We can use these notations to rewrite the PB_θ constraints as bounds on the strike price:

$$v_\theta^\theta \geqslant p^\theta \geqslant u_\theta^\theta. \tag{2}$$

3.1 Relaxed problem

In a dynamic mechanism design problem with exogenous information (e.g., Courty and Li, 2000), the true value ω is revealed in period two, and the buyer reporting type θ buys if and only if ω exceeds price p^θ, both on and off the truthful reporting path. As a result, under the weaker order of first-order stochastic dominance, IR_H follows from IR_L and IC_H, and this is used to show that IR_L and IC_H bind while IC_L is satisfied. In contrast, in the present optimal disclosure problem, the buyer's value estimate in period two depends on his true type and the assigned information policy through his reported type. For IR_H to follow from IR_L and IC_H, we need

$$\max\left\{\int_{\underline{\omega}}^{\overline{\omega}} (\omega - p^L)\sigma^L(\omega) f_H(\omega) d\omega, \int_{\underline{\omega}}^{\overline{\omega}} (\omega - p^L) f_H(\omega) d\omega\right\} \geq \int_{\underline{\omega}}^{\overline{\omega}} (\omega - p^L)\sigma^L(\omega) f_L(\omega) d\omega.$$

[11] This is because for each $\theta = H, L$, the density function $\sigma^\theta(\omega) f_H(\omega) / \int_{\underline{\omega}}^{\overline{\omega}} \sigma^\theta(w) f_H(w) dw$ dominates in likelihood ratio order the density function $\sigma^\theta(\omega) f_L(\omega) / \int_{\underline{\omega}}^{\overline{\omega}} \sigma^\theta(w) f_L(w) dw$, implying $v_H^\theta \geq v_L^\theta$; a similar argument shows that $u_H^\theta \geq u_L^\theta$.

If $u_H^L \leq p^L$ so that in deviation type H buys only after receiving the "buy" outcome, the above becomes

$$\int_{\underline{\omega}}^{\overline{\omega}} (\omega - p^L)\sigma^L(\omega)(f_H(\omega) - f_L(\omega))d\omega \geq 0, \tag{3}$$

which does not necessarily hold even under the stronger assumption of likelihood ratio dominance. However, if

$$\int_{\underline{\omega}}^{\overline{\omega}} \sigma^L(\omega)(f_H(\omega) - f_L(\omega))d\omega \geq 0, \tag{4}$$

that is, if the information policy σ^L for type L is such that a true type L buyer buys the good with a smaller probability than a deviating type H buyer, then (3) holds for $p^L \leq v_L^L$. This is because (3) is equivalent to

$$(v_H^L - p^L) \int_{\underline{\omega}}^{\overline{\omega}} \sigma^L(\omega) f_H(\omega) d\omega \geq (v_L^L - p^L) \int_{\underline{\omega}}^{\overline{\omega}} \sigma^L(\omega) f_L(\omega) d\omega,$$

which follows from (4) and $v_H^L \geq v_L^L$. In particular, if σ^L is given by a monotone partition and is therefore weakly increasing, then (4) holds, and thus IR_H is implied by IR_L, IC_H and PB_L.

Following the standard approach to dynamic mechanism design problem with exogenous information, we consider a "relaxed problem" by dropping IC_L. Since information policy σ^L is endogenously chosen and is not necessarily increasing, we have to retain IR_H. As in the standard relaxed problem, we first establish that any solution to the relaxed problem has both IR_L and IC_H binding. The argument for why IC_H is binding is slightly complicated by the fact that we have retained IR_H in the relaxed problem.

Lemma 1 *At any solution to the relaxed problem, both IR_L and IC_H bind.*

Proof. See appendix. ∎

The next hurdle in analyzing our relaxed problem is that we need to deal with the possibility of "double deviation" by type H: as already mentioned, a type H buyer who deviates and reports L may buy at both signals. This is tackled in the result below. We show that in characterizing the solution to the relaxed problem, we can restrict to no double deviation by type H.

Lemma 2 *At any solution to the relaxed problem, $u_H^L \leq p^L$.*

Proof. See appendix. ∎

The idea behind Lemma 2 is simple. If double deviation by type H occurs at the solution to the relaxed problem, so that type H buys the good even after the "don't-buy" outcome after the first deviation of misreporting as type L, the information policy for type L must be a monotone partition. But then double deviation by type H means that type L strictly prefers to buy after the "buy" outcome. As a result, the seller could raise the profit by increasing p^L without violating IR_H. A corollary of Lemma 2 is that $u_H^L \leq v_L^L$.

Combining Lemma 1 and Lemma 2, we can rewrite the objective (1) in the relaxed problem as

$$\begin{aligned}\phi_H \int_{\underline{\omega}}^{\overline{\omega}} (\omega - c)\, \sigma^H(\omega) f_H(\omega) d\omega \\ + \int_{\underline{\omega}}^{\overline{\omega}} \left(\phi_L (\omega - c) f_L(\omega) - \phi_H \left(\omega - p^L\right) (f_H(\omega) - f_L(\omega))\right) \sigma^L(\omega) d\omega. \end{aligned} \tag{5}$$

By Lemma 2, IR_H becomes (3). In choosing the two information policies σ^H and σ^L and two strike prices p^H and p^L, the seller also faces the two PB_H and PB_L constraints, and the constraint of no double deviation by type H

$$u_H^L \leq p^L. \tag{ND_H}$$

Since $u_H^L \geq u_L^L$ and given constraint ND_H, the only part of PB_L constraints that still remains to be considered is $v_L^L \geq p^L$.

Since we have dropped IC_L in the relaxed problem, from the first integral in the the objective function (5), we have that the solution in σ^H is "efficient," given by $\sigma^H(\omega) = \mathbb{1}\{\omega \geq c\}$. The choice of the strike price p^H for type H is indeterminate as it does not appear in (5). However, it must satisfy PB_H and, together with the advance payment a^H, keep the truth-telling payoff of type H at the same level given by IC_H:

$$-a^H + \int_c^{\overline{\omega}} (\omega - p^H) f_H(\omega) d\omega = \int_{\underline{\omega}}^{\overline{\omega}} (\omega - p^L) \sigma^L(\omega)(f_H(\omega) - f_L(\omega)) d\omega. \tag{6}$$

Our next result establishes that there is a solution to the relaxed problem that satisfies the dropped constraint of IC_L, and is thus a solution to the original problem.[12]

[12]Since p^H and a^H are indeterminate given that $\sigma^H(\omega) = \mathbb{1}\{\omega \geq c\}$, not all solutions to the relaxed problem satisfy IC_L. For example, if we set p^H to the conditional expectation of type H's value above c, then the solution to the relaxed problem may have $a^H < 0$, which clearly violates IC_L because IR_L binds by Lemma 1.

Lemma 3 *Any solution to the relaxed problem such that $p^H \leq v_L^H$ satisfies IC_L.*

Proof. See appendix. ■

The intuition behind the argument is simple. If a solution to the relaxed problem has the property that a deviating type L will buy only after receiving the "buy" outcome (e.g., with $p^H = c$), and that IC_L is not satisfied, then the seller's profit would be strictly higher under an efficient and non-discriminatory disclosure policy with $p^H = p^L = c$ and $\sigma^H(\omega) = \sigma^L(\omega) = \mathbb{1}\{\omega \geq c\}$. This of course contradicts the assumption that we have found a solution to the relaxed problem. It follows from this lemma that any solution to the relaxed problem with $p^H = c$ also solves the original problem. Therefore, from now on, we will set $p^H = c$.

We can now focus on the following "residual" relaxed problem, which is choosing the information policy σ^L and the strike price p^L for type L to maximize the second integral in (5), or

$$\int_{\underline{\omega}}^{\overline{\omega}} \left(\phi_L \left(\omega - c\right) f_L(\omega) - \phi_H \left(\omega - p^L\right) \left(f_H(\omega) - f_L(\omega)\right)\right) \sigma^L(\omega) d\omega, \tag{7}$$

subject to the IR_H constraint (equation (3)) and the combined PB_L and ND_H constraints of

$$u_H^L \leq p^L \leq v_L^L. \tag{8}$$

3.2 Optimal mechanisms

Li and Shi (2017) use monotone partitions to show that full disclosure is suboptimal in general. Although monotone partitions can be effective in both creating trade surplus and extracting information rent, the following example shows that a monotone partition may not be optimal.

Example 1 *Suppose that $\phi_L = \phi_H = 1/2$. and the seller's reservation value $c = 1/2$. Type L has a uniform value distribution over $[0, 1]$. The value distribution of type H is also uniform except for an atom of size $1/4$ at the top:*

$$F_H(\omega) = \begin{cases} \frac{3}{4}\omega & \text{if} \quad \omega \in [0, 1) \\ 1 & \text{if} \quad \omega = 1. \end{cases}$$

Consider the following disclosure policy and pricing schemes. For type H, choose information policy $\sigma^H(\omega) = \mathbb{1}\{\omega \geq c\}$, set strike price $p^H = c$, and set advance *payment $a^H = 7/32$. For type L, choose $\sigma^L(\omega) = \mathbb{1}\{\omega \in (1/2, 1)\}$, set strike price $p^L = 3/4$, and charge advance payment $a^L = 0$. Under this menu of contracts, type L will not mimic type H, and he buys only upon observing the "buy" outcome and receives zero expected payoff. A type H buyer will not mimic type L because, after deviation, he buys only at the "buy" outcome and gets zero expected payoff since his posterior estimate when observing the "buy" outcome is 3/4. The disclosure policy and pricing schemes together extract the full surplus.*

In Example 1, the atom in the value distribution of type H means that the likelihood ratio $f_H(\omega)/f_L(\omega)$ explodes at the top. It is straightforward to show that, if the seller is restricted to monotone partitions for type L, the optimal partition threshold is equal to $5/8$, leaving an information rent of $3/128$ to type H. If the seller is allowed to exclude the top realization of $\omega = 1$ from the low type's buy interval (and hence the low type's information policy is no longer a monotone partition), she can cut the information rent of type H to zero without incurring any loss in the trading surplus with type L, because $\omega = 1$ occurs with probability $1/4$ for a misreporting type H while $\omega = 1$ occurs with probability zero for type L. Indeed, the seller can extract the full surplus by setting the low type's buy interval as $(1/2, 1)$.

Monotone partitions can be optimal with suitable upper bounds on the likelihood ratio, as we show now. To simplify notation, we write the (point) likelihood ratio at $\omega \in [\underline{\omega}, \overline{\omega}]$ as

$$\lambda(\omega) = \frac{f_H(\omega)}{f_L(\omega)}$$

and the average likelihood ratio over an interval $[k_1, k_2] \subseteq [\underline{\omega}, \overline{\omega}]$ as

$$\Lambda(k_1, k_2) = \frac{F_H(k_2) - F_H(k_1)}{F_L(k_2) - F_L(k_1)}.$$

Proposition 1 *Suppose that $\lambda(\overline{\omega}) \leq \phi_L/\phi_H$ and $\max_\omega \lambda'(\omega) \leq 1/(\overline{\omega} - \underline{\omega})$. The optimal disclosure policy is a pair of monotone partitions.*

Proof. We show that under the conditions stated in the proposition, the solution in $\sigma^L(\cdot)$ to the relaxed problem is a two-step function, with $\sigma^L(\omega) = \mathbb{1}\{\omega \geq \underline{k}\}$ for some $\underline{k}$. The objective is (7). We relax the problem further by dropping (3) and the constraint $u_H^L \leq p^L$. The remaining constraint $p^L \leq v_L^L$ can be written as

$$\int_{\underline{\omega}}^{\overline{\omega}} \left(\omega - p^L\right) \sigma^L(\omega) f_L(\omega) d\omega \geq 0.$$

Let $\beta \geq 0$ be the Lagrange multiplier associated with the above constraint and write the Lagrangian as

$$\mathcal{L} = \int_{\underline{\omega}}^{\overline{\omega}} \left(\phi_L (\omega - c) - \phi_H \left(\omega - p^L \right) \left(\frac{f_H(\omega)}{f_L(\omega)} - 1 \right) + \beta \left(\omega - p^L \right) \right) \sigma^L(\omega) f_L(\omega) d\omega.$$

Hence, the solution is $\sigma^L(\omega) = \mathbb{1}\{\Upsilon(\omega) \geq 0\}$, where

$$\Upsilon(\omega) = \phi_L(\omega - c) + (\omega - p^L) \left(\phi_H \left(1 - \lambda(\omega) \right) + \beta \right). \tag{9}$$

From (9), for any fixed p^L, using the two assumptions in the proposition and $\beta \geq 0$, we have

$$\begin{aligned} \Upsilon'(\omega) &= \phi_L + \phi_H \left(1 - \lambda(\omega) \right) + \beta - \phi_H (\omega - p^L) \lambda'(\omega) \\ &\geq \phi_L + \phi_H \left(1 - \phi_L / \phi_H \right) + \beta - \phi_H \left| \omega - p^L \right| / (\overline{\omega} - \underline{\omega}) \\ &= \beta + \phi_H \left(1 - \left| \omega - p^L \right| / (\overline{\omega} - \underline{\omega}) \right) \\ &\geq 0 \end{aligned}$$

for all $\omega \in [\underline{\omega}, \overline{\omega}]$. It follows that there exists some $\underline{k}$ such that $\sigma^L(\omega) = \mathbb{1}\{\omega \geq \underline{k}\}$.

