Wealth Dynamics in Communities

Daniel Barron, Yingni Guo, and Bryony Reich*

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Abstract

This paper develops a model to explore how favor exchange influences wealth dynamics. We identify a key obstacle to wealth accumulation: wealth crowds out favor exchange. Therefore, households must choose between growing their wealth and accessing favor exchange. We show that low-wealth households rely on favor exchange at the cost of having tightly limited long-term wealth. As a result, initial wealth disparities persist and can even grow worse. We then explore how communities and policymakers can overcome this obstacle. Using simulations, we show that community benefits and place-based policies can stimulate both saving and favor exchange, and in some cases, can even transform favor exchange into a force that accelerates wealth accumulation.

*Barron (Corresponding Author): Kellogg School of Management, Northwestern University, Kellogg Global Hub, 2211 Campus Drive, Evanston, IL 60208 USA, d-barron@kellogg.northwestern.edu; Guo: Department of Economics, Northwestern University, yingni.guo@northwestern.edu; Reich: Kellogg School of Management, Northwestern University, bryony.reich@kellogg.northwestern.edu. We thank Nageeb Ali, Renee Bowen, Matthias Fahn, George Georgiadis, Ben Golub, Sanjeev Goyal, Maria Guadalupe, Marina Halac, Oliver Hart, Rachel Kranton, Judith Levi, Dilip Mookherjee, Andrew Newman, Michael Powell, Benjamin Roth, Kathryn Spier, and the audiences at Aalto, Berkeley, SIOE 2020, the Harvard Contracts Lunch, the Canadian Economic Theory Conference virtual seminar series, SITE 2021, ThReD 2021, Johannes Kepler University, Northwestern University, and Pennsylvania State University for comments. We thank Edwin Muñoz-Rodriguez for excellent research assistance.
1 Introduction

Rising economic tides do not lift all households equally. Even in growing economies, some communities are left behind, with persistently lower wealth than surrounding areas. Such left-behind communities can be found in both rich and developing countries, and in both rural and urban areas.¹

Faced with limited wealth, members of left-behind communities rely on one another for practical support (e.g., Kranton 1996, Ali and Miller 2016). Community members engage in all kinds of favor exchange, from the trade of food, lodging, and childcare within poor neighborhoods in Milwaukee (Desmond 2012, 2016) to the exchange of rice and kerosene among villagers in India (Jackson et al. 2012). By allowing households to procure goods and services without spending money, in-kind favor exchange frees up resources and therefore has the potential to lead to faster wealth accumulation. Yet in practice, favor exchange does not readily translate into growing wealth. Households instead struggle to “get by” (Warren et al. 2001), even when they have access to high-return savings opportunities like making productive investments or paying off high-interest debts (Ananth et al. 2007, Stegman 2007, Bernheim et al. 2015). Given that favor exchange frees up money that can then be saved, and given the presence of high-return savings opportunities, why doesn’t community support translate into growing wealth?

This paper develops a model to study how favor exchange shapes wealth dynamics. While favor exchange could, in principle, encourage saving, it suffers from a commitment problem: households can renege on promised favors and instead use money alone to meet their needs. We show that this commitment problem prevents favor exchange from encouraging saving. Even worse, wealth actually crowds out favor exchange, so that households must keep their wealth artificially low in order to access favor exchange. The result is a persistent wealth gap between left-behind communities and the rest of the economy. We then explore how commu-

nities and policymakers can mitigate this tension between favor exchange and saving, and potentially even transform favor exchange into a force that encourages wealth accumulation.

The foundation of our analysis is a model that introduces endogenous wealth dynamics into favor-exchange relationships. A household faces a standard consumption-saving problem, with the twist that it can “purchase” consumption using not only money but also promises of future in-kind favors. The household cannot commit to following through on these promises, so they must be credible in the context of an ongoing favor-exchange relationship. In contrast to the literature on informal lending (e.g., Bulow and Rogoff 1989, Ligon et al. 2000), favors are in-kind, so they add directly to the consumption utility of the household.

Using this model, we show that wealth undermines the trust that is essential for favor exchange. The reason is that even after losing access to favor exchange, households can use money to buy consumption. Wealthier households therefore have less to lose from not reciprocating favors; namely, they have better outside options to favor exchange. These households therefore have a weaker incentive to return favors. Recognizing this, community members are less willing to do favors for households that are expected to become wealthy in the future. In short, households that accumulate wealth become “too big for their boots” and lose access to community support.

Our main result characterizes how this “too-big-for-their-boots” mechanism constrains wealth accumulation and exacerbates long-term inequality. Equilibrium wealth dynamics are shaped by two forces. First, there is an initial selection effect: wealthy households opt out of favor exchange, while low-wealth households rely on it. Second, in response to the “too-big-for-their-boots” mechanism, households that rely on favor exchange engage in sharply constrained saving. Some households are so constrained that they even decrease their wealth over time. Consequently, for households that rely on favor exchange, long-term wealth remains substantially below what it would be without favor exchange. Together, the selection effect and the “too-big-for-their-boots” mechanism imply that initial wealth
disparities between households persist and can even grow worse over time.

The “too-big-for-their-boots” mechanism resonates with sociological and ethnographic evidence on favor exchange in left-behind communities. Support for this mechanism dates back at least to Stack [1975]’s classic study of favor exchange in a low-wealth U.S. community. Stack [1975, p43] notes that the wealthiest members of that community are most at risk of being excluded from favor exchange:

As people say, “The poorer you are, the more likely you are to pay back.” This criterion often determines which kin and friends are actively recruited into exchange networks.

To explain why wealthier households are excluded, Stack [1975] suggests that everyone knows that these households can more easily leave the community and move to a nearby city. Portes and Sensenbrenner [1993], Briggs [1998], Domínguez and Watkins [2003], and others echo this tension between favor exchange and outside options. Our main result characterizes the dynamic implications of the “too-big-for-their-boots” mechanism for households’ saving decisions.

Building on this result, we then explore how community leaders and policymakers can mitigate the “too-big-for-their-boots” mechanism. This mechanism arises because households cannot commit to repaying favors. We first prove that, if a household were able to commit, then favor exchange would unambiguously encourage wealth accumulation instead of discouraging it. This result suggests that easing the commitment problem can transform favor exchange into a force that accelerates wealth accumulation.

Using simulations, we show that communities can unlock this transformative effect by making non-favor-exchange benefits, such as participation in family, social, or religious activities, contingent on repaying past favors. For communities that provide these types of benefits, favor exchange can help at least some low-wealth households catch up to their wealthier peers. We link this analysis to examples of communities that appear to have successfully transformed favor exchange in this way.
Policymakers can similarly mitigate the “too-big-for-their-boots” mechanism using place-based policies, which provide benefits that are localized to a particular community (Austin et al. 2018). Using simulations, we show that these policies can relax the commitment problem and lead to higher long-term wealth in the community. While such policies cannot completely eliminate the wealth gap between low-wealth households and their wealthier peers, they can help left-behind communities by narrowing that gap.

A key feature of our model is that favor exchange is in-kind: community members trade consumption goods and services rather than borrowing and lending money. This distinction separates our paper from the literature on informal lending (e.g., Bulow and Rogoff 1989, Ligon et al. 2000). We show that it is exactly this in-kind nature that allows favor exchange to flourish. Indeed, if favors were instead monetary, then favor exchange would be impossible in our setting. This impossibility result is almost identical to the impossibility result in Bulow and Rogoff [1989]; it follows from the fact that when a household borrows money (i.e., receives monetary favors), it necessarily gets wealthier and so becomes less reliant on monetary favor exchange in the future. Monetary favor exchange is therefore self-defeating. In contrast, in-kind favor exchange is self-sustaining, because the household can engage in substantial favor exchange without becoming any wealthier.