Given that $\sigma(\cdot)$ is a monotone partition with a threshold $\underline{k}$, the objective (7) is increasing in p^L for any $\underline{k}$. Thus, we have $p^L = v_L^L$. The dropped constraint of $u_H^L \leq p^L$ is also satisfied, as $u_H^L < v_L^L$. Finally, the solution to the relaxed problem satisfies (3) because $\sigma^L(\cdot)$ is weakly increasing. The proposition then follows from Lemma 3. ■

Although the sufficient conditions stated in Proposition 1 are restrictive, we provide an analytical example below to show how they can be satisfied. This example features two linear density functions f_H and f_L over the unit interval.

Example 2 *Let $f_L(\omega) = 1 + (2\omega - 1)t_L$ and $f_H(\omega) = 1 + (2\omega - 1)t_H$ for $\omega \in [0, 1]$, with $-1 \leq t_L < t_H \leq 1$. We have*

$$\lambda(\overline{\omega}) = \frac{1 + t_H}{1 + t_L}, \quad \max_{\omega \in [0,1]} \lambda'(\omega) = \frac{2(t_H - t_L)}{(1 - |t_L|)^2}.$$

Therefore, the sufficient conditions in Proposition 1 can be written as

$$t_H \leq t_L + \min \left\{ \frac{\phi_L - \phi_H}{\phi_H} \left(1 + t_L \right), \frac{1}{2}(1 - |t_L|)^2 \right\}.$$

As long as $\phi_L > \phi_H$, the right hand side of the above inequality is always strictly *larger than t_L. Therefore, for any $t_L > -1$, there always exist values of t_H that satisfy this condition. However, for $t_L = -1$ the condition can never hold because $\lambda(\omega)$ is unbounded at $\omega = \overline{\omega}$.*

In Example 1, a monotone partition is not optimal for type L. Instead, type L is recommended to buy if ω lies in the interval $(1/2, 1)$ and not to buy otherwise, and moreover, the interval $(1/2, 1)$ is nested by the interval $[1/2, 1]$ that type H is recommended to buy. To answer the question of when a pair of such "nested intervals" is optimal, we will focus on the "regular" case where condition (4) holds.

Definition 1 *A solution to the residual relaxed problem is regular if condition (4) holds and is irregular otherwise.*

Whether a solution is regular or not depends only on the information policy σ^L for type L in the solution. When the solution is regular, IR_H is slack. This case is closer to the classical sequential screening problem with exogenous information. If (4) is strict, then since the residual objective function (7) increases with p^L, the solution must have $p^L = v_L^L$; if (4) holds with an equality, setting $p^L = v_L^L$ gives another solution to the residual relaxed problem. Moreover, $p^L \geq c$ in any regular solution, because if $p^L = v_L^L < c$, then the value of the objective (7) is necessarily negative, as the trade surplus from type L is negative while the rent to type H is non-negative.

We show that in any regular solution, the information policy σ^L is partitional, the buy region is an interval $[\underline{k}, \overline{k}] \subset [\underline{\omega}, \overline{\omega}]$, and the interval $[\underline{k}, \overline{k}]$ is a subset of the buy interval $[c, \overline{\omega}]$ of type H. The optimal partition for the low type may be either monotone ($\overline{k} = \overline{\omega}$) or non-monotone ($\overline{k} < \overline{\omega}$). In other words, a pair of monotone parititions is a special case of a pair of nested intervals with $\overline{k} = \overline{\omega}$. This "nested-interval" characterization of optimal disclosure in regular solutions is presented in the following lemma.

Lemma 4 *At any regular solution, $p^L = v_L^L \geq c$. Furthermore, there exist $\underline{k}$ and $\overline{k}$ satisfying $c < \underline{k} < \overline{k} \leq \overline{\omega}$ such that $\sigma^L(\omega) = \mathbb{1}\{\omega \in [\underline{k}, \overline{k}]\}$.*

Proof. See appendix. ■

The proof of Lemma 4 in the appendix is based on a perturbation argument. If the buy region of the low type in a regular solution is not an interval, we can perturb σ^L marginally, keeping v_L^L and hence p^L unchanged. The key is to show that there is always a perturbation that increases the trade surplus from the low type, which is the first term in (7), while simultaneously decreases the information rent to the high type, which is the second term in (7). The intuitive reason for the existence such perturbations when the buy region of the low type is not an interval, can be obtained from the same Lagrangian function in the proof of Proposition 1. If we drop ND_H from the residual relaxed problem and impose the single remaining constraint $p^L \leq v_L^L$, with $\beta \geq 0$ being the associated Lagrangian multiplier, then as in the proof of Proposition 1, the solution is $\sigma^L(\omega) = \mathbb{1}\{\Upsilon(\omega) \geq 0\}$, where $\Upsilon(\omega)$ is given in (9). Given that $p^L \geq c$, we have

$$\Upsilon(p^L) = \phi_L(p^L - c) \geq 0.$$

Further, $\Upsilon(\omega)$ can cross 0 only once for all $\omega > p^L$. To see the latter claim, note that for $\omega > p^L$, $\Upsilon(\omega)$ has the same sign as

$$\frac{\Upsilon(\omega)}{\omega - p^L} = \phi_L \frac{\omega - c}{\omega - p^L} + \phi_H \left(1 - \lambda(\omega)\right) + \beta.$$

The second term on the right-hand side of the above expression is decreasing in ω by likelihood ratio dominance, while the first term is non-decreasing because $p^L \geq c$. Therefore, $\Upsilon(\omega)$ can cross 0 only once and only from above for all $\omega > p^L$. Similarly, $\Upsilon(\omega)$ can cross 0 only once and only from below for all $\omega < p^L$. It follows that there exists an interval of values $[\underline{k}, \overline{k}] \subset [\underline{\omega}, \overline{\omega}]$ such that $\Upsilon(\omega) \geq 0$ if and only if $\omega \in [\underline{k}, \overline{k}]$. In other words, $\sigma^L(\omega) = \mathbb{1}\{\omega \in [\underline{k}, \overline{k}]\}$.[13]

We are ready to present the main results in this section that establish the optimality of nested intervals. We do so by providing sufficient conditions for the solution to be regular. Since we cannot exploit the characterization of regular solutions given by Lemma 4, in the first set of sufficient conditions we directly tackle the inequality (4), using only the fact that the seller does not exclude the low type completely in an optimal mechanism.

Proposition 2 *If there exists $\gamma > 0$ such that $\lambda(\omega) \geq 1 + \gamma(\omega - c)$ for all $\omega \in [\underline{\omega}, \overline{\omega}]$, then the optimal disclosure policy is a pair of nested intervals.*

Proof. By assumptions in the proposition, we have

$$\int_{\underline{\omega}}^{\overline{\omega}} \sigma^L(\omega)(f_H(\omega) - f_L(\omega))d\omega \geq \gamma \int_{\underline{\omega}}^{\overline{\omega}} \sigma^L(\omega)(\omega - c) f_L(\omega)\, d\omega \geq 0.$$

[13]If we assume that $\mu_H \leq c$, then it is never profitable for a deviating type H buyer to always buy. The dropped ND_H constraint is slack at any regular solution, and thus the above argument provides an alternative proof for Lemma 4. The perturbation argument in the appendix is more general, and covers both when ND_H is slack and when it is binding.

The last inequality follows because in any optimal solution the trade surplus with type L – the first term in the residual objective function (7) – must be non-negative. Otherwise, the seller can exclude type L altogether and be better off. Thus, condition (4) for regular solutions holds. The proposition follows immediately from Lemma 4. ■

The sufficient condition in Proposition 2 is satisfied if we can identify some value $k \leq c$ such that

$$\lambda(\omega) \geq 1 + \gamma(\omega - k)$$

for some $\gamma > 0$. One candidate for such value is where the density functions f_H and f_L intersect: by likelihood ratio dominance there exists a unique $\omega_o \in (\underline{\omega}, \overline{\omega})$ such that $f_H(\omega_o) = f_L(\omega_o)$, or $\lambda(\omega_o) = 1$. Note that the condition in Proposition 2 requires $\omega_o \leq c$. It is helpful to compare two increasing functions $\lambda(\omega) - 1$ and $\omega - \omega_o$ for $\omega \in [\underline{\omega}, \overline{\omega}]$. Both functions pass 0 at $\omega = \omega_o$. For there to exist $\gamma > 0$ such that $\lambda(\omega) - 1 \geq \gamma(\omega - \omega_o)$, we must be able to "rotate" the function $\omega - \omega_o$ around ω_o such that it stays below $\lambda(\omega) - 1$ for $\omega \in [\underline{\omega}, \overline{\omega}]$. If $\lambda(\omega)$ is continuously differentiable at $\omega = \omega_o$, a necessary condition for this to happen is that $\lambda(\omega)$ is convex at $\omega = \omega_o$. Indeed, if $\lambda(\omega)$ is convex for all $\omega \in [\underline{\omega}, \overline{\omega}]$, the sufficient condition is satisfied by setting $\gamma = \lambda'(\omega_0)$. Example 2 can be used to illustrate that a convex likelihood ratio function λ satisfies the sufficient condition in Proposition 2; it also illustrates that convex likelihood functions are sufficient but not necessary for the condition in Proposition 2.

Example 2 continued *We have $\omega_o = 1/2$, and*

$$\lambda(\omega) = \frac{1 - t_H + 2t_H\omega}{1 - t_L + 2t_L\omega}.$$

It can be easily verified that

$$\lambda''(\omega) = -\frac{8t_L(t_H - t_L)}{(1 - t_L + 2t_L\omega)^3}.$$

Therefore, $\lambda(\omega)$ is convex if $t_L \leq 0$ and concave otherwise. If $\lambda(\omega)$ is convex, the sufficient condition in Proposition 2 is satisfied if and only if $\lambda(c) \geq 1$, which is equivalent to $c \geq \omega_o = 1/2$. If $\lambda(\omega)$ is concave, the sufficient condition is satisfied if and only if there exists $\gamma > 0$ such that $\lambda(\underline{\omega}) \geq 1 + \gamma(\underline{\omega} - c)$ and $\lambda(\overline{\omega}) \geq 1 + \gamma(\overline{\omega} - c)$, which is equivalent to $t_L \leq 2c - 1$. Thus, under the assumption of $c \geq 1/2$, the sufficient condition in Proposition 2 always holds when $t_L \leq 0$, i.e., when $\lambda(\omega)$ is convex, but it can also hold when $t_L > 0$ so long as it is small relative to c.

Now we provide a second set of sufficient conditions for solutions to the residual relaxed problem to be regular. This is achieved by first providing a characterization of candidate irregular solutions. Since condition (4) fails for an irregular solution σ^L, the objective function (7) of the residual relaxed problem is strictly decreasing in p^L and thus the solution has $p^L = u_H^L$. That is, the no double deviation constraint for the high type is binding at any irregular solution. For value distribution F_H of the high type that satisfies $\mu_H \leq c$, we have $p^L < c$. This means that the strike price for the low type at any irregular solution is lower than the seller's cost. The following lemma establishes $\sigma^L(\omega) = 0$ for all $\omega < u_H^L$. The proof is in the appendix; it follows a similar argument as in Lemma 2 by showing that if the seller allocates the good to the low type with a positive probability for values lower than the strike price, there is a profitable perturbation to σ^L.

Lemma 5 *Suppose that $\mu_H \leq c$. If σ^L is an irregular solution to the residual relaxed problem, then $\sigma^L(\omega) = 0$ for all $\omega < u_H^L$.*

Proof. See appendix. ■

Proposition 3 below shows that, given Lemma 5, if in addition $\omega_o \leq \mu_H$, then there is no irregular solution to the residual relaxed problem. The idea is the following. An irregular solution requires the high type to buy with a smaller probability after misreporting as the low type than the truthful low type. Since $f_H(\omega) > f_L(\omega)$ if and only if $\omega > \omega_o$, an irregular solution σ^L has to put relatively large weights on values $\omega < \omega_o$ and correspondingly relatively small weights on values $\omega > \omega_o$. At the same time, among values $\omega < \omega_o$, Lemma 5 requires σ^L to assign zero weights on those smaller than u_H^L. This becomes impossible when ω_o is relatively small.

Proposition 3 *Suppose that $\omega_o \leq \mu_H \leq c$. Any solution to the residual relaxed problem is regular.*

Proof. See appendix. ■

The sufficient condition in Proposition 3 for solutions to be regular is complementary to the one in Proposition 2. Note that both sets of conditions require $c \geq \omega_o$. Example 2 illustrates this complementarity.

Example 2 continued *Since $\omega_o = 1/2$, and*

$$\mu_H = \frac{1}{6}t_H + \frac{1}{2},$$

the sufficient condition of Proposition 3 is satisfied if

$$0 \leq t_H \leq 6c - 3.$$

Recall that the sufficient condition for ruling out irregular solutions from Proposition 2 is $t_L \leq 2c - 1$. *By assumption* $t_L < t_H$, *and* $2c - 1 \leq 6c - 3$ *when* $c \geq 1/2$, *with equality if* $c = 1/2$. *Thus, when* $t_L > 2c - 1$, *the condition in Proposition 2 fails, but there are values of* t_H *that satisfy the sufficient condition in Proposition 3 so long as* $c > 1/2$. *Conversely, when* $t_H < 0$, *the condition in Proposition 3 fails, but there are values of* t_L *that satisfy the sufficient condition in Proposition 2 so long as* $c \geq 1/2$.

In addition to Propositions 2 and Proposition 3, Proposition 1 can also be viewed as providing sufficient conditions for solutions to the residual relaxed problem to be regular. The partitional information disclosure policy implied by Proposition 1 is monotone. A natural question is then when the optimal information policy for type L is a non-monotone partition with $\overline{k} < \overline{\omega}$. To answer this question, we can apply Lemma 4 to rewrite the residual objective function (7) as

$$\Gamma(\underline{k}, \overline{k}) \equiv \phi_L \int_{\underline{k}}^{\overline{k}} (\omega - c) f_L(\omega) d\omega - (1 - \phi_L) \int_{\underline{k}}^{\overline{k}} (\omega - v_L^L)(f_H(\omega) - f_L(\omega)) d\omega. \tag{10}$$

Then we have

$$\frac{\partial \Gamma(\underline{k}, \overline{k})}{\partial \underline{k}} = \left[-\phi_L(\underline{k} - c) + (1 - \phi_L)\left(v_L^L - \underline{k}\right)\left(\Lambda(\underline{k}, \overline{k}) - \lambda(\underline{k})\right)\right] f_L(\underline{k});$$
$$\frac{\partial \Gamma(\underline{k}, \overline{k})}{\partial \overline{k}} = \left[\phi_L(\overline{k} - c) - (1 - \phi_L)\left(\overline{k} - v_L^L\right)\left(\lambda(\overline{k}) - \Lambda(\underline{k}, \overline{k})\right)\right] f_L(\overline{k}).$$

The first-order conditions for optimal $\underline{k}$ and $\overline{k}$ are

$$\frac{\partial \Gamma(\underline{k}, \overline{k})}{\partial \underline{k}} = 0, \ \frac{\partial \Gamma(\underline{k}, \overline{k})}{\partial \overline{k}} \geq 0 \text{ and } \overline{k} \leq \overline{\omega} \text{ with complementary slackness.} \tag{11}$$

Although it is intuitive that, if the likelihood ratio $\lambda(\omega)$ increases sharply in the neighborhood of $\overline{\omega}$, the optimal information policy assigned to type L will be a non-monotone partition, it is difficult to find general sufficient conditions for the optimality of non-monotone partitions. We need an upperbound for $\underline{k}$ to show $\frac{\partial \Gamma(\underline{k}, \overline{k})}{\partial \overline{k}}|_{\overline{k}=\overline{\omega}} < 0$ but a good upperbound for $\underline{k}$ is not available. However, as one can see from the expression of $\frac{\partial \Gamma(\underline{k}, \overline{k})}{\partial \overline{k}}$, the condition $\frac{\partial \Gamma(\underline{k}, \overline{k})}{\partial \overline{k}}|_{\overline{k}=\overline{\omega}} < 0$ is more likely to hold if ϕ_L is sufficiently small. The following corollary provides sufficient conditions for a non-montone partition to be optimal for type L.

Corollary 1 *At any regular solution, if $\lambda''(\overline{\omega})/\lambda'(\overline{\omega}) > 3/(\overline{\omega}-c)+2f_L'(\overline{\omega})/f_L(\overline{\omega})$, then for sufficiently small ϕ_L, the optimal σ^L has $\overline{k} < \overline{\omega}$.*

Proof. See appendix. ■

We use Example 2 again to illustrate that the conditions in Corollary 1 are sufficient but not necessary. Indeed, for this example, when the likelihood ratio function $\lambda(\omega)$ is unbounded at $\overline{\omega}$, which ensures it is convex and therefore the sufficient condition of Proposition 2 is satisfied, the first order conditions (11) imply that $\overline{k} < \overline{\omega}$ regardless of the value of ϕ_L. Conversely, whenever the likelihood ratio function is concave, in any regular solution to the residual relaxed problem we have $\overline{k} = \overline{\omega}$.

Example 2 continued *Suppose that $c \geq 1/2$. We have $\overline{\omega} = 1$, and*

$$\frac{\lambda''(1)}{\lambda'(1)} = -\frac{4t_L}{1+t_L}; \ \frac{f_L'(1)}{f_L(1)} = \frac{2t_L}{1+t_L}.$$

Recall in this example the sufficient condition for regular solutions in Proposition 2 is satisfied if $t_L \leq 0$. By Corollary 1, for $t_L < -3/(11-8c)$, the optimal σ^L has $\overline{k} < \overline{\omega}$ when ϕ_L is sufficiently small.

If $t_L = -1$, the likelihood ratio function $\lambda(\omega)$ is unbounded at $\overline{\omega}$, and we have

$$\frac{\partial\Gamma(\underline{k},\overline{k})}{\partial\overline{k}} = 2\phi_L(\overline{k}-c)(1-\overline{k}) - \frac{1}{3}(1-\phi_L)(t_H+1)\left(\frac{\overline{k}-\underline{k}}{2-\overline{k}-\underline{k}}\right)^2(3-\overline{k}-2\underline{k}).$$

The above is strictly negative when $\overline{k} = 1$, violating the first order conditions (11). Thus, regardless of the value of ϕ_L, the optimal σ^L has $\overline{k} < 1$ if $t_L = -1$.

Recall that $\lambda(\omega)$ is concave if $\lambda_L \geq 0$. Recall also that the solution to residual relaxed problem is regular so long as $\lambda \leq 2c-1$. The first order conditions (11) become

$$\underline{k} - c - \frac{(1-\phi_L)(t_H-t_L)}{6\phi_L}\left(\frac{\overline{k}-\underline{k}}{1-t_L+t_L\overline{k}+t_L\underline{k}}\right)^2 \frac{3(1-t_L)+4t_L\overline{k}+2t_L\underline{k}}{1-t_L+2t_L\underline{k}} = 0;$$
$$\overline{k} - c - \frac{(1-\phi_L)(t_H-t_L)}{6\phi_L}\left(\frac{\overline{k}-\underline{k}}{1-t_L+t_L\overline{k}+t_L\underline{k}}\right)^2 \frac{3(1-t_L)+2t_L\overline{k}+4t_L\underline{k}}{1-t_L+2t_L\overline{k}} \geq 0,$$

and $\overline{k} \leq 1$, with complementary slackness. When $\lambda_L \geq 0$, the second condition above holds with a strict inequality whenever the first one holds. This implies that $\overline{k} = 1$.

So far in this subsection we have focused on regular solutions to the residual relaxed problem, and have shown that optimal disclosure policies feature a pair of nested intervals. Lemma 5 provides a preliminary characterization of irregular solutions, but a complete characterization of irregular solutions is beyond the scope of this paper. We conclude this subsection by noting that regularity is not necessary for a pair of nested intervals to be optimal. This is illustrated with Example 1 from the beginning of this subsection.

Example 1 continued *The optimal disclosure in the full-surplus extraction mechanism features a pair of nested intervals. Type L buys when $\omega \in (1/2, 1)$, with a probability of 1/2, while after a deviation type H also buys when $\omega \in (1/2, 1)$, with a probability of* 3/8. *Thus, the optimal mechanism as a solution to the residual relaxed problem is not regular.*

3.3 Necessity of information discrimination

We have shown that optimal discriminatory disclosure is a pair of nested intervals. Is information discrimination necessary for profit maximization? As shown in Guo and Shmaya (2019), although the optimal information policies σ^H and σ^L are different, the optimal mechanism may nonetheless be implemented with a non-discriminatory disclosure policy. Throughout this subsection, we will hold it as given that the optimal information policy assigned to type H is a monotone partition with threshold c.

We first show that replication is achieved in the special case where the optimal information policy assigned to type L is also a monotone binary partition with threshold $\underline{k} \in [c, \overline{\omega})$. To see this, consider the non-discriminatory disclosure with common partition refined from monotone partition $\{[\underline{\omega}, c], [c, \overline{\omega}]\}$ assigned to type H and monotone partition $\{[\underline{\omega}, \underline{k}], [\underline{k}, \overline{\omega}]\}$ assigned to type L under optimal discriminatory disclosure:

$$\{[\underline{\omega}, c], [c, \underline{k}], [\underline{k}, \overline{\omega}]\},$$

and set $p^H = c$ and $p^L = \mathbb{E}_L[\omega | \omega \in [\underline{k}, \overline{\omega}]]$. Under this common partition, the on-path and off-path behavior of the two buyer types is the same as under optimal discriminatory disclosure.[14] Therefore, non-discriminatory disclosure with common refined partition can replicate both on- and off-path behavior for both buyer types, and thus attain the same revenue as the optimal discriminatory disclosure.

[14]For on-path behavior, type H will buy if and only if $\omega \in [c, \underline{k}] \cup [\underline{k}, \overline{\omega}]$ and type L will buy if and only if $\omega \in [\underline{k}, \overline{\omega}]$, which is the same as before. For off-path behavior, suppose type H deviates and pretends to be type L. By definition of p^L, $p^L > \underline{k}$ and thus the deviating type H buys if and only if $\omega \in [\underline{k}, \overline{\omega}]$, which is the same as before. Finally, a deviating type L will buy off-path if and only if $\omega \in [c, \underline{k}] \cup [\underline{k}, \overline{\omega}]$, which is again the same as before.

Replication may fail, however, if the optimal information policy assigned to type L is a non-monotone partition, with $\overline{k} < \overline{\omega}$. The reason for the failure is as follows. Consider the following non-discriminatory disclosure with common partition refined from $\{[\underline{\omega}, c], [c, \overline{\omega}]\}$ and $\left\{[\underline{\omega}, \underline{k}] \cup [\overline{k}, \overline{\omega}], [\underline{k}, \overline{k}]\right\}$:

$$\left\{[\underline{\omega}, c], [c, \underline{k}] \cup [\overline{k}, \overline{\omega}], [\underline{k}, \overline{k}]\right\}.$$

Type H follows the "don't-buy" recommendation off path only if

$$\mathbb{E}_H\left[\omega | \omega \in [c, \underline{k}] \cup [\overline{k}, \overline{\omega}]\right] \leq p^L.$$

In contrast, under discriminatory disclosure, type H follows the "don't-buy" recommendation off path only if

$$\mathbb{E}_H\left[\omega | \omega \in [\underline{\omega}, \underline{k}] \cup [\overline{k}, \overline{\omega}]\right] \leq p^L.$$

Since

$$\mathbb{E}_H\left[\omega | \omega \in [c, \underline{k}] \cup [\overline{k}, \overline{\omega}]\right] > \mathbb{E}_H\left[\omega | \omega \in [\underline{\omega}, \underline{k}] \cup [\overline{k}, \overline{\omega}]\right],$$

it is easier under discriminatory disclosure to provide type H incentives to follow recommendation off path. In particular, if

$$\mathbb{E}_H\left[\omega | \omega \in [c, \underline{k}] \cup [\overline{k}, \overline{\omega}]\right] > p^L \geq \mathbb{E}_H\left[\omega | \omega \in [\underline{\omega}, \underline{k}] \cup [\overline{k}, \overline{\omega}]\right], \tag{12}$$

the deviating type H buyer will buy more often off path and have higher deviation payoff under non-discriminatory disclosure. The information rent for type H will be higher under non-discriminatory disclosure, leading to a lower revenue for the seller. Therefore, replication with the common refined partition fails. If we further assume that the optimal information policy for type L is essentially unique in the sense that any other optimal policy leads to the same purchasing behavior of type L who buys if and only if $\omega \in \left[\underline{k}, \overline{k}\right]$ with $\overline{k} < \overline{\omega}$,[15] then replications through any other non-discriminatory disclosure policy must also fail if condition (12) holds, because any non-discriminatory disclosure policy can always be implemented with a discriminatory disclosure policy. Hence, we have

Proposition 4 *Suppose that the optimal information policy for type L is essentially unique where type L buys if and only if* $\omega \in \left[\underline{k}, \overline{k}\right]$ *with* $\overline{k} < \overline{\omega}$ *and that condition (12)*

[15] As we pointed out earlier, there are experiments with three or more outcomes that are equivalent to experiments with binary outcomes. Hence, the optimal information policy for type L is not unique.

is satisfied. Then the optimal mechanism cannot be implemented without information discrimination.