Related Literature

This paper makes three contributions. First, we build a model that incorporates endogenous wealth dynamics into in-kind favor exchange. Second, we characterize the “too-big-for-their-boots” mechanism and demonstrate how it discourages saving and exacerbates wealth inequality. Third, we identify policies and community characteristics that can mitigate the “too-big-for-their-boots” mechanism and encourage wealth accumulation. We discuss the literature related to the first two contributions here, deferring the discussion of the third contribution to Section 5.

Our modeling approach draws from the literature on community enforcement and favor
exchange (e.g., Hauser and Hopenhayn 2008, Jackson et al. 2012, Ambrus et al. 2014, Ali and Miller 2016 and 2020, Miller and Tan 2018, Sugaya and Wolitzky 2021), and the literature on relational contracting (e.g., Levin 2003, Malcolmson 2012). Our key departure from these papers is to introduce wealth as an endogenous state variable. This addition gives rise to the wealth dynamics that are at the core of our analysis.

In our model, the “too-big-for-their-boots” mechanism arises because market exchange acts as an endogenous outside option to favor exchange. Thus, our analysis builds on papers that study the role of outside options in relationships (e.g., Baker et al. 1994, Kovriyukh 2013), as well as papers that study how favor and market exchange interact (e.g., Kranton 1996, Banerjee and Newman 1998, Gagnon and Goyal 2017, Banerjee et al. 2020, Jackson and Xing 2020). Unlike those papers, the outside option in our setting evolves based on the household’s saving decisions. This dynamic feedback is what leads to the “too-big-for-their-boots” mechanism. More distantly related are papers that study other types of distortions in communities, such as those that arise from signaling (e.g., Austen-Smith and Fryer 2005) or hold-up (e.g., Hoff and Sen 2006).

Our main result shows how persistent wealth inequality arises from the combination of a selection effect and the “too-big-for-their-boots” mechanism. The selection effect says that wealthy households opt out of favor exchange. By identifying how the value of favor exchange varies with wealth, this effect relates to papers that consider the costs and benefits of being part of a close-knit community (e.g., Kranton 1996, Banerjee and Newman 1998) and the role of wealth in migration decisions (e.g., Munshi and Rosenzweig 2016). On its own, however, the selection effect is silent about how favor exchange influences saving decisions within communities. The “too-big-for-their-boots” mechanism addresses this gap by identifying a hidden cost of saving, which is that it undermines favor exchange.

Our analysis provides an explanation for the well-documented fact that poverty can persist even in the presence of high-return savings opportunities (e.g., Ananth et al. 2007, Karlan et al. 2019). In contrast to other explanations of under-saving, our mechanism does
not rely on fixed costs of investment (e.g., Nelson 1956, Advani 2019), monopolistic credit markets (e.g., Mookherjee and Ray 2002, Liu and Roth 2022), or behavioral preferences (e.g., Banerjee and Mullainathan 2010, Bernheim et al. 2015). Thus, while we share Advani [2019]'s emphasis on favor exchange and Liu and Roth [2022]'s focus on outside options, our mechanism leads to persistently low wealth even in the absence of frictions like fixed-cost investments (in contrast to Advani 2019) or monopolistic credit markets (in contrast to Liu and Roth 2022). By connecting a household’s saving decisions to its ability to engage in favor exchange, our mechanism leads to novel empirical predictions and policy prescriptions.

2 Model

A long-lived household (“it”) has initial wealth $w_0 \geq 0$ and discount factor $\delta \in (0, 1)$. The household starts in a community. At the beginning of each period $t \in \{0, 1, \ldots\}$, if the household still lives in the community, then it can choose to either stay or move to a city. Once it moves to the city, it remains there forever.

If the household is in the community in period $t$, then it plays the following community game with a short-lived neighbor $t$ (“she”), who is another member of the community:

1. The household requests a consumption level $c_t \geq 0$ and offers a payment $p_t \geq 0$ in exchange.\footnote{Restricting $p_t$ to be non-negative means that the household cannot borrow money from neighbor $t$. This restriction is not important for our main result; Proposition 2 still holds if we allow $p_t < 0$, and the proof of this result goes through without change.} The payment cannot exceed the household’s wealth, $p_t \leq w_t$.

2. Neighbor $t$ either accepts or rejects this exchange, $d_t \in \{1, 0\}$. If she accepts ($d_t = 1$), then she receives $p_t$ and incurs the cost of providing $c_t$. If she rejects ($d_t = 0$), then no trade occurs.

3. The household decides how much of a favor, $f_t \geq 0$, to perform for neighbor $t$. The household incurs the cost of providing $f_t$. 
4. The household invests its remaining wealth, \( w_t - p_t d_t \), to generate \( w_{t+1} \). Let \( R(\cdot) \) denote the return on investment, so that \( w_{t+1} = R(w_t - p_t d_t) \).

Let \( U(\cdot) \) be the household’s consumption utility in the community. The household’s period-\( t \) payoff in the community is \( \pi_t = U(c_t d_t) - f_t \). Neighbor \( t \)’s payoff is \( (p_t - c_t) d_t + f_t \). The community is tight-knit and so all actions are observed by all neighbors.

We assume that consumption utility \( U(\cdot) \) and investment returns \( R(\cdot) \) are strictly increasing, with \( U''(\cdot) \) and \( R'(\cdot) \) continuous, \( U(0) = R(0) = 0 \), \( U(\cdot) \) strictly concave, \( R(\cdot) \) concave, \( \lim_{c \downarrow 0} U'(c) = \infty \), and \( \lim_{c \to \infty} U'(c) = 0 \). We say that investment gives positive returns at investment level \( w \) if \( R'(w) \geq \frac{1}{\delta} \). We assume that \( R'(w) > \frac{1}{\delta} \) for \( w < \bar{w} \) and \( R'(w) = \frac{1}{\delta} \) for \( w \geq \bar{w} \), so that investment generates positive returns at all investment levels and strictly so below a threshold \( \bar{w} > 0 \). Investment returns are deterministic in this model; we discuss the case with stochastic returns in Section 7 and analyze this case in Online Appendix C.

If the household has moved to the city by period \( t \), then it plays the city game with a short-lived vendor \( t \) (“she”), who has the same actions and payoff as neighbor \( t \). The city game is identical to the community game in all but two ways. First, each vendor observes only her own interaction with the household, so interactions are anonymous in the city. Second, the household’s marginal utility of consumption is weakly higher in the city. Formally, the household’s period-\( t \) payoff in the city is \( \pi_t = \hat{U}(c_t d_t) - f_t \), where \( \hat{U}(\cdot) \) satisfies the same regularity conditions as \( U(\cdot) \), with \( \hat{U}'(c) \geq U'(c) \) for all \( c > 0 \).

The household’s continuation payoff at the beginning of period \( t \) is

\[
\Pi_t = (1 - \delta) \sum_{s=t}^{\infty} \delta^{s-t} \pi_s.
\]

We characterize household-optimal equilibria, which are the perfect Bayesian equilibria that maximize the household’s \textit{ex ante} expected payoff. Without loss of generality, we assume that the household leaves the community if it is indifferent between staying and leaving.

The following assumption ensures that in equilibrium, households that stay in the com-
munity have access to strictly positive-return investments.

**Assumption 1** Define \( \bar{c} > 0 \) as the solution to \( U'(\bar{c}) = 1 \). Then, \( R(\bar{w} - \bar{c}) > \bar{w} \).