We again use Example 2 from the previous subsection to illustrate Proposition 4.

Example 2 continued *Recall that when $c \geq 1/2$ and $t_L = -1$, any solution to the residual relaxed problem is regular with $\overline{k} < \overline{\omega} = 1$. The two first order conditions (11) both hold as equations:*

$$\underline{k} - c = \frac{(1-\phi_L)(t_H+1)}{6\phi_L}\left(\frac{\overline{k}-\underline{k}}{2-\overline{k}-\underline{k}}\right)^2 \frac{3-2\overline{k}-\underline{k}}{1-\underline{k}};$$
$$\overline{k} - c = \frac{(1-\phi_L)(t_H+1)}{6\phi_L}\left(\frac{\overline{k}-\underline{k}}{2-\overline{k}-\underline{k}}\right)^2 \frac{3-\overline{k}-2\underline{k}}{1-\overline{k}}.$$

To obtain an analytical solution, define $q = (1-\overline{k})/(1-\underline{k})$, and combine the above two first order conditions into an equation in q:

$$\frac{6(1-c)\phi_L}{(1-\phi_L)(t_H+1)} = \frac{(1-q)^2}{1+q}\left(\frac{2}{q} + \frac{1+2q}{1+q}\right).$$

It can be verified that there is a unique value of $q \in (0,1)$ that solves the above equation. For $t_H = 0$, $c = 1/2$ and $\phi_L = 8/35$, we have $q = 1/2$. From the first order conditions, we obtain $\underline{k} = 5/8$ and $\overline{k} = 13/16$. At the solution, $p^L = 17/24$, and $\mathbb{E}_H\left[\omega|\omega \in [c,\underline{k}] \cup [\overline{k},\overline{\omega}]\right] = 123/160 > 17/24$. Condition (12) is satisfied and replication fails.

Proposition 4 does not require the solution to be regular. As we have mentioned at the end of the previous subsection, a solution to the residual relaxed problem does not need to be regular for it to feature a nested-interval structure. This is illustrated with Example 1 from the previous subsection.

Example 1 continued *We have $\mathbb{E}_H\left[\omega|\omega \in [c,\underline{k}] \cup [\overline{k},\overline{\omega}]\right] = 1$, $p^L = 3/4$, and $\mathbb{E}_H\left[\omega|\omega \in [\underline{\omega},\underline{k}] \cup [\overline{k},\overline{\omega}]\right] = 11/20$. Therefore, condition (12) holds and replication fails.*

4 Discussion

We have shown that the optimal discriminatory disclosure has an nested interval structure, and information discrimination is in general necessary to obtain the maximal profit. These results are obtained under the assumption that the information controlled by the seller is correlated with the buyer's type and that the seller can price discriminate against different buyer types. In this section we solve two variations of the model and comparing optimal disclosure policy here to those in Eső and Szentes (2007) and Guo and Shmaya (2019).

4.1 Independent Information

Suppose that the additional private information controlled by the seller is independent of the buyer's ex ante type. Formally, we follow Eső and Szentes (2007) to assume that the seller's signal z is equal to the random variable $F_\theta(\omega)$, so that z is uniformly distributed on $Z = [0, 1]$, independent of θ. For analytical convenience, we write $\omega_\theta(z) = F_\theta^{-1}(z)$ as type θ's value for the product conditional on a signal realization z, for each $\theta \in \{H, L\}$. It is straightforward to verify that $\omega_\theta(z)$ is strictly increasing in z for each θ, and $\omega_H(z) \geq \omega_L(z)$ for all z because by assumption type H first-order stochastically dominates type L.

Following Eső and Szentes (2007), we adopt an indirect approach by first solving a *hypothetical* full-disclosure problem in which the seller can release, *and observe*, the realization of z to the buyer. In this hypothetical setting, the seller cannot infer anything about the buyer's ex ante type θ by observing z because z and θ are independent, while the buyer has the same private ex ante information as in the original setting but none of private ex post information. The seller's hypothetical problem is to find a menu of contracts, $\left(x^\theta(z), y^\theta(z)\right)_{\theta \in \{H,L\}}$, to maximize her hypothetical revenue, where $x^\theta(z)$ and $y^\theta(z)$ are allocation and transfer for each reported buyer type θ conditional on the realized z, respectively.

Proposition 5 *Suppose the optimal allocation in the hypothetical problem is given by $x^\theta(z) = 1\left\{z \geq z^\theta\right\}$ with $z^H \leq z^L$, and the following condition holds:*[16]

$$\mathbb{E}[\omega_H(z)|z \leq z^L] \leq \mathbb{E}[\omega_L(z)|z \geq z^L]. \tag{13}$$

Then in the seller's original problem, the optimal disclosure policy is a pair of monotone binary partitions $\left(\sigma^H, \sigma^L\right)$ with threshold z^H and z^L, respectively. The maximal profit can also be achieved through non-discriminatory disclosure.

Proof. See appendix. ■

[16] The sufficient condition (13) is rather mild if ex ante types are not too different. It implies that a type-H buyer who mimics type L would buy only if he learns that z is above z^L.

Let us compare Proposition 5 to results obtained in Eső and Szentes (2007) who consider a model where the buyer's ex ante type is continuous. They show that full disclosure is optimal if the seller is restricted to disclose only information that is orthogonal to the buyer's ex ante type. Since full disclosure is non-discriminatory, we can also interpret their result as an equivalence result between optimal discriminatory and non-discriminatory disclosure. Even though full disclosure is not optimal in our binary type setting, the equivalence result – a robust property with independent information – holds in both settings.

4.2 Uniform pricing

Now we return to the setting with correlated information and suppose that the seller has to offer the same pricing scheme (a, p) to both types. Would optimal disclosure remain discriminatory if we do not allow the seller to price discriminate? In general, the answer is yes, in contrast to Wei and Green (2020) who show that information and price discrimination are complementary with posterior participation constraint and independent information.

Formally, for each $\theta = H, L$, let $\sigma^\theta : \Omega \to [0, 1]$ be the probability that type θ receives the "buy" outcome. The seller chooses disclosure policy (σ^L, σ^H) and a contract (a, p) to maximize her profit, subject to: (i) the interim participation constraint for each type $\theta = H, L$:

$$\int_{\underline{\omega}}^{\overline{\omega}} f_\theta(\omega)\sigma^\theta(\omega)(\omega - p)d\omega \geq a; \tag{IR$_\theta$}$$

(ii) two obedience constraints, so the low type is willing to buy after the "buy" outcome and the high type is willing to pass after the "don't-buy" outcome:[17]

$$\int_{\underline{\omega}}^{\overline{\omega}} f_L(\omega)\sigma^L(\omega)(\omega - p)d\omega \geqslant 0 \geqslant \int_{\underline{\omega}}^{\overline{\omega}} f_H(\omega)(1 - \sigma^H(\omega))(\omega - p)d\omega; \tag{OB$_\theta$}$$

[17]Since the price p is the same for the two types, by MLRP the interim participation constraints imply that the type is willing to buy after the "buy" outcome and the low type is willing to pass after the "don't-buy" outcome.

and (iii) two incentive compatibility constraints:[18]

$$\int_{\underline{\omega}}^{\overline{\omega}} f_H(\omega)\sigma^H(\omega)(\omega - p)d\omega \geqslant \int_{\underline{\omega}}^{\overline{\omega}} f_H(\omega)\sigma^L(\omega)(\omega - p)d\omega, \tag{IC$_H$}$$

$$\int_{\underline{\omega}}^{\overline{\omega}} f_L(\omega)\sigma^L(\omega)(\omega - p)d\omega \geqslant \max\left\{\int_{\underline{\omega}}^{\overline{\omega}} f_L(\omega)\sigma^H(\omega)(\omega - p)d\omega, 0\right\}. \tag{IC$_L$}$$

Proposition 6 *Suppose $c \leq \underline{\omega}$. Then the optimal disclosure policy consists of a pair of nested intervals. Moreover, there is a mechanism without information discrimination that gives the seller the same profit.*

Proof. See appendix. ■

With $c \leq \underline{\omega}$, the gain from trade is certain and it is optimal to set $p > c$ in the optimal mechanism. Hence, maximizing the seller's profit is equivalent to maximizing the trading probability with the buyer. Then a logic similar to the one in Guo and Shmaya (2019) applies and the optimal disclosure is a pair of nested intervals. Alternatively, if the buyer's participation constraints are ex post so that $a = 0$, then it is necessarily true that $p > c$ in the optimal solution and again the optimal disclosure is a pair of nested intervals.

The optimality of nested intervals relies on the assumption of $c \leq \underline{\omega}$ which implies $p > c$ in the optimal pricing scheme. But can $p > c$ hold in the optimal solution if $c > \underline{\omega}$? The following example demonstrates that the optimal solution may feature $p < c$ and optimal disclosure may not be a pair of nested intervals.

Example 3 *Suppose $c = \frac{1}{2}$ and consider the following distributions*

	$\omega = 0$	$\omega = \frac{1}{2}$	$\omega = \frac{3}{4}$	$\omega = 1$
f_H	$\frac{7}{16}$	$\frac{7}{16}(1-\varepsilon)$	$\frac{7}{16}\varepsilon$	$\frac{1}{8}$
f_L	$\frac{1}{2}$	$\frac{1}{2}(1-\varepsilon)$	$\frac{1}{2}\varepsilon$	0

for some small $\varepsilon > 0$. Consider the following two information policies that induce the efficient allocation:

$$\sigma^L(\omega) = \begin{cases} 1 & \text{if } \omega \in \left\{\frac{1}{2}, \frac{3}{4}\right\} \\ 0 & \text{if } \omega \in \{0, 1\} \end{cases}, \quad \text{and } \sigma^H(\omega) = \begin{cases} 1 & \text{if } \omega \in \left\{\frac{3}{4}, 1\right\} \\ 0 & \text{if } \omega \in \left\{0, \frac{1}{2}\right\} \end{cases}.$$

[18] In IC$_H$ we can ignore the possibility that type H deviates and buys at all signals. This is because if a deviating type H always buys, then he will also always buy when he reports his type truthfully.

The price p is chosen so that both types have the same on-path payoff:

$$-a+\frac{1-\varepsilon}{2}\left(\frac{1}{2}-p\right)+\frac{\varepsilon}{2}\left(\frac{3}{4}-p\right)=-a+\frac{7}{16}\varepsilon\left(\frac{3}{4}-p\right)+\frac{1}{8}(1-p).$$

This implies that

$$p=\frac{8-13\varepsilon}{24-28\varepsilon},$$

and $p<c$ for small $\varepsilon>0$. The advance payment a is set to extract the full surplus:

$$a=\frac{1}{2}(1-\varepsilon)\left(\frac{1}{2}-\frac{8-13\varepsilon}{24-28\varepsilon}\right)+\frac{1}{2}\varepsilon\left(\frac{3}{4}-\frac{8-13\varepsilon}{24-28\varepsilon}\right)=\frac{4+5\varepsilon-7\varepsilon^2}{48-56\varepsilon}.$$

Hence, both IR_H and IR_L bind. Furthermore, any price $p>c$ cannot fully extract the surplus because such a price cannot equalize the on-path payoffs of different types.

It remains to verify that all other constraints hold for small $\varepsilon>0$. First, both price bounds

$$p\geq\mathbb{E}_H\left[\omega|\sigma^H(\omega)=0\right]\Leftrightarrow\frac{8-13\varepsilon}{24-28\varepsilon}\geq\frac{1-\varepsilon}{4-2\varepsilon}$$

and

$$p\geq\mathbb{E}_H\left[\omega|\sigma^L(\omega)=0\right]\Leftrightarrow\frac{8-13\varepsilon}{24-28\varepsilon}\geq\frac{2}{9}$$

hold for small ε. IC_H constraint is satisfied for small ε because $p<c$ and type L on-path trading probability (1/2) is higher than type H off-path trading probability (7/16). Finally, IC_L constraint is also satisfied because for small ε,

$$0\geq-\frac{4+5\varepsilon-7\varepsilon^2}{48-56\varepsilon}+\frac{1}{2}\varepsilon\left(\frac{3}{4}-p\right).$$

In this example, the gain from trade is uncertain and optimal strike price $p<c$. Why is it useful to set $p<c$ for the seller? By pricing below cost, the seller can subsidize trade with type L and raise the advance payment. Such a pricing scheme can reduce information rent from type H, if it does not lead to substantial efficiency loss in allocation. Discriminatory information disclosure can exactly help with that. In particular, excluding value 1 from trade for type L has no efficiency loss since it has zero probability, and excluding value 1/2 from trade for type H also has no efficiency loss since the gain from trade is zero, while including value 1/2 for trade for type L has no efficiency implication but can help boost up the advance payment.

There are two interesting takeaways from this example. First, even in the absence of price discrimination, information discrimination can help reduce information rent and hence increase profit. Second, without price discrimination, the buy regions of the two types may not be nested in the optimal disclosure policy.[19]

5 Concluding Remarks

Our characterization of optimal price and information discriminations restricts to deterministic pricing mechanisms. In the sequential screening model of Courty and Li (2000), deterministic contracts are optimal with binary types, but randomization can be optimal with three or more types. In Li and Shi (2021), we provide necessary and sufficient conditions for randomization, and a characterization of optimal stochastic sequential mechanisms. With binary types, but with an endogenous information policy, it is an open question whether the assumption of deterministic pricing schemes is restrictive or not.

A natural question for future research is how to generalize our approach and characterization to a model with more than two types or even a continuum of types. We conjecture that the optimal information disclosure policy still has a nested-interval structure at any regular solution to a suitably constructed "residual relaxed problem." At any such regular solution, a higher type has a greater probability of buying the good after misreporting as a lower type than the truthful lower type, so that the individual rationality constraint of the higher type follows from the individual rationality constraint of the lower type and the "downward" incentive compatibility constraint. Finding sufficient conditions on the primitives of the mechanism design problem to ensure that solutions are regular would be a challenge. The other important issue is that, with more than two types, the residual relaxed problem has to drop global incentive compatibility constraints. Our approach has to be validated by showing that solutions to the residual relaxed problem have nested-interval structures and satisfy the dropped global incentive compatibility constraints.

6 Appendix: Omitted Proofs

6.1 Proof of Lemma 1

First, IR_L binds; otherwise raising a^L slightly would not affect any constraint in the relaxed problem and increase the profit given in the objective (1). Second, IC_H binds. Suppose not. Since IR_L binds, the profit from type L in the objective (1) can be

[19] In Example 3, the values are discrete. Since all inequalities are strict, however, one can approximate this example by a continuous one where both observations hold.

rewritten as

$$\int_{\underline{\omega}}^{\overline{\omega}} (\omega - c)\sigma^L(\omega) f_L(\omega) d\omega.$$

Since IC_H is slack, the solution to the relaxed problem must have $\sigma^L(\omega) = 1$ for all $\omega \geq c$ and 0 otherwise. Given that IR_L binds, the deviation payoff for type H is then at least

$$\int_{c}^{\overline{\omega}} (\omega - p^L)(f_H(\omega) - f_L(\omega)) d\omega,$$

obtained by buying only after the "buy" outcome. The above is strictly positive because $F_H(\omega)$ first-order stochastic dominates $F_L(\omega)$. Thus, IR_H is also slack. But then the seller's profit can be increased by raising a^H, a contradiction.

6.2 Proof of Lemma 2

Suppose that $u_H^L > p^L$ at some solution to the relaxed problem. First, we claim that in this case, the optimal information policy $\sigma^L(\omega)$ is a monotone partition such that $\sigma^L(\omega) = 1$ for all $\omega \geq k^L$ and 0 for $\omega < k^L$ for some threshold $k^L \in (\underline{\omega}, \overline{\omega})$. Suppose this is not the case. Then, we can find k_1 and k_2 with $k_1 < k_2$, such that $\sigma^L(\omega) > 0$ for all $\omega \in (k_1, k_1 + dk_1)$ for $dk_1 > 0$, and $\sigma^L(\omega) < 1$ for all $\omega \in (k_2, k_2 + dk_2)$ for $dk_2 > 0$. Consider $\tilde{\sigma}^L$ such that $\tilde{\sigma}^L(\omega) = \sigma^L(\omega)$ except that, for some sufficiently small $\epsilon > 0$, $\tilde{\sigma}^L(\omega) = \sigma^L(\omega) - \epsilon$ for $\omega \in (k_1, k_1 + dk_1)$ and $\tilde{\sigma}^L(\omega) = \sigma^L(\omega) + \epsilon$ for $\omega \in (k_2, k_2 + dk_2)$, where dk_1 and dk_2 satisfy

$$-f_L(k_1)dk_1 + f_L(k_2)dk_2 = 0.$$

By construction,

$$\int_{\underline{\omega}}^{\overline{\omega}} \tilde{\sigma}^L(\omega) f_L(\omega) d\omega = \int_{\underline{\omega}}^{\overline{\omega}} \sigma^L(\omega) f_L(\omega) d\omega.$$

This implies the total change in v_L^L is given by

$$dv_L^L = -\frac{(k_1 - v_L^L) f_L(k_1) dk_1}{\int_{\underline{\omega}}^{\overline{\omega}} \sigma^L(\omega) f_L(\omega) d\omega} + \frac{(k_2 - v_L^L) f_L(k_2) dk_2}{\int_{\underline{\omega}}^{\overline{\omega}} \sigma^L(\omega) f_L(\omega) d\omega} = \frac{(k_2 - k_1) f_L(k_1) dk_1}{\int_{\underline{\omega}}^{\overline{\omega}} \sigma^L(\omega) f_L(\omega) d\omega} > 0.$$

Similarly,

$$du_L^L = \frac{(k_1 - k_2) f_L(k_1) dk_1}{\int_{\underline{\omega}}^{\overline{\omega}} (1 - \sigma^L(\omega)) f_L(\omega) d\omega} < 0.$$

It follows that by keeping p^L unchanged, the seller can ensure that PB_L is still satisfied under $\tilde{\sigma}^L$. Changing a^L to continue to bind IR_L, we have

$$da^L = -(k_1 - p^L)f_L(k_1)dk_1 + (k_2 - p^L)f_L(k_2)dk_2 = (k_2 - k_1)f_L(k_1)dk_1 > 0.$$

Since $u_H^L > p^L$, under $\tilde{\sigma}^L$ type H continues to strictly prefer to buy regardless of the outcome after the deviation. Type H's deviation payoff is thus $\mu_H - p^L - a^L$, which is decreased when a^L is increased, and so IC_H remains satisfied. But after the modifications, the seller's profit from type L in the objective (1) would increase, because a^L is increased. This is a contradiction to optimality. Thus, σ^L is given by a two-step function with some threshold k^L.

By Lemma 1, IR_L and IC_H bind at any solution to the relaxed problem. Given that σ^L is a two-step function with k^L, using $u_H^L > p^L$ we can now write the seller's profit as

$$\phi_H \int_{\underline{\omega}}^{\overline{\omega}} (\omega - c)\,\sigma^H(\omega) f_H(\omega)d\omega + \phi_L \int_{k^L}^{\overline{\omega}} (\omega - c)\, f_L(\omega)d\omega$$
$$- \phi_H \left(\mu_H - p^L - \int_{k^L}^{\overline{\omega}} \left(\omega - p^L\right) f_L(\omega)d\omega \right).$$

The above is increasing in p^L. A slight increase in p^L does not violate PB_L, because σ^L is a monotone partition with threshold k^L, which implies that $v_L^L \geq k^L \geq u_H^L > p^L$. IR_H remains satisfied too, because type H could always misreport his type and then buy only after the buy outcome, achieving the deviation payoff given by the left-hand side of (4). This deviation payoff is non-negative regardless of p^L, because σ^L is a two-step function with k^L and F_H first order stochastically dominates F_L. This is a contradiction to optimality.

6.3 Proof of Lemma 3

Consider any solution to the relaxed problem with p^H such that $p^H \leq v_L^H$. Then we can use binding IR_L and binding IC_H (6) implied by Lemma 1 to rewrite IC_L as

$$\int_{\underline{\omega}}^{\overline{\omega}} (\omega - p^L)(f_H(\omega) - f_L(\omega))\sigma^L(\omega)d\omega \leq \int_{c}^{\overline{\omega}} (\omega - p^H)(f_H(\omega) - f_L(\omega))d\omega.$$

Suppose the above is violated. Then, consider the alternative of setting $\hat{\sigma}^L(\omega) = 1$ for $\omega \geq c$ and 0 otherwise, and setting $\hat{p}^L = p^H$. Together with $\hat{a}^L$ that binds IR_L, and $\hat{a}^H$ that binds IC_H, this alternative satisfies (2), as well as (3) because $\hat{\sigma}^L$ is weakly increasing. However, given that $\sigma^L(\cdot)$ and p^L violate IC_L, we have

$$\int_{\underline{\omega}}^{\overline{\omega}} (\omega - p^L)\sigma^L(\omega)(f_H(\omega) - f_L(\omega))d\omega > \int_{\underline{\omega}}^{\overline{\omega}} \left(\omega - \hat{p}^L\right) \hat{\sigma}^L(\omega)(f_H(\omega) - f_L(\omega))d\omega.$$

From the second integral of the objective (5), the seller's profit under $\hat{\sigma}^L(\cdot)$ and $\hat{p}^L$ is higher than under $\sigma^L(\cdot)$ and p^L. This contradicts the assumption that $\sigma^L(\cdot)$ and p^L solve the relaxed problem.

6.4 Proof of Lemma 4

We show by contradiction that, if p^L and σ^L are a regular solution to the residual relaxed problem, then σ^L is partitional with an interval buy region that contains p^L. Suppose not. Then, there exist $k_1 < k_2 \in (\underline{\omega}, \overline{\omega})$, with $dk_1, dk_2 > 0$, such that, (i) if $k_1 > p^L$ then $\sigma^L(\omega) < 1$ for all $\omega \in (k_1, k_1+dk_1)$ and $\sigma^L(\omega) > 0$ for all $\omega \in (k_2, k_2+dk_2)$, and (ii) if $k_2 < p^L$ then $\sigma^L(\omega) > 0$ for all $\omega \in (k_1, k_1 + dk_1)$ and $\sigma^L(\omega) < 1$ for all $\omega \in (k_2, k_2 + dk_2)$.[20] Define a new information policy $\tilde{\sigma}^L$ that coincides with σ^L, except $\tilde{\sigma}^L(\omega) = \sigma^L(\omega) + \epsilon_1$ for all $\omega \in (k_1, k_1 + dk_1)$ and $\tilde{\sigma}^L(\omega) = \sigma^L(\omega) + \epsilon_2$ for all $\omega \in (k_2, k_2 + dk_2)$. Such ϵ_1 and ϵ_2 can be always be found for $\tilde{\sigma}^L$ to be a valid information policy: in case (i), we choose $\epsilon_1 > 0 > \epsilon_2$, and in case (ii), we choose $\epsilon_1 < 0 < \epsilon_2$. Furthermore, in either case, we choose ϵ_1 and ϵ_2 such that $dv_L^L = 0$, and hence $v_L^L = p^L$. This is equivalent to

$$\sum_{i=1,2} (k_i - p^L) f_L(k_i)\epsilon_i dk_i = 0.$$

The change to the first term in (7) has the same sign as

$$\sum_{i=1,2} (k_i - c) f_L(k_i)\epsilon_i dk_i > \frac{k_2 - c}{k_2 - p^L} \sum_{i=1,2} (k_i - p^L) f_L(k_i)\epsilon_i dk_i = 0,$$

where the inequality follows because $p^L > c$ implies that $(k - c)/(k - p^L)$ is decreasing in k, and $k^1 > p^L$ with $\epsilon_1 > 0$ in case (i) and $k^1 < p^L$ with $\epsilon_1 < 0$ in case (ii). If $u_H^L < v_L^L$, then $u_H^L < p^L = v_L^L$. Thus, both before and after the change from σ^L to $\tilde{\sigma}^L$, type H strictly prefers to buy only upon getting the buy outcome after misreporting