In the context of Stack [1975], the household and neighbors are members of a low-income Midwestern community called “the Flats.” Members of this community regularly exchange food, clothing, childcare, lodging, and other goods and services \((c_t)\). To compensate one another, households can pay with money \((p_t)\) and “pay” with future favors \((f_t)\). For example, the recipient of childcare \((c_t > 0)\) can reciprocate with future childcare \((f_t > 0)\). These favors are in-kind, in the sense that they involve directly trading goods and services. Households accumulate wealth \((R(\cdot))\) by repaying high-interest debt or making other investments. The Flats is a tight-knit community and “everyone knows who is working, when welfare checks arrive, and when additional resources are available” (p. 37), as well as who has reneged on promised favors. Households in the Flats can move to a nearby city, Chicago, which harbors greater opportunities \((\hat{U}' \geq U')\) but separates them from their favor-exchange network.

In Online Appendices D.1, D.2, and D.3, we prove that our main findings are robust to relaxing several modeling assumptions. While we assume that moving to the city is irreversible, a version of our main result holds even if the household can return to the community after leaving. Neither is it essential for the household’s cost of providing \(f_t\) to be linear; we prove a similar result if the cost of providing \(f_t\) is convex.\(^3\) Finally, we show that our analysis still holds if we replace the sequence of short-lived neighbors in the community with a single long-lived neighbor.

We model the city as an alternative location to the community. In Online Appendix D.4, we prove that our main result holds if we eliminate the city and instead assume that deviations are punished by reversion to a Markov perfect equilibrium in the community. Note that Markov perfect equilibrium is not necessarily the harshest way to punish the

\(^3\)As a special case, the cost of providing \(f_t\) could be \(U^{-1}(f_t)\), in which case neighbors value the favor in the same way as the household values consumption.
household.\(^4\) Subject to this caveat, we show that wealth crowds out favor exchange even in a model without the city, since wealthier households are better able to meet their needs using money alone. Given the equivalence between the two models, we include the city in our analysis in order to match our applications, which typically include mobility across locations, and to derive empirical and policy implications. In applications that do not have mobility across locations, one can interpret “leaving for the city” as opting out of favor exchange in the community.

3 Life in the City

We begin by characterizing wealth dynamics in the city. The household faces a standard consumption-saving problem. It takes full advantage of investment opportunities and accumulates wealth.

Interactions are anonymous in the city, so \(f_t = 0\) in equilibrium. Vendor \(t\) is therefore willing to accept an offer only if the payment covers her cost (i.e., \(p_t \geq c_t\)), and strictly prefers to do so if \(p_t > c_t\). Consequently, every equilibrium entails \(p_t = c_t\) in every \(t \geq 0\), so that \(w_{t+1} = R(w_t - c_t)\). For a household with wealth \(w\), the resulting optimal payoff and consumption are given, respectively, by:

\[
\hat{\Pi}(w) = \max_{c \in [0,w]} \left( (1 - \delta)\hat{U}(c) + \delta \hat{\Pi}(R(w - c)) \right) \quad \text{and} \\
\hat{C}(w) \in \arg \max_{c \in [0,w]} \left( (1 - \delta)\hat{U}(c) + \delta \hat{\Pi}(R(w - c)) \right).
\]

Our first result shows that in any equilibrium of the city, the household consumes \(\hat{C}(w_t)\) and its payoff is \(\hat{\Pi}(w_t)\). Moreover, both consumption and wealth increase over time, with long-term wealth above \(R(\bar{w})\).

\(^4\)Online Appendix D.4 also presents an alternative model that allows us to dispense with the assumption that deviations are punished by reversion to Markov perfect equilibrium.
Proposition 1 Both $\hat{\Pi}(\cdot)$ and $\hat{C}(\cdot)$ are strictly increasing, with $\hat{\Pi}(\cdot)$ continuous. In any equilibrium of the city, $\Pi_t = \hat{\Pi}(w_t)$ and $c_t = \hat{C}(w_t)$ in any $t \geq 0$, with $(w_t)_{t=0}^{\infty}$ increasing and

$$\lim_{t \to \infty} w_t \geq R(\bar{w})$$
on the equilibrium path.

The proof of Proposition 1 is routine and relegated to Appendix B. Since $R'(w) > \frac{1}{\delta}$ for $w < \bar{w}$, the standard Euler equation,

$$\hat{U}'(\hat{C}(w_t)) = \delta R'(w_t - \hat{C}(w_t))\hat{U}'(\hat{C}(w_{t+1})), \ \forall \ t,$$

(Euler)

implies that the household’s long-term wealth is at least $R(\bar{w})$ in the city. Figure 1 simulates equilibrium outcomes in the city.\(^5\)

![Figure 1: Left panel: the household’s equilibrium payoff and consumption in the city as a function of $w$. Right panel: consumption and wealth over time in the city, starting at $w_0 = 0.006$.](image)

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\(^5\)Parameters in Figures 1 to 3 are $\delta = \frac{8}{10}, U(c) = \frac{\sqrt{c}}{2}, \hat{U}(c) = \frac{13\sqrt{c}}{25}$, and

$$R(w) = \begin{cases} 
3\left(\sqrt{w+1} - 1\right) & w \leq \frac{11}{25}; \\
\frac{3w}{4} + \frac{1}{20} & \text{otherwise}.
\end{cases}$$
4 Wealth Dynamics in the Community

This section characterizes household-optimal equilibria for a household starting in the community. Section 4.1 presents our main result, which shows that households that rely on favor exchange have sharply limited long-term wealth. Section 4.2 discusses empirical implications of this result.

4.1 Underinvestment and Persistent Inequality

Our main result identifies two reasons why wealth in the community remains substantially below wealth in the city. First, there is a selection effect: sufficiently wealthy households leave the community, whereas poorer households stay. Second, the “too-big-for-their-boots” mechanism constitutes an extra cost of investment in the community, resulting in sharply limited long-term wealth for households that stay.

The community’s sole advantage over the city is that favor exchange can augment consumption in the community but not in the city. In particular, neighbors are able to observe and punish a household that reneges on \( f_t > 0 \). Consequently, the household can credibly promise \( f_t > 0 \) to repay neighbor \( t \) for providing consumption. Given a credibly promised favor \( f_t > 0 \), neighbor \( t \) is willing to accept any request that satisfies \( c_t \leq p_t + f_t \).

To understand the selection effect, note that the opportunity to engage in favor exchange is most attractive to low-wealth households, which would otherwise have low consumption and a high marginal utility of consumption. Conversely, wealthy households already consume a lot, so their marginal utility from further increasing \( c_t \) is low. Thus, wealthy households leave the community while low-wealth households stay.

To understand the “too-big-for-their-boots” mechanism, consider a household with wealth \( w_t \) that stays in the community, consumes \( c_t = p_t + f_t \), and invests \( I_t = w_t - p_t \). Since the household can renge on \( f_t \), leave the community, and earn \( \Pi(R(I_t)) \) in the city, the maximum favor that can be sustained in equilibrium depends on the investment \( I_t \). Denote
a household’s maximum continuation payoff if it is in the community and has wealth $R(I_t)$ by $\Pi^*(R(I_t))$. Then, $f_t$ must satisfy the following dynamic enforcement constraint:

$$f_t \leq \bar{f}(I_t) = \frac{\delta}{1 - \delta} (\Pi^*(R(I_t)) - \tilde{\Pi}(R(I_t))). \quad (DE)$$

This constraint ensures that the household prefers to do favor $f_t$ and earn continuation payoff $\Pi^*(R(I_t))$, rather than reneging on $f_t$ and earning punishment payoff $\tilde{\Pi}(R(I_t))$.