[20] Note that if the buy region for type L is not an interval, then there must exist a "hole" (ω^-, ω^+) inside of the buy region such that $\sigma^L(\omega) \in [0, 1)$ for all $\omega \in (\omega^-, \omega^+)$. If p^L lies below or inside the hole, then we can pick $k_1 \in (\omega^-, \omega^+)$ and $k_2 > \omega^+$ and hence conditions in case (i) are fulfilled. If p^L lies above the hole, then we can pick $k_1 < \omega^-$ and $k_2 \in (\omega^-, \omega^+)$ so that conditions in case (ii) are fulfilled.

as type L. The information rent of type H is therefore given by the second term in (7) (without the negative sign). The change in the information rent has the same sign as

$$\sum_{i=1,2}(k_i - p^L)(f_H(k_i) - f_L(k_i))\epsilon_i dk_i < \frac{f_H(k_2) - f_L(k_2)}{f_L(k_2)} \sum_{i=1,2}(k_i - p^L) f_L(k_i)\epsilon_i dk_i = 0,$$

where the inequality follows because $(f_H(k) - f_L(k))/f_L(k)$ is increasing in k by likelihood ratio ranking, and $k_1 > p^L$ with $\epsilon_1 > 0$ in case (i) and $k_1 < p^L$ with $\epsilon_1 < 0$ in case (ii). If $u_H^L = v_L^L$, we have $u_H^L = p^L$. Then, with the change from σ^L to $\tilde{\sigma}^L$, the sign of du_H^L is the same as

$$-\sum_{i=1,2}(k_i - p^L) f_H(k_i)\epsilon_i dk_i > -\frac{f_H(k_2)}{f_L(k_2)} \sum_{i=1,2}(k_i - p^L) f_L(k_i)\epsilon_i dk_i = 0,$$

where the inequality follows because $-f_H(k)/f_L(k)$ is decreasing in k by likelihood ratio ranking, and $k_1 > p^L$ with $\epsilon_1 > 0$ in case (i) and $k_1 < p^L$ with $\epsilon_1 < 0$ in case (ii). This implies that, after misreporting as type L, type H weakly prefers to buy regardless of the recommendation by the seller, both before and after the change from σ^L to $\tilde{\sigma}^L$. Thus, the information rent of type H is instead

$$\phi_H \left(\mu_H - p^L - \int_{\underline{\omega}}^{\overline{\omega}} \left(\omega - p^L\right) \sigma^L(\omega) f_L(\omega) d\omega \right).$$

The above is unchanged when σ^L changes to $\tilde{\sigma}^L$. Further, for infinitesimal changes from σ^L to $\tilde{\sigma}^L$, Lemma 3 continues to hold. Thus, regardless of whether $u_L^L < v_L^L$ or $u_L^L = v_L^L$, the change from σ^L to $\tilde{\sigma}^L$ increases the value of the objective function (7).

Finally, to show $\underline{k} > c$, suppose by contradiction that $\underline{k} \leq c$. We use the interval form to rewrite the residual objective function as

$$\phi_L \int_{\underline{k}}^{\overline{k}} (\omega - c) f_L(\omega) d\omega - \phi_H \int_{\underline{k}}^{\overline{k}} (\omega - p^L)(f_H(\omega) - f_L(\omega)) d\omega, \tag{14}$$

Consider increasing $\underline{k}$ marginally and at the same time we increase p^L so as to keep it equal to v_L^L. The effect of the proposed change on the first term in the objective is

$$-\phi_L(\underline{k} - c) f_L(\underline{k}) \geq 0.$$

The effect on the second term in the objective without the negative sign is

$$-\phi_H \left(v_L^L - \underline{k}\right) \left(\Lambda(\underline{k}, \overline{k}) - \lambda(\underline{k})\right) f_L(\underline{k}).$$

The above expression is negative, because $v_L^L > \underline{k}$, and because likelihood ratio dominance implies that the difference in the last bracket is positive, implying that the rent to type H is decreased. Therefore, the seller's profit increases, which contradicts optimality. Hence, $\underline{k} > c$ and the optimal disclosure policy in the regular solution is a pair of nested intervals.

6.5 Proof of Lemma 5

As we argued in the text, if solution σ^L is irregular, then $p^L = u_H^L < c$. Moreover, there must exist $k_2 > c$ such that $\sigma^L(\omega) > 0$ for all $\omega \in (k_2, k_2 + dk_2)$ for $dk_2 > 0$. Otherwise, the trade surplus with type L – the first term in the residual objective function (7) – would be negative, and the seller can exclude type L altogether and be better off.

Suppose by contradiction that there exists $k_1 < u_H^L = p^L$ such that $\sigma^L(\omega) > 0$ for all $\omega \in (k_1, k_1 + dk_1)$ for $dk_1 > 0$. Now, consider $\tilde{\sigma}^L$ such that $\tilde{\sigma}^L(\omega) = \sigma^L(\omega)$ except that, for some sufficiently small $\epsilon > 0$, $\tilde{\sigma}^L(\omega) = \sigma^L(\omega) - \epsilon$ for $\omega \in (k_1, k_1 + dk_1)$ and $\tilde{\sigma}^L(\omega) = \sigma^L(\omega) - \epsilon$ for $\omega \in (k_2, k_2 + dk_2)$, where dk_1 and dk_2 satisfy

$$(k_1 - u_H^L) f_H(k_1) dk_1 + (k_2 - u_H^L) f_H(k_2) dk_2 = 0.$$

Note that the above is feasible because $k_1 < u_H^L = p^L < c < k_2$. By construction, there is no change to u_H^L. By keeping p^L the same, from binding IR_L we have that the change to a^L is given by

$$da^L = -(k_1 - p^L) f_L(k_1) dk_1 - (k_2 - p^L) f_L(k_2) dk_2.$$

Using the condition on dk_1 and dk_2, and $u_H^L = p^L$, we have that da^L has the same sign as

$$\frac{f_H(k_2)}{f_L(k_2)} - \frac{f_H(k_1)}{f_L(k_1)},$$

which is strictly positive by likelihood ratio dominance. Given that u_H^L and p^L are unchanged, under $\tilde{\sigma}^L$ type H will buy the good with probability 1 after misreporting as type L. Since a^L is increased, IC_H is satisfied. Finally, the trade surplus with type

L – the first term in the residual objective function (7) – is changed by

$$-(k_1 - c) f_L(k_1) dk_1 - (k_2 - c) f_L(k_2) dk_2.$$

Using the condition on dk_1 and dk_2, the change to the trading surplus has the same sign as

$$\frac{c - k_1}{p^L - k_1} \frac{f_H(k_2)}{f_L(k_2)} - \frac{k_2 - c}{k_2 - p^L} \frac{f_H(k_1)}{f_L(k_1)}.$$

The above is strictly positive, because of likelihood ratio dominance and because

$$\frac{c - k_1}{p^L - k_1} > 1 > \frac{k_2 - c}{k_2 - p^L}.$$

This then contradicts the assumption that σ^L is a solution to the residual relaxed problem.

6.6 Proof of Proposition 3

Let σ^L be any signal structure that fails (4) and satisfies $\sigma^L(\omega) = 0$ for all $\omega < u_H^L$. Define

$$k = \inf_{\omega} \{\omega : \sigma^L(\omega) > 0\}.$$

Since σ^L fails (4), we have $k < \omega_o$. By assumption, $u_H^L \leq k$.

Suppose that $\sigma^L(\omega) < 1$ for all $\omega \in (k_1, k_1 + dk_1)$ for some $k_1 \in (k, \omega_o)$ and $dk_1 > 0$. Consider $\tilde{\sigma}^L$ such that $\tilde{\sigma}^L(\omega) = \tilde{\sigma}^L(\omega)$ except that, for some sufficiently small $\epsilon > 0$, $\tilde{\sigma}^L(\omega) = \sigma^L(\omega) + \epsilon$ for $\omega \in (k_1, k_1 + dk_1)$. The change to the difference in the probabilities of trade by type H after misreporting as type L and by the true type L is given by

$$(f_H(k_1) - f_L(k_1)) dk_1 < 0,$$

because $k_1 < \omega_o$. The change to u_H^L has the same sign as

$$-(k_1 - u_H^L) f_H(k_1) dk_1 < 0,$$

because $u_H^L \leq k < k_1$. Thus, there exists a signal structure $\tilde{\sigma}^L$ for the low type, such that for some $k < \omega_o$, we have $\tilde{\sigma}^L(\omega) = 0$ for all $\omega \in [\underline{\omega}, k)$ and $\tilde{\sigma}^L(\omega) = 1$ for all $\omega \in (k, \omega_o]$, (4) fails and $\tilde{u}_H^L \leq k$.

Next, we show that we can further restrict $\tilde{\sigma}^L$ to satisfy the condition of equal trading probabilities for a deviating high type and the true low type. If this is not true, then we have

$$\int_{\omega_o}^{\overline{\omega}} (f_H(\omega) - f_L(\omega))(1 - \tilde{\sigma}^L(\omega))d\omega > F_L(k) - F_H(k).$$

Then there exists $k_2 > \omega_o$ such that $\sigma^L(\omega) < 1$ for all $\omega \in (k_2, k_2 + dk_2)$ for some $dk_2 > 0$. Consider $\hat{\sigma}^L$ such that $\hat{\sigma}^L(\omega) = \tilde{\sigma}^L(\omega)$ except that, for some sufficiently small $\epsilon > 0$, $\hat{\sigma}^L(\omega) = \tilde{\sigma}^L(\omega) + \epsilon$ for $\omega \in (k_2, k_2 + dk_2)$. The change to the difference in the probabilities of trade by type H after misreporting as type L and by the true type L is given by

$$-(f_H(k_2) - f_L(k_2))dk_2 < 0,$$

because $k_2 > \omega_o$. For sufficiently small ϵ, condition (4) continues to fail. The change to $\tilde{u}_H^L$ has the same sign as

$$-(k_2 - \tilde{u}_H^L)f_H(k_2)dk_2 < 0,$$

because $\tilde{u}_H^L \leq k < k_2$. Therefore, we can assume that under $\tilde{\sigma}^L$ condition (4) holds as an equality.

To summarize, we have shown that if there is an irregular solution to the residual relaxed problem that satisfies Lemma 5, then there exists σ^L with $\sigma^L(\omega) = 0$ for all $\omega \in [\underline{\omega}, k]$ and $\sigma^L(\omega) = 1$ for all $\omega \in (k, \omega_o]$ for some $k < \omega_o$, such that (4) holds as an equality, and $u_H^L \leq k$. Now, fix any $k \in [\underline{\omega}, \omega_o]$. Let $\Sigma(k)$ be the set of all signal structures σ^L with $\sigma^L(\omega) = 0$ for all $\omega \in [\underline{\omega}, k]$ and $\sigma^L(\omega) = 1$ for all $\omega \in (k, \omega_o]$, such that

$$\int_{\omega_o}^{\overline{\omega}} (f_H(\omega) - f_L(\omega))(1 - \sigma^L(\omega))d\omega = F_L(k) - F_H(k).$$

Let

$$u_H^L(\sigma^L; k) = \frac{\int_{\underline{\omega}}^{k} \omega f_H(\omega)d\omega + \int_{\omega_o}^{\overline{\omega}} \omega(1 - \sigma^L(\omega))f_H(\omega)d\omega}{F_H(k) + \int_{\omega_o}^{\overline{\omega}} (1 - \sigma^L(\omega))f_H(\omega)d\omega},$$

and define

$$\sigma^L(\omega; k) \in \arg\min_{\sigma^L \in \Sigma(k)} u_H^L(\sigma^L; k).$$

When $k = \underline{\omega}$, any $\sigma^L \in \Sigma(\underline{\omega})$ satisfies

$$\int_{\omega_o}^{\overline{\omega}} (f_H(\omega) - f_L(\omega))(1 - \sigma^L(\omega))d\omega = 0.$$

Since $f_H(\omega) > f_L(\omega)$ for all $\omega \in (\omega_o, \overline{\omega})$, any σ^L in $\Sigma(\underline{\omega})$ satisfies $\sigma^L(\omega) = 1$ for all

$\omega > \omega_o$. We can therefore set $\sigma^L(\cdot;\underline{\omega})$ to $\sigma^L(\omega;\underline{\omega}) = 1$ for all $\omega \in [\underline{\omega},\overline{\omega}]$. We have $u_H^L(\sigma^L(\cdot;\underline{\omega});\underline{\omega}) \geq \underline{\omega}$.