If (DE) binds, then changing $I_t$ affects the size of the favor, $f_t$, which affects period-$t$ consumption because $c_t = p_t + f_t$. The “too-big-for-their-boots” mechanism holds when $\bar{f}(I_t)$ is decreasing in $I_t$, so that investment crowds out favor exchange. When this occurs, the standard cost-benefit trade-off captured by the Euler equation, (Euler), is augmented by an extra cost: the favor $f_t$, and so consumption $c_t$, decrease as investment increases. Consequently, the household optimally invests less than the investment that would satisfy (Euler). This is the sense in which a household in the community underinvests.

This intuition elides a key complication: $\bar{f}(\cdot)$ depends on both $\Pi^*(\cdot)$ and $\tilde{\Pi}(\cdot)$, which in turn depend on the household’s future decisions about consumption, favor exchange, and investment. Wealth affects all of these decisions, rendering a full characterization of household-optimal equilibria intractable. The central difficulty is that the presence of equilibrium continuation payoffs in (DE) rules out standard dynamic programming techniques.\(^6\)

Given this challenge, our main result, Proposition 2, focuses on the selection effect and the “too-big-for-their-boots” mechanism. Selection is summarized by a wealth threshold, $w^{se} < \bar{w}$, and a set $W \subseteq [0, w^{se}]$ with $\sup W = w^{se}$. The household stays forever if $w_0 \in W$ and otherwise leaves the community immediately. The “too-big-for-their-boots” mechanism is summarized by a wealth level, $w^{tr} < w^{se}$, such that the long-term wealth of a household that stays in the community is below $w^{tr}$. Since $w^{tr} < w^{se} < R(\bar{w})$, the long-term wealth in the

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\(^6\)Marcet and Marimon [2011] develop recursive techniques that can handle endogenous state variables (i.e., wealth) and forward-looking constraints (i.e., (DE)). While these techniques are a powerful tool for computation and certain kinds of partial characterization, it is not clear to us how to apply them to strengthen our characterization. Our focus on the selection effect and the “too-big-for-their-boots” mechanism leads us to take a different approach to the proof.
community is substantially below the long-term wealth in the city. Moreover, a household that stays with wealth $w_0 \in (w^{tr}, w^{se})$ has strictly declining wealth over time. This is the sense in which wealth inequality persists and can grow worse over time.

**Proposition 2** Impose Assumption 1. There exist wealth levels $w^{tr}$ and $w^{se}$ satisfying $0 \leq w^{tr} < w^{se} < \bar{w}$, and a positive-measure set $\mathcal{W} \subseteq [0, w^{se}]$ with $\sup \mathcal{W} = w^{se}$, such that in any household-optimal equilibrium:

1. **Selection.** The household stays in the community forever if $w_0 \in \mathcal{W}$, and otherwise leaves in period 0.

2. **“Too-big-for-their-boots” mechanism.** If the household stays in the community, then $(w_t)_{t=0}^\infty$ is monotone, with

$$\lim_{t \to \infty} w_t \leq w^{tr}.$$  

Moreover, $\mathcal{W} \cap [w^{tr}, w^{se}]$ has a positive measure.

The proof of this result is in Appendix A. We have already argued that sufficiently wealthy households leave the community, giving us the selection threshold $w^{se}$, while some poorer households stay, giving us the set $\mathcal{W}$. Moreover, any household that stays in $t = 0$ stays forever. The reason is that a household that leaves in period $t > 0$ cannot engage in favor exchange during period $(t - 1)$, so it might as well leave in period $(t - 1)$. Iterating this argument, such a household might as well leave in period 0.

To understand why long-term wealth in the community is below $w^{tr}$, consider a household that stays with wealth just below $w^{se}$. Such a household is close to indifferent between leaving and staying. Since staying implies that wealth remains below $w^{se}$, which is lower than long-term wealth in the city, this household optimally stays only if it engages in significant favor exchange in some period; that is, only if $f_t \gg 0$ in some $t$. If the household’s wealth always remains near $w^{se}$, then the right-hand side of the dynamic enforcement constraint, (DE), is close to zero, so $f_t \approx 0$ in every $t \geq 0$. Therefore, for $f_t \gg 0$ to be sustainable in equilibrium,
the household must underinvest so severely that its wealth decreases. We conclude that long-term wealth in the community is below some level, \( w^{tr} \), which is below the selection threshold \( w^{se} \) and so sharply below the long-term wealth in the city. The proof of Proposition 2 strengthens this result by showing that \((w_t)_{t=0}^\infty\) is monotone in the community and that a positive measure of households, \( W \cap [w^{tr}, w^{se}] \), stay in the community. These households have decreasing wealth.

Figure 2: Household-optimal equilibrium payoffs and wealth dynamics.

Figure 2 summarizes Proposition 2. In this simulation, the household moves to the city if \( w_0 \geq w^{se} \) and otherwise stays in the community. Among those that stay, households with \( w_0 < w^{tr} \) grow their wealth, but only up to \( w^{tr} \). Those with \( w_0 \in (w^{tr}, w^{se}) \) have declining wealth over time.\(^7\)

4.2 Empirical Implications

An immediate implication of Proposition 2 is that one-time transfers do not necessarily improve long-term wealth. In Figure 2, consider transferring money to a household with initial wealth \( w_0 < w^{se} \). If the household’s post-transfer wealth is still below \( w^{se} \), then this

\(^7\)Note that in this simulation, \( \Pi^*(w) > \hat{\Pi}(w) \) for all \( w < w^{se} \) and \( \Pi^*(w) = \hat{\Pi}(w) \) for all \( w \geq w^{se} \). This property holds in every simulation that we have run, but we have not been able to prove that it is true in general.
extra money is consumed rather than saved, and long-term wealth remains below $w^{tr}$. This implication resonates with Karlan et al. [2019], who study what happens when indebted individuals receive one-time debt relief. Consistent with Figure 2, they find that recipients tend to quickly fall back into debt. In contrast, a transfer that is large enough to bring wealth above $w^{se}$ induces further investment, but only by spurring the household to opt out of favor exchange.

Figure 3: Investment in the city versus in the community.

We use numerical simulations to explore further implications of our model. Figure 3 identifies a key non-monotonicity in how favor exchange affects wealth accumulation. This figure plots investment as a function of the household’s current wealth both in the city (the blue sparsely-dotted curve) and in the community (the green densely-dotted curve). As expected, the wealthier households in the community underinvest relative to the city; however, the poorest households invest strictly more than they would in the city. The reason is that the outside option for the poorest households is so low that the dynamic enforcement constraint, (DE), does not bind. These households optimally invest all of their wealth and rely on favor exchange to meet their needs. Of course, these households eventually reach wealth levels where (DE) binds, at which point the “too-big-for-their-boots” mechanism kicks in and limits their long-term wealth. Nevertheless, this simulation result shows that favor exchange has the potential to free up resources and thereby accelerate wealth accumulation. We explore this idea further in Section 5.
Figure 4: Simulated comparative statics with respect to \( \hat{U}(\cdot) \). To create “higher \( \hat{U} \),” we multiply “lower \( \hat{U} \)” by a constant strictly larger than 1.

Our model also sheds light on how changes in more prosperous areas can spill over into left-behind communities. For instance, productivity has diverged across localities in the United States over the past 20 years, with the most productive metropolitan areas growing even more productive relative to the rest of the country (Parilla and Muro 2017). Figure 4 illustrates how growing productivity in the city, which we model by multiplying \( \hat{U} \) by a constant strictly larger than 1, affects wealth dynamics in the community.\(^8\) While nothing material has changed within the community, increasing \( \hat{U} \) in this way translates into a higher outside option and so tightens (DE). Households in the community respond by investing less, leading to lower long-term wealth, \( w^{tr} \).

Figure 5 shows that increasing the returns to investment, \( R(\cdot) \), can tighten (DE) in a way that is similar to increasing \( \hat{U} \).\(^9\) The reason is that increasing \( R(\cdot) \) disproportionately benefits

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\(^8\)The simulation for “lower \( \hat{U} \)” uses parameters identical to those in Figures 1-3. For “higher \( \hat{U} \),” parameters are the same except that \( \hat{U}(c) = \frac{2\sqrt{c}}{30} \).

\(^9\)The line labeled “lower \( R(\cdot) \)” uses parameters identical to those in Figures 1-3. For “higher \( R(\cdot) \),” the only difference is that \( R(\cdot) \) is given by

\[
R(w) = \begin{cases} 
\frac{31}{10} \left( \sqrt{w + \frac{1}{2}} - 1 \right) + \frac{33}{125} & w \leq \frac{336}{625} \\
\frac{2}{5} \left( w - \frac{336}{625} \right) + \frac{93}{125} & \text{otherwise.}
\end{cases}
\]

This investment return function is obtained by multiplying the investment return function for “lower \( R(\cdot) \)” by a constant strictly greater than 1, then adjusting the resulting function so that it has a well-defined wealth level \( \bar{w} \) above which \( R'(w) = 1/\delta \).
a household in the city, since investment is higher in the city than in the community. For many households in the community, increasing $R(\cdot)$ therefore decreases equilibrium payoffs. Note, however, that this effect is not uniformly negative in the community. A household with $w_0$ close to 0 benefits from higher $R(\cdot)$, since it invests substantially while facing a relatively slack (DE) constraint.

5 Transforming Favor Exchange

While favor exchange increases consumption in left-behind communities, the accompanying “too-big-for-their-boots” mechanism is a serious obstacle to wealth accumulation. In this section, we explore how communities and policymakers can mitigate this obstacle, or even transform favor exchange into a force that accelerates wealth accumulation. Section 5.1 proves that with commitment, favor exchange leads to faster wealth accumulation than would occur in the city. Informed by this result, Section 5.2 explores practical ways to relax the household’s commitment problem.
5.1 With Commitment, Favor Exchange Accelerates Wealth Accumulation

Define the model with commitment identically to the baseline model in Section 2, except that the household can commit to following through on promised favors. Formally, if the household is still in the community at the start of period \( t \geq 0 \), then it chooses a promised favor, \( f_t^* \geq 0 \), at the same time as it chooses the payment, \( p_t \), and the consumption request, \( c_t \). The promised favor is observed by all neighbors. If neighbor \( t \) accepts the request, then the household is committed to following through on its promised favor: \( f_t = f_t^* \). If neighbor \( t \) rejects the request, then \( f_t = 0 \). We assume that favor exchange is not possible in the city, so as in Section 3, the household earns continuation payoff \( \hat{\Pi}(w_t) \) if it leaves the community at the start of period \( t \).

Recall that \( \hat{C}(w_t) \) is the household’s equilibrium consumption in the city, so that investment in the city equals \( w_t - \hat{C}(w_t) \). We say that favor exchange accelerates wealth accumulation if a household that stays in the community invests strictly more than it would in the city. Our next result shows that favor exchange unambiguously accelerates wealth accumulation in the model with commitment.

**Proposition 3** Consider a household in the community with initial wealth \( w_0 > 0 \). In the model with commitment, there exists a household-optimal equilibrium in which the household leaves at the start of period \( \tau < \infty \). For any \( t < \tau \), the household invests more than it would in the city by choosing \( I_t = w_t - p_t > w_t - \hat{C}(w_t) \). Moreover, for any \( t < \tau - 1 \), the household invests its entire wealth by choosing \( p_t = 0 \), so that \( I_t = w_t \).

The proof of Proposition 3 is in Appendix B. Favor exchange allows the household to meet its needs with favors and invest the money it would have otherwise used for consumption. Indeed, the household optimally invests its entire wealth in every period until it is about to leave the community. To see why, consider the problem of maximizing the household’s payoff while in the community, subject to holding fixed its continuation payoff upon leaving,
\( \hat{\Pi}(w\tau) \), and its total discounted cost of favors, \( \sum_{t=0}^{t-1} \delta^t f_t \). Holding this “budget” of favors fixed, the household can decrease \( f_t \) at rate 1 in order to increase \( f_{t+1} \) at rate \( \frac{1}{3} \). Holding \( w\tau \), and hence \( \hat{\Pi}(w\tau) \), fixed, the household can decrease payment \( p_t \) at rate 1 in order to increase \( p_{t+1} \) at rate \( R'(w_t - p_t) \). Since \( R'(w_t - p_t) \geq \frac{1}{3} \), the household gets higher returns from delaying monetary payments relative to delaying favors. It therefore relies entirely on favor exchange in early periods and entirely on monetary payments in later periods. Once it starts relying on monetary payments alone, the household optimally leaves the community.

5.2 Ways to Increase Long-term Wealth

Proposition 3 implies that relaxing the household’s commitment problem can mitigate the tension between favor exchange and investment. In this section, we show that both community leaders and policymakers have tools to accomplish this goal, although they do so in different ways.

The unifying feature of these tools is that they relax the dynamic enforcement constraint, (DE), by increasing the household’s on-path payoff relative to its payoff from reneging and leaving for the city. Communities can relax this constraint by directly conditioning non-favor-exchange benefits (e.g., family, social, or religious ties) on the household’s favor-exchange behavior. Policies cannot condition on a household’s favor-exchange behavior in this way, but they can still relax (DE) by increasing the benefits of staying in the community rather than leaving.

**Community Benefits:** Thus far in our model, the only reason for the household to repay favors is to access future favors. In reality, households sometimes repay favors to avoid losing access to other benefits that communities provide (Ambrus et al. 2014). They might do so to maintain warm relationships with their friends, to keep their family happy, or to remain a respected member of their social or religious community.

These types of community benefits have two properties that help transform favor ex-
change. First, benefits can be conditioned on the household’s behavior, and in particular, they can be withheld if the household reneges on a promised favor. Second, a household values these benefits even after it no longer relies on favor exchange. We therefore treat these benefits as being available in both the community and the city if and only if the household has not reneged.\footnote{As discussed in the final paragraph of Section 2, “moving to the city” can be interpreted as staying in the community but no longer engaging in favor exchange, rather than moving to a different location. Under this interpretation, these benefits include benefits that are available only within the community.}

Formally, we model community benefits as providing a per-period additive utility benefit, \( B > 0 \), so long as the household has not deviated. These benefits are lost following a deviation, so the household’s punishment payoff remains the same as in the baseline model. These benefits therefore relax the commitment problem by adding \( B \) to the right-hand side of the dynamic enforcement constraint, (DE).

Figure 6 presents simulated equilibrium wealth dynamics for various levels of \( B \).\footnote{These simulations use parameters identical to those in Figures 1-3, with values of \( B \) given in Figure 6.} Each figure gives wealth in period \( t+1 \) as a function of wealth in period \( t \), so that the household’s wealth is increasing in the community whenever the green densely-dotted line is above the orange dashed line and decreasing otherwise. The right-hand boundary of these figures corresponds to the selection threshold \( w^{se} \).

Panel I sets \( B = 0 \). The “too-big-for-their-boots” mechanism is apparent from the fact that \( w_{t+1} < w_t \) whenever \( w_t \in (w^{tr}, w^{se}) \). As we increase \( B \) in Panel II, new wealth dynamics emerge: wealthier households in the community now accumulate wealth until they eventually leave. Even when they are about to leave, these households can engage in some favor exchange, because they prefer repaying small favors to losing \( B \). This feature is what allows households to smoothly transition from the community to the city. As we will show, this feature is a key difference between community benefits and place-based policies.