When $k = \omega_o$, any $\sigma^L \in \Sigma(\omega_o)$ satisfies

$$\int_{\omega_o}^{\overline{\omega}} (f_H(\omega) - f_L(\omega))(1 - \sigma^L(\omega))d\omega = F_L(\omega_o) - F_H(\omega_o).$$

This implies that σ^L in $\Sigma(\omega_o)$ satisfies $\sigma^L(\omega) = 0$ for all $\omega > \omega_o$. We can therefore set $\sigma^L(\cdot;\omega_o)$ to $\sigma^L(\omega;\underline{\omega}) = 0$ for all $\omega \in [\underline{\omega},\overline{\omega}]$. By assumption, we have $u_H^L(\sigma^L(\cdot;\omega_o);\omega_o) = \mu_H > \omega_o$.

By the envelope theorem, the derivative of $u_H^L(\sigma^L(\cdot;k);k)$ with respect to k has the same sign as

$$k - u_H^L(\sigma^L(\cdot;k);k).$$

This means that as a function of k, $u_H^L(\sigma^L(\cdot;k);k)$ can cross k only from above. Since $u_H^L(\sigma^L(\cdot;\underline{\omega});\underline{\omega}) \geq \underline{\omega}$ and $u_H^L(\sigma^L(\cdot;\omega_o);\omega_o) > \omega_o$, the above implies that $u_H^L(\sigma^L(\cdot;k);k) > k$ for all $k \in (\underline{\omega},\omega_o)$. This is a contradiction. The proposition follows from Lemma 4.

6.7 Proof of Corollary 1

Suppose by contradiction that we have $\overline{k} = \overline{\omega}$ for all sufficiently small ϕ_L. Recall from (10) the residual objective function $\Gamma(\underline{k},\overline{k})$. Note that in the limit of $\phi_L = 0$, we have $\underline{k} = \overline{k}$; otherwise, the first term of $\Gamma(\underline{k},\overline{k})$ is 0 in the limit, but the second term is strictly positive, which would be a contradiction. Then, the first-order condition with respect to $\underline{k}$ becomes $\frac{\partial\Gamma(\underline{k},\overline{k})}{\partial\underline{k}}|_{\overline{k}=\overline{\omega}} \geq 0$ and $\underline{k} \leq \overline{\omega}$ with complementary slackness. It must hold with equality for ϕ_L sufficiently close to 0. If not, we have $\underline{k} = \overline{k} = \overline{\omega}$ and hence

$$\frac{\partial\Gamma(\underline{k},\overline{k})}{\partial\underline{k}}|_{\underline{k}=\overline{k}=\overline{\omega}} = -\phi_L(\overline{\omega} - c)f_L(\overline{\omega}) < 0,$$

contradicting the assumption that $\underline{k} = \overline{k} = \overline{\omega}$ in the limit. Therefore, we can rewrite the first-order condition for $\underline{k}$ as

$$\frac{\phi_L}{1-\phi_L}(\underline{k} - c) - \left(v_L^L - \underline{k}\right)\left(\Lambda(\underline{k},\overline{\omega}) - \lambda(\underline{k})\right) = 0. \tag{15}$$

The corollary follows immediately from the following claim: when ϕ_L is sufficiently small, for any $\underline{k}$ satisfying first-order condition (15), the first-order condition for $\overline{k}$

evaluated at $\overline{k} = \overline{\omega}$,

$$\frac{\phi_L}{1-\phi_L}(\overline{\omega}-c) - \left(\overline{\omega} - v_L^L\right)\left(\lambda(\overline{\omega}) - \Lambda(\underline{k},\overline{\omega})\right) \geq 0, \tag{16}$$

is violated if $\lambda''(\overline{\omega})/\lambda'(\overline{\omega}) > 3/(\overline{\omega}-c) + 2f_L'(\overline{\omega})/f_L(\overline{\omega})$. To prove the above claim, define

$$\Psi(\underline{k}) \equiv (\underline{k}-c)\left(\overline{\omega}-v_L^L\right)\left(\lambda(\overline{\omega})-\Lambda(\underline{k},\overline{\omega})\right) - (\overline{\omega}-c)\left(v_L^L-\underline{k}\right)\left(\Lambda(\underline{k},\overline{\omega})-\lambda(\underline{k})\right).$$

It follows from (15) that condition (16) is violated if $\Psi(\underline{k}) > 0$ for $\underline{k}$ sufficiently close to but strictly below $\overline{\omega}$. Note that $\Psi(\overline{\omega}) = 0$ and

$$\begin{aligned}\Psi'(\underline{k}) = &\left(\overline{\omega}-v_L^L\right)\left(\lambda(\overline{\omega})-\Lambda(\underline{k},\overline{\omega})\right) - (\underline{k}-c)\left(\left(\lambda(\overline{\omega})-\Lambda(\underline{k},\overline{\omega})\right)\frac{\partial v_L^L}{\partial \underline{k}} + \left(\overline{\omega}-v_L^L\right)\frac{\partial \Lambda(\underline{k},\overline{\omega})}{\partial \underline{k}}\right) \\ &- (\overline{\omega}-c)\left(\left(\Lambda(\underline{k},\overline{\omega})-\lambda(\underline{k})\right)\left(\frac{\partial v_L^L}{\partial \underline{k}}-1\right) + \left(v_L^L-\underline{k}\right)\left(\frac{\partial \Lambda(\underline{k},\overline{\omega})}{\partial \underline{k}} - \lambda'(\underline{k})\right)\right),\end{aligned}$$

where

$$\frac{\partial v_L^L}{\partial \underline{k}} = \frac{f_L(\underline{k})}{1-F_L(\underline{k})}(v_L^L-\underline{k}); \quad \frac{\partial \Lambda(\underline{k},\overline{\omega})}{\partial \underline{k}} = \frac{f_L(\underline{k})}{1-F_L(\underline{k})}\left(\Lambda(\underline{k},\overline{\omega})-\lambda(\underline{k})\right).$$

Using L'Hopital's rule, we have

$$\lim_{\underline{k}\to\overline{\omega}} \frac{\partial v_L^L}{\partial \underline{k}} = \frac{1}{2}; \quad \lim_{\underline{k}\to\overline{\omega}} \frac{\partial \Lambda(\underline{k},\overline{\omega})}{\partial \underline{k}} = \frac{1}{2}\lambda'(\overline{\omega}).$$

Thus, $\Psi'(\overline{\omega}) = 0$. Moreover,

$$\begin{aligned}\Psi''(\underline{k}) = &-2\left(\lambda(\overline{\omega})-\Lambda(\underline{k},\overline{\omega})\right)\frac{\partial v_L^L}{\partial \underline{k}} - 2\left(\overline{\omega}-v_L^L\right)\frac{\partial \Lambda(\underline{k},\overline{\omega})}{\partial \underline{k}} + 2(\underline{k}-c)\frac{\partial v_L^L}{\partial \underline{k}}\frac{\partial \Lambda(\underline{k},\overline{\omega})}{\partial \underline{k}} \\ &- (\underline{k}-c)\left(\left(\lambda(\overline{\omega})-\Lambda(\underline{k},\overline{\omega})\right)\frac{\partial^2 v_L^L}{\partial(\underline{k})^2} + \left(\overline{\omega}-v_L^L\right)\frac{\partial^2 \Lambda(\underline{k},\overline{\omega})}{\partial(\underline{k})^2}\right) \\ &- 2(\overline{\omega}-c)\left(\frac{\partial v_L^L}{\partial \underline{k}}-1\right)\left(\frac{\partial \Lambda(\underline{k},\overline{\omega})}{\partial \underline{k}} - \lambda'(\underline{k})\right) \\ &- (\overline{\omega}-c)\left(\left(\Lambda(\underline{k},\overline{\omega})-\lambda(\underline{k})\right)\frac{\partial^2 v_L^L}{\partial \underline{k}^2} + \left(v_L^L-\underline{k}\right)\left(\frac{\partial^2 \Lambda(\underline{k},\overline{\omega})}{\partial \underline{k}^2} - \lambda''(\underline{k})\right)\right),\end{aligned}$$

where

$$\frac{\partial^2 v_L^L}{\partial(\underline{k})^2} = \frac{f_L'(\underline{k})}{f_L(\underline{k})}\frac{\partial v_L^L}{\partial \underline{k}} + \frac{f_L(\underline{k})}{1-F_L(\underline{k})}\left(2\frac{\partial v_L^L}{\partial \underline{k}} - 1\right);$$
$$\frac{\partial^2 \Lambda(\underline{k},\overline{\omega})}{\partial(\underline{k})^2} = \frac{f_L'(\underline{k})}{f_L(\underline{k})}\frac{\partial \Lambda(\underline{k},\overline{\omega})}{\partial \underline{k}} + \frac{f_L(\underline{k})}{1-F_L(\underline{k})}\left(2\frac{\partial \Lambda(\underline{k},\overline{\omega})}{\partial \underline{k}} - \lambda'(\underline{k})\right).$$

Using L'Hopital's rule, the limits of $\partial v_L^L/\partial \underline{k}$ and $\partial\Lambda(\underline{k},\overline{\omega})/\partial\underline{k}$, we have

$$\lim_{\underline{k}\to\overline{\omega}} \frac{\partial^2 v_L^L}{\partial \underline{k}^2} = \frac{f_L'(\overline{\omega})}{6 f_L(\overline{\omega})};\quad \lim_{\underline{k}\to\overline{\omega}} \frac{\partial^2 \Lambda(\underline{k},\overline{\omega})}{\partial(\underline{k})^2} = \frac{f_L'(\overline{\omega})\lambda'(\overline{\omega})}{6 f_L(\overline{\omega})} + \frac{\lambda''(\overline{\omega})}{3}.$$

Thus, $\Psi''(\overline{\omega}) = 0$. Taking derivatives of $\Psi''(\underline{k})$ and evaluating at $\underline{k} = \overline{\omega}$, using the limits of $\partial v_L^L/\partial\underline{k}$ and $\partial^2 v_L^L/\partial(\underline{k})^2$, and the limits of $\partial\Lambda(\underline{k},\overline{\omega})/\partial\underline{k}$ and $\partial^2\Lambda(\underline{k},\overline{\omega})/\partial\underline{k}^2$, we have

$$\Psi'''(\overline{\omega}) = \left(\frac{3}{2} + (\overline{\omega}-c)\frac{f_L'(\overline{\omega})}{f_L(\overline{\omega})}\right)\lambda'(\overline{\omega}) - \frac{1}{2}(\overline{\omega}-c)\lambda''(\overline{\omega}).$$

Under the condition stated in the lemma, we have $\Psi'''(\overline{\omega}) < 0$, and thus $\Psi(\underline{k}) > 0$ for $\underline{k}$ sufficiently close to $\overline{\omega}$.

6.8 Proof of Proposition 5

Following the standard procedure, we can write the seller's revenue in the hypothetical problem as

$$\begin{aligned}&\phi_H \int_{\underline{z}}^{\overline{z}} \left(\omega_H(z) - c\right) x^H(z)\, dz \\ &+\phi_L \int_{\underline{z}}^{\overline{z}} \left(\omega_L(z) - c - \frac{\phi_H}{\phi_L}\left(\omega_H(z) - \omega_L(z)\right)\right) x^L(z)\, dz \end{aligned} \tag{17}$$

If optimal allocation in the hypothetical problem is given by $x^\theta(z) = \mathbb{1}\left\{z \geq z^\theta\right\}$, then (z^L, z^H) must solve

$$\begin{aligned}\omega_L(z) - c - \frac{\phi_H}{\phi_L}\left(\omega_H(z) - \omega_L(z)\right) &= 0,\\ \omega_H(z) - c &= 0.\end{aligned}$$

These two equations, together with the fact of $\omega_H(z) \geq \omega_L(z)$ for all z, imply that $z^H \leq z^L$. It is straightforward to verify that this allocation is incentive compatible for the hypothetical problem, and the maximal hypothetical revenue is thus

$$\sum_{\theta=L,H}\phi_\theta\int_{z^\theta}^{\overline{z}}\left(\omega_\theta\left(z\right)-c\right)dz-\phi_H\int_{z^L}^{\overline{z}}\left(\omega_H\left(z\right)-\omega_L\left(z\right)\right)dz. \tag{18}$$

Now consider the solution (z^H, z^L) to the hypothetical problem in the original problem. Suppose that the seller in the original setting commits to a disclosure policy $\left(\sigma^H, \sigma^L\right)$ where both σ^H and σ^L are monotone partitions with thresholds z^H and z^L, respectively. The proof below constructs a menu of option contracts and verifies that the seller attains the hypothetical revenue given in (18). Consider a menu of option contracts $\left(a^\theta, p^\theta\right)$, where strike price $p^\theta = \omega_\theta(z^\theta)$ and advanced payments a^L and a^H are chosen to bind IR_L and IC_H. Under condition (13), all deviating buyer types buy only if they are recommended to buy. Then, the advanced payments are given by

$$\begin{aligned} a^L &= \int_{z^L}^{\overline{z}}\left(\omega_L\left(z\right)-p^L\right)dz \\ a^H &= a^L+\int_{z^H}^{\overline{z}}\left(\omega_H\left(z\right)-p^H\right)dz-\int_{z^L}^{\overline{z}}\left(\omega_H\left(z\right)-p^L\right)dz \end{aligned}$$

It is straightforward to verify that IR_H and IC_L are also satisfied. The binding IC_H constraint implies that the information rent is

$$\int_{z^L}^{\overline{z}}\left(\omega_H\left(z\right)-p^L\right)dz-\int_{z^L}^{\overline{z}}\left(\omega_L\left(z\right)-p^L\right)dz=\int_{z^L}^{\overline{z}}\left(\omega_H\left(z\right)-\omega_L\left(z\right)\right)dz.$$

The seller's revenue is then the difference between the expected total trading surplus over all ex ante types and the information rent. It is immediate from the expression (18) that the hypothetical revenue is attained by the same pair of monotone partitions. Since the seller can always discard information about z, the hypothetical revenue is clearly a revenue upper-bound for the original setting. Hence, this pair of monotone partitions is optimal among all disclosure policies.