Panel II shows that, as in Proposition 3, wealthier households in the community invest some of the money that is freed up by favor exchange and so grow their wealth strictly faster than they would in the city. In contrast, while low-wealth households also benefit
Figure 6: Wealth dynamics for various levels of community benefits. The x-axis is the household’s current wealth. The orange dashed line is the 45-degree line, corresponding to current wealth. Wealth is increasing in the community if the green densely-dotted line is above the orange dashed line and decreasing otherwise. Investment is higher than it would be in the city if the green densely-dotted line is above the blue sparsely-dotted line.

from $B > 0$, they still suffer from the “too-big-for-their-boots” mechanism and so still have substantially lower long-term wealth than in the city.

Panel III shows that large enough community benefits can completely transform favor exchange, so that the household invests more than it would in the city for any $w_t \in (0, w^{se})$. In this case, for any initial wealth $w_0$, the household has the same long-term wealth as it
would in the city. Large enough community benefits can therefore turn favor exchange into a force that helps left-behind communities catch up.

In practice, community benefits are likely to be large when members rely heavily on the community for social relationships. Portes and Sensenbrenner [1993] suggest that members of the Dominican community in New York were able to accumulate wealth because the community provided a valuable safe haven, a place of shared language and culture that was free from discrimination. In describing the economic success of the Chinese immigrant community in New York, Zhou [1992] highlights that members received similarly large benefits from the community. Religious organizations can also be a source of community benefits. Coleman [1988] notes that for the New York community of Jewish diamond merchants, the threat of social ostracism from synagogues and other religious activities supported business relationships.

**Place-Based Policies:** Policymakers also have tools that can relax the household’s commitment problem. However, unlike community benefits, public policies typically cannot condition on a household’s behavior in its favor-exchange relationships. The closest that a policy can come is to condition on whether the household locates in the community or the city. The following simulations show that such “place-based” policies can lead to higher long-term wealth in the community, albeit without the transformative effects of community benefits.

Place-based policies include local infrastructure improvements, job subsidies, and other policies that provide localized benefits; see Austin et al. [2018] for a detailed discussion. We model these policies as providing a per-period additive utility benefit, \( B > 0 \), *so long as the household stays in the community*. Unlike community benefits, place-based policies cannot condition on whether or not the household has deviated. Instead, a household loses the benefit \( B \) if and only if it leaves the community.

Figure 7 simulates the equilibrium effects of a place-based policy,\(^{12}\) where we assume that

\(^{12}\)This figure uses the same parameters as those in Figures 1 to 3, with \( B = \frac{1}{250} \).
players play a Markov perfect equilibrium following a deviation. This policy increases $w^{se}$ by inducing the household to stay for a wider range of wealth levels. It also relaxes the commitment problem and so increases long-term wealth in the community, $w^{tr}$. However, unlike community benefits, place-based policies do not induce households to smoothly transition from the community to the city; instead, households that choose to stay in the community stay forever, with long-term wealth substantially below that in the city. This difference arises because unlike community benefits, place-based policies cannot induce a household that was already planning to leave to repay favors. Nevertheless, because place-based policies relax the commitment problem, they have a role to play in decreasing the wealth gap between left-behind communities and the rest of the economy.

Place-based policies are most effective if the household leaves the community following a deviation. In particular, the household must prefer to leave rather than staying and continuing to benefit from the policy. Therefore, place-based policies tend to have a large effect when the utility gain from moving to the city, $\hat{U} - U$, is large relative to the policy

\[ \text{Figure 7: Household-optimal equilibrium payoffs with a place-based policy (blue solid line) versus without the policy (green dashed line).} \]

---

\[ \text{Place-based policies might induce a household to stay in the community following a deviation. Thus, in contrast to our baseline model, Markov perfect equilibria do not necessarily min-max the household. This simulation therefore gives an upper bound on punishment payoffs and so a lower bound on household-optimal equilibrium payoffs.} \]
benefit, $B$, since then a deviating household is likely to actually move to the city.

6 The Impossibility of Monetary Favor Exchange

A key feature of our model is that favors are in-kind rather than monetary, which distinguishes it from the literature on informal lending (e.g., Bulow and Rogoff 1989, Ligon et al. 2000). In this section, we show that this distinction is crucial: in sharp contrast to in-kind favors, no favor exchange would occur if favors were monetary.

To make this point, we consider a \textbf{model with monetary favors}. A long-lived household with initial wealth $w_0 \geq 0$ starts in the community. At the start of each period $t \in \{0, 1, 2, \ldots \}$, it chooses to either stay in the community or permanently leave for the city. If it remains in the community in period $t$, then the household plays the following community game with a \textit{long-lived} neighbor:

1. The household requests a payment $s_t \in [-w_t, \infty)$ from the neighbor, where $s_t < 0$ indicates that it pays the neighbor.

2. The neighbor accepts ($d_t = 1$) or rejects ($d_t = 0$) this request. If it accepts, then it pays $s_t$. If it rejects, then no transfer is made.

3. The household chooses an amount to consume, $c_t \geq 0$, where consumption cannot exceed its total wealth including the transfer: $c_t \leq w_t + s_t d_t$. Any remaining wealth is invested, so $I_t = w_t + s_t d_t - c_t$, resulting in period-$(t+1)$ wealth $w_{t+1} = R(I_t)$.

The household’s stage-game payoff is $U(c_t)$, while the neighbor’s stage-game payoff is $-s_t d_t$. The parties share a common discount factor $\delta \in [0, 1)$.

Once the household leaves the community, it thereafter plays the city game, which is identical to the community game except that $s_t = 0$ in every period and the household’s stage-game payoff is $\hat{U}(c_t)$. We maintain the assumptions on $U(\cdot)$, $\hat{U}(\cdot)$, and $R(\cdot)$ from Section 2.
This model makes two changes to the model from Section 2. First, and most importantly, it replaces in-kind favors with monetary favors, $s_t$, which capture both borrowing and repayments by the household. Borrowed money can either be consumed or invested to generate returns according to $R(\cdot)$. The second change is that the neighbor is long-lived, which ensures that the household can delay repayments in order to invest borrowed money. Making the neighbor long-lived makes it easier to sustain favor exchange and so strengthens our impossibility result.

We show that favor exchange cannot occur in any equilibrium of this model.

**Proposition 4** In any equilibrium of the model with monetary favors, $s_t = 0$ in all $t \geq 0$.

See Appendix B for the proof of this result, which is nearly identical to the proof of the impossibility result in Bulow and Rogoff [1989]. The intuition is that monetary favor exchange is inherently self-defeating; the household becomes wealthier when it borrows money, which necessarily improves its outside option. In particular, if the household ever borrows money, then there must exist a period in which the discounted sum of future repayments strictly exceeds the discounted sum of future loans. In that period, the household prefers to renege on future repayments and leave for the city.

Why are Propositions 2 and 4 so different? The key is that in-kind favor exchange can increase consumption utility without increasing wealth. As the “too-big-for-their-boots” mechanism shows, the household can access favor exchange only if it keeps its wealth from increasing, which is possible with in-kind favors. Thus, while monetary favor exchange is self-defeating, in-kind favor exchange can be self-sustaining. Our result also suggests that in Bulow and Rogoff [1989], it might be possible to sustain a lending relationship if lending and repayment were (at least partially) in-kind.\footnote{One important feature of Bulow and Rogoff [1989]'s impossibility result is that the borrower has access to state-contingent formal contracts following a deviation. The borrower might not be able to use in-kind favors in formal contracts in the same way. See, e.g., Kletzer and Wright [2000] for an analysis of lending contracts if the borrower loses access to state-contingent contracts following a deviation. See also, e.g., Ligon et al. [2000], especially their footnote 3.}
We view Proposition 4 as highlighting an important difference between in-kind and monetary favor exchange in settings where households have investment opportunities and the commitment problem is at play. This result is particularly stark because investment returns are deterministic. With stochastic returns, monetary favor exchange can insure the household in the same way that in-kind favor exchange does. If households cannot purchase insurance in the market, then the insurance role of monetary favor exchange can be enough to make it self-sustaining. See, e.g., Ligon et al. [2000].

7 Considerations Beyond the Model

Our paper focuses on a simple model to highlight the tension between accumulating wealth and relying on favor exchange. In this section, we discuss how other considerations may affect our results and empirical implications. We argue that the impact of these considerations is potentially similar to that of community benefits or place-based policies. We also discuss how our results extend to a setting with risky investments.

Why might wealthy households engage in favor exchange?

The reason that wealth undermines favor exchange in our model is that wealthier households have less to lose from reneging on a favor. This section highlights two key assumptions underpinning this result and discusses what happens if they do not hold. The first assumption is that consumption purchased using money is a substitute for consumption procured via favor exchange. The second assumption is that the household can continue accessing its wealth after moving to the city or reneging on a favor.

First, we assume that consumption goods bought with money are substitutes for those acquired by favor exchange. While this is a reasonable assumption for many goods and services traded within communities, certain goods might be “community-specific,” in the sense that they are available only from other community members. For example,
childcare from neighbors, friends, or family might provide flexibility or benefits that cannot be replicated by childcare from a daycare. Even wealthy households value community-specific goods, so they may be willing to repay favors to avoid losing them. By providing an extra reason to repay favors, community-specific goods have the potential to play a role similar to community benefits in Section 5.2. Like community benefits, such goods might relax the commitment problem for wealthy households, increase long-term wealth, and potentially even transform favor exchange into a force for faster wealth accumulation.

**Second, we assume that the household keeps its wealth after leaving for the city or reneging on a favor.** In some settings, however, a household's wealth might be “localized,” in the sense that it would be lost if the household leaves the community, or “favor-exchange specific,” in the sense that it would be lost if the household fails to repay a favor. We discuss each of these possibilities in turn.

Localized investments are lost if the household leaves the community. For example, a household might invest in starting or expanding a local business, which might be difficult to sell or relocate upon moving. Such localized investments give the household an extra reason to stay in the community and therefore resemble the place-based policies discussed in Section 5.2. We would therefore expect households with localized investments to have higher long-term wealth. Since the cost of losing a localized investment is increasing in wealth, those households that would be most tempted to leave face the largest such loss. Therefore, localized investments might encourage even more wealth accumulation than place-based policies that provide benefits independent of wealth.

Favor-exchange specific investments are lost if the household reneges on a favor. For example, the household might own a business that serves other members of the community, who might be able to threaten a boycott if the household reneges on a favor. The more profitable the business, the more the household loses from a boycott, giving wealthy households an additional reason to repay favors. Indeed, favor-exchange specific investments resemble
the community benefits discussed in Section 5.2, since both are lost following a deviation. Like community benefits, this type of investment might have the potential to transform favor exchange so that it encourages faster wealth accumulation for at least some households.\textsuperscript{15}

This discussion suggests two testable implications. First, the wealth gap should be smaller when the household can invest in ways that yield returns only within the community or when those returns rely on the cooperation of the community. Second, when households have access to both localized and non-localized investments, they should disproportionately choose investments with localized returns. This latter implication resonates with Zhou [1992], which describes New York’s Chinatown as comprising many small entrepreneurs whose businesses predominantly serve those within the community. This discussion also suggests that policies aimed at expanding access to localized or favor-exchange specific investments might be especially effective at encouraging wealth accumulation. Such policies include programs that fund entrepreneurs in underserved communities.\textsuperscript{16}

For a localized or favor-exchange specific investment to provide such benefits, it must be difficult to convert into liquid wealth. For instance, if the household could easily sell its business when it moves, then it could keep its wealth after leaving the community and so its investment would not be localized. Investments must also remain difficult to convert to liquid wealth in the long run; otherwise, favor exchange could unravel from the point at which an investment becomes liquid.\textsuperscript{17}

\textsuperscript{15}Note that favor-exchange specific investments resemble relationship-specific investments, which are valuable within but not outside of a relationship. Klein and Leffler [1981], among others, show how such investments can encourage cooperation. Favor-exchange specific investments might similarly encourage cooperation in our setting, leading to more wealth accumulation.

\textsuperscript{16}Examples of such policies include the New Economic Initiative, which provides business grants and place-based support to entrepreneurs in underserved communities in southeast Michigan, and NetWork Kansas, which organizes community-administered funds for entrepreneurs and small businesses.

\textsuperscript{17}For this reason, policies that provide households with “lock boxes” for saving, whose proceeds can only be accessed following a delay, might not be enough to overcome the “too-big-for-their-boots” mechanism.
Stochastic Investment Returns

While our baseline model imposes the simplification that investment is riskless, real-world investments sometimes entail substantial risk. In Online Appendix C, we enrich our model to study how risky investment returns affect our main results. Favor exchange has an additional role in this setting, because it can provide insurance against negative shocks.

If returns are not too risky, then we prove that our main result still holds: the “too-big-for-their-boots” mechanism depresses long-term wealth for households that rely on favor exchange. If returns are extremely noisy, then qualitatively different dynamics can emerge. Using numerical simulations, we show that noisy returns relax the commitment problem, since even wealthy households value access to future favors following negative wealth shocks. Thus, extremely noisy returns can transform favor exchange as in Section 5.1, so that it becomes a force that accelerates wealth accumulation rather than discouraging it.

8 Conclusion

Helping left-behind communities requires understanding the social constraints faced by those experiencing poverty. This paper argues that while favor exchange is an essential source of support in left-behind communities, it imposes hidden costs that can constrain wealth accumulation and deepen long-term inequality. More work remains to be done to understand how heterogeneity within communities—whether from heterogeneous access to favor-exchange networks, heterogeneous ability to repay favors, or another source—influences the constraints imposed by favor exchange and thereby affects economic outcomes.
References


A Appendix: Proof of Proposition 2

Let $\Pi^*(w)$ be the maximum equilibrium payoff of a household with wealth $w$. Define

$$
\Pi_c(w) = \max_{c \geq 0, f \geq 0} \{(1 - \delta)(U(c) - f) + \delta \Pi^*(R(w + f - c))\}
$$

s.t. $0 \leq c - f \leq w$

$$
\tag{1}
f \leq \frac{\delta}{1 - \delta} \left( \Pi^*(R(w + f - c)) - \hat{\Pi}(R(w + f - c)) \right).
$$

We show that $\Pi_c(w)$ is the household’s maximum payoff conditional on staying in the community in the current period. Hence, the household’s maximum equilibrium payoff, $\Pi^*(w)$, is the maximum of $\hat{\Pi}(w)$ and $\Pi_c(w)$.

**Lemma 1** The household’s maximum equilibrium payoff is $\Pi^*(w_0) = \max \left\{ \hat{\Pi}(w_0), \Pi_c(w_0) \right\}$, where $\Pi_c(\cdot)$ and $\Pi^*(\cdot)$ are strictly increasing.