To attain the maximal profit by non-discriminatory disclosure, the seller can reveal to all buyer types the partition of $\left\{\left[\underline{z}, z^H\right], \left[z^H, z^L\right], \left[z^L, \overline{z}\right]\right\}$, and set $p^H = \omega_H\left(z^H\right)$ and $p^L = \mathbb{E}[\omega_L\left(z\right)|z \geq z^L]$.

6.9 Proof of Proposition 6

Let (a, p) and (σ^H, σ^L) denote an optimal mechanism without price discrimination. We argue that we must have $p > c$. Suppose by contradiction $p \leq c$. By assumption of

$c \leq \underline{\omega}$, both buyer types would buy regardless of the disclosure policy, so the seller's profit under (a,p) with $p \leq c$ is $a+p-c$. It follows from IR_L that $a+p-c$ is bounded above by

$$\int_{\underline{\omega}}^{\overline{\omega}} (\omega - p) f_L(\omega) d\omega + p - c = \int_{\underline{\omega}}^{\overline{\omega}} \omega f_L(\omega) d\omega - c.$$

Consider an alternative mechanism with $\hat{\sigma}^H(\omega) = 1$ for all ω, $\hat{\sigma}^L(\omega) = \mathbf{1}\{\omega \geq k\}$, $\hat{a} = 0$ and $\hat{p} = \mathbb{E}_L[\omega | \omega \geq k]$, which is feasible and incentive compatible. The seller's profit is

$$\phi_L(\hat{p} - c)[1 - F_L(k)] + \phi_H(\hat{p} - c)$$

If $k = c$, it replicate the profit under mechanism $\{(a,p), (\sigma^H, \sigma^L)\}$. Its derivative with respect to k, evaluated at $k = c$, is

$$\phi_H \frac{f_L(c)}{1 - F_L(c)} (\mathbb{E}_L[\omega | \omega \geq c] - c) > 0.$$

Therefore, the alternative mechanism $\{(\hat{a}, \hat{p}), (\hat{\sigma}^H, \hat{\sigma}^L)\}$ with $k > c$ can generate a strictly higher profit than mechanism $\{(a,p), (\sigma^H, \sigma^L)\}$. A contradiction to the optimality of $\{(a,p), (\sigma^H, \sigma^L)\}$. Thus, we have $p > c$ in the optimal pricing scheme.

If only the high type participates in this optimal scheme, then the seller can recommend that the high type buys if and only if $\omega \geqslant c$ and can let a be zero and let p be the high type's expected value conditional on $\omega \geqslant c$. This optimal scheme can be implemented without information discrimination. Hence, from now on, we focus on the parameter region in which it is optimal to serve both types.

First, consider the relaxed problem without IC_L constraint. For each $\theta \in \{H, L\}$, we let $\underline{t}^\theta$ and $\bar{t}^\theta$ be such that $\underline{t}^\theta \leqslant p \leqslant \bar{t}^\theta$, and

$$\int_{\underline{t}^\theta}^{p} f_\theta(\omega)(\omega - p) d\omega = \int_{\underline{\omega}}^{p} f_\theta(\omega) \sigma^\theta(\omega)(\omega - p) d\omega,$$
$$\int_{p}^{\bar{t}^\theta} f_\theta(\omega)(\omega - p) d\omega = \int_{p}^{\overline{\omega}} f_\theta(\omega) \sigma^\theta(\omega)(\omega - p) d\omega.$$

By definition, the high type's payoff stays the same if he buys when the value is in the interval $[\underline{t}^H, \bar{t}^H]$, and the low type's payoff stays the same if he buys when the value is in the interval $[\underline{t}^L, \bar{t}^L]$. The IR constraints ($\text{IR}_\theta$) and the obedience constraints (OB_θ) are unaffected.

We claim that the above concentration makes the high type less willing to mimic low type:

$$\int_{\underline{\omega}}^{\overline{\omega}} f_H(\omega)\sigma^L(\omega)(\omega-p)d\omega \geq \int_{\underline{t}_L}^{\bar{t}_L} f_H(\omega)(\omega-p)d\omega.$$

To see this, suppose that σ^L has two values $p<\omega_1<\omega_2$ such that $\sigma^L(\omega_1)<1$ and $\sigma^L(\omega_2)>0$. We can now change the probabilities of buying at (ω_1,ω_2) to

$$(\sigma^L(\omega_1)+\varepsilon_1, \sigma^L(\omega_2)-\varepsilon_2),\ \text{for some } \varepsilon_1>0, \varepsilon_2>0,$$

so that type L is indifferent. This means that

$$\varepsilon_1(\omega_1-p)f_L(\omega_1)-\varepsilon_2(\omega_2-p)f_L(\omega_2)=0,$$

which implies that

$$\varepsilon_2=\varepsilon_1\frac{(\omega_1-p)f_L(\omega_1)}{(\omega_2-p)f_L(\omega_2)}.$$

The high type's payoff from mimicking the low type, under this new mechanism, will change by

$$\varepsilon_1(\omega_1-p)f_H(\omega_1)-\varepsilon_2(\omega_2-p)f_H(\omega_2)=\varepsilon_1(\omega_1-p)\left(f_H(\omega_1)-f_H(\omega_2)\frac{f_L(\omega_1)}{f_L(\omega_2)}\right)\leqslant 0,$$

where the inequality follows from the MLRP because by assumption $\omega_1<\omega_2$. Hence, IC_H constraint is relaxed after concentration.

The change from $\sigma^\theta(\omega)$ to $[\underline{t}^\theta, \bar{t}^\theta]$ also increases the seller's profit from each type $\theta=H,L$ by increasing the probability that type θ buys the good. To see this, note that from the definition of $\bar{t}^\theta$ we have

$$\int_p^{\bar{t}^\theta}(\omega-p)(1-\sigma^\theta(\omega))f_\theta(\omega)d\omega=\int_{\bar{t}^\theta}^{\overline{\omega}}(\omega-p)\sigma^\theta(\omega)f_\theta(\omega)d\omega.$$

Thus,

$$(\bar{t}^\theta-p)\int_p^{\bar{t}^\theta}(1-\sigma^\theta(\omega))f_\theta(\omega)d\omega \geq (\bar{t}^\theta-p)\int_{\bar{t}^\theta}^{\overline{\omega}}\sigma^\theta(\omega)f_\theta(\omega)d\omega,$$

which implies that

$$\int_p^{\bar{t}^\theta} f_\theta(\omega)d\omega \geq \int_p^{\overline{\omega}}\sigma^\theta(\omega)f_\theta(\omega)d\omega.$$

Similarly, from the definition of $\underline{t}^\theta$ we have

$$\int_{\underline{t}^\theta}^{p} f_\theta(\omega)d\omega \geq \int_{\underline{\omega}}^{p} \sigma^\theta(\omega) f_\theta(\omega)d\omega,$$

and thus

$$\int_{\underline{t}^\theta}^{\bar{t}^\theta} f_\theta(\omega)d\omega \geq \int_{\underline{\omega}}^{\overline{\omega}} f_\theta(\omega)\sigma^\theta(\omega)d\omega.$$

Since $p \geq c$, the seller's profit is higher when type θ buys more often. It follows that in the relaxed problem the solution in σ^θ is given by $\sigma^\theta(\omega) = 1$ for $\omega \in [\underline{t}^\theta, \bar{t}^\theta]$ for each $\theta = H, L$.

Second, we argue that the solution to the relaxed problem has the nested interval structure with $\underline{t}^H \leqslant \underline{t}^L$ and $\bar{t}^H \geqslant \bar{t}^L$. Suppose by contradiction that the intervals are not nested as claimed. It is sufficient to rule out the following three cases:

(i) If $\underline{t}^H \leqslant \underline{t}^L$ and $\bar{t}^H < \bar{t}^L$, we can extend the buy interval for the high type to $[\underline{t}^H, \bar{t}^L]$. Since we added some signal realizations above p to the high type's buy interval, both IR_H and IC_H are still satisfied. Furthermore, the OB_L constraint is unaffected while the OB_H constraint is relaxed after the change. The seller's profit increases, contradicting the assumption that (σ^L, σ^H) is part of the solution to the relaxed problem.

(ii) If $\underline{t}^H > \underline{t}^L$ and $\bar{t}^H \geqslant \bar{t}^L$, we can extend the intervals for both types to $[\underline{t}^L, \overline{\omega}]$. The IR_L constraint is still satisfied, since we added some sigal realizations above p to the low type's interval, while the IR_H constraint is also satisfied because the high type's buying probability is weakly higher than the low type's due to likelihood ratio dominance. The IC_H constraint is trivally satisfied, and both obedience constraints are weakly relaxed. The seller's profit increases, contradicting the assumption that (σ^L, σ^H) is part of the solution to the relaxed problem.

(iii) If $\underline{t}^H \geq \underline{t}^L$ and $\bar{t}^H \leqslant \bar{t}^L$ with at least one inequalities holding strictly, we can again extend the interval for both types to $[\underline{t}^L, \overline{\omega}]$. As in case (ii), all constraints are still satisfied. The seller's profit increases, contradicting the assumption that (σ^L, σ^H) is part of the solution to the relaxed problem.

Third, we claim that the solution to the relaxed problem satisfies the dropped IC_L constraint and hence also solves the original problem. Suppose that IC_L is violated.

Since the relaxed solution satisfies OB_L, a violation of IC_L implies that

$$\int_{\underline{t}_H}^{\bar{t}_H} f_L(\omega)(\omega - p)d\omega > \int_{\underline{t}_L}^{\bar{t}_L} f_L(\omega)(\omega - p)d\omega \geq 0.$$

Then the seller could just change σ^L to σ^H, which satisfies all constraints. Since type L now buys more often and since $p > c$, the seller's revenue is increased, a contradiction to assumption that we have a solution to the relaxed problem.

Finally, we argue that the seller's maximal profit generated by optimal discriminatory disclosure can be achieved by a non-discriminatory information disclosure policy. Since the buy intervals are nested with $\underline{t}^H \leqslant \underline{t}^L$ and $\bar{t}^H \geqslant \bar{t}^L$, the seller can reveal to both buyer types whether ω is in $[\underline{t}^L, \bar{t}^L]$ or in $[\underline{t}^H, \underline{t}^L] \cup [\bar{t}^L, \bar{t}^H]$ or in $[\underline{\omega}, \overline{\omega}] \setminus [\underline{t}^H, \bar{t}^H]$. Both types are willing to buy when $\omega \in [\underline{t}^L, \bar{t}^L]$. By the IC_H constraint, we have

$$\int_{\omega \in [\underline{t}^H, \underline{t}^L] \cup [\bar{t}^L, \bar{t}^H]} f_H(\omega)(\omega - p)d\omega \geq 0,$$

so the high type is also willing to buy if $\omega \in [\underline{t}^H, \underline{t}^L] \cup [\bar{t}^L, \bar{t}^H]$. Hence, for each type, this non-discriminatory information policy induces a trading probability weakly higher than the one under the optimal discriminatory disclosure. Since $p > c$ and the optimal discriminatory disclosure weakly dominates the non-discriminatory one, this non-discriminatory information policy must yield the same profit as the optimal discriminatory disclosure.

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