**Proof of Lemma 1:** In any equilibrium, neighbor 0 accepts only if $c_0 \leq p_0 + f_0$. The household’s continuation payoff is at most $\Pi^*(R(w_0 - p_0))$ and at least $\hat{\Pi}(R(w_0 - p_0))$. Hence, it is willing to do favor $f_0$ only if

$$
\tag{2}
f_0 \leq \frac{\delta}{1 - \delta} \left( \Pi^*(R(w_0 - p_0)) - \hat{\Pi}(R(w_0 - p_0)) \right).
$$

Setting $c_0 = p_0 + f_0$ yields $\Pi_c(w_0)$ as an upper bound on the household’s payoff from staying.

This bound is tight. For any $(c_0, f_0)$ that satisfies (1) and (2), it is an equilibrium to set $p_0 = c_0 - f_0 \geq 0$, play a household-optimal continuation equilibrium on-path, and respond to any deviation with the household leaving and $f_t = 0$ in all future periods.\(^\text{18}\) Thus, $\Pi_c(\cdot)$ is the household’s maximum equilibrium payoff conditional on staying. It follows that $\Pi^*(w) = \max \{\hat{\Pi}(w), \Pi_c(w)\}$. Since $\Pi_c(\cdot)$ is strictly increasing by inspection and $\hat{\Pi}(\cdot)$ is strictly increasing by Proposition 1, $\Pi^*(\cdot)$ is strictly increasing.■

\(^{18}\)If $f_t = 0$ in all $t \geq 0$, then the household is willing to leave because $U \leq \hat{U}$.
The next three lemmas characterize household-optimal equilibria in the community. First, we show that households that stay in the community, stay forever.

**Lemma 2** If \( w_0 \geq 0 \) is such that \( \Pi^*(w_0) > \hat{\Pi}(w_0) \), then in any \( t \geq 0 \) of any household-optimal equilibrium, \( \Pi^*(w_t) > \hat{\Pi}(w_t) \) on the equilibrium path.

**Proof of Lemma 2:** Suppose \( t > 0 \) is the first period in which \( \Pi^*(w_t) = \hat{\Pi}(w_t) \), so \( \Pi^*(w_{t-1}) = \Pi_c(w_{t-1}) > \hat{\Pi}(w_{t-1}) \). Let \( \{c_{t-1}, f_{t-1}\} \) achieve \( \Pi_c(w_{t-1}) \). Since \( \Pi^*(w_t) = \hat{\Pi}(w_t) \), (2) implies \( f_{t-1} = 0 \). Therefore, \( \hat{\Pi}(w_{t-1}) \geq \Pi_c(w_{t-1}) \), since if the household exits in \( t - 1 \), it could choose the same \( p_{t-1} \) and \( c_{t-1} \) and earn continuation payoff \( \hat{\Pi}(w_t) = \Pi^*(w_t) \). This contradicts the presumption that \( \Pi_c(w_{t-1}) > \hat{\Pi}(w_{t-1}) \). ■

Second, we show that wealthy households leave the community, while poorer households stay.

**Lemma 3** The set \( W = \{w : \Pi^*(w) > \hat{\Pi}(w)\} \) has positive measure. Moreover, \( w^{se} = \sup \{w : \Pi^*(w) > \hat{\Pi}(w)\} \) satisfies \( 0 < w^{se} < \bar{w} \).

**Proof of Lemma 3:** First, we show that \( \Pi^*(0) > \hat{\Pi}(0) = 0 \). Because \( \lim_{c \to 0} U'(c) = \infty \), there exists a \( c > 0 \) such that \( c \leq \delta U(c) \). Suppose that in all \( t \geq 0 \), \( f_t = c_t = c \) and \( p_t = 0 \) on the equilibrium path, so the household’s equilibrium payoff is \( U(c) - c \). Any deviation is punished by \( f_t = 0 \) in all future periods and the household immediately exiting. This strategy delivers a strictly positive payoff. It is an equilibrium because \( c \leq \delta U(c) \) implies (2). Thus, \( \Pi^*(0) > 0 \). Since \( \Pi^*(w) \) is increasing and \( \hat{\Pi}(w) \) is continuous, there exists an interval around 0 such that \( \Pi^*(w) > \hat{\Pi}(w) \). So \( \{w : \Pi^*(w) > \hat{\Pi}(w)\} \) has positive measure.

Next, we show that \( w^{se} < \bar{w} \). Let \( \bar{c} \) satisfy \( U'(\bar{c}) = 1 \), and let \( w_0 \) be such that \( \Pi^*(w_0) > \hat{\Pi}(w_0) \). By Lemma 2, \( \Pi^*(w_t) > \hat{\Pi}(w_t) \) in any \( t \geq 0 \) of any household-optimal equilibrium. Suppose that \( c_t > \bar{c} \) in period \( t \geq 0 \). If \( f_t > 0 \), then we can perturb the equilibrium by decreasing \( c_t \) and \( f_t \) by \( \epsilon > 0 \), which increases the household’s payoff at rate \( 1 - U'(c_t) > 0 \) as \( \epsilon \to 0 \). So, \( f_t = 0 \).
Let $\tau > t$ be the first period after $t$ such that $f_{\tau} > 0$. Consider decreasing $p_t$ and $c_t$ by $\epsilon > 0$, increasing $p_{\tau}$ by $\chi(\epsilon)$, and decreasing $f_{\tau}$ by $\chi(\epsilon)$, where $\chi(\epsilon)$ is chosen so that $w_{\tau+1}$ remains constant. Then, $\chi(\epsilon) \geq \frac{\epsilon}{\delta t}$ because $R'(\cdot) \geq \frac{1}{\delta}$. As $\epsilon \to 0$, this perturbation increases the household’s payoff by at least $\delta^{\tau-t} \frac{1}{\delta t} - U'(c_t) > 0$. It is an equilibrium because $f_s = 0$ for all $s \in [t, \tau - 1]$, so (2) still holds in these periods.

The above argument implies that if $c_t > \bar{c}$, then $f_{\tau} = 0$ for all $\tau \geq t$. But then $\Pi^*(w_t) \leq \hat{\Pi}(w_t)$, contradicting Lemma 2. Therefore, if $\Pi^*(w_0) > \hat{\Pi}(w_0)$, then $c_t \leq \bar{c}$ in every $t \geq 0$ and so $\Pi^*(w_0) \leq U(\bar{c})$. Since $R(\bar{w} - \bar{c}) > \bar{w}$ by Assumption 1, it follows that $\Pi^*(\bar{w}) \geq \hat{\Pi}(\bar{w}) > \hat{U}(\bar{c}) \geq U(\bar{c})$. By the definition of $w^{se}$, there exists a sequence of initial wealth levels in $W$ which are arbitrarily close to $w^{se}$ such that the household strictly prefers to stay in the community with those initial wealth levels. If $w^{se} \geq \bar{w}$, the equilibrium payoffs at those initial wealth levels would be strictly above $U(\bar{c})$ due to the continuity of $\hat{\Pi}(w)$. This leads to a contradiction, so $w^{se} < \bar{w}$. ■

Finally, we show that household-optimal equilibria exhibit monotone wealth dynamics.

**Lemma 4** In any household-optimal equilibrium, $(w_t)_{t=0}^\infty$ is monotone.

The proof of Lemma 4 is presented after we complete the proof of the main result. The key step of this proof shows that household-optimal investment, $w_t - p_t$, increases in $w_t$. Thus, if $w_1 \geq w_0$, then $w_2 = R(w_1 - p_1) \geq R(w_0 - p_0) = w_1$ and so on, and similarly if $w_1 \leq w_0$.

We can now prove Proposition 2. **Selection** is implied by Lemma 2 and Lemma 3. For the “too-big-for-their-boots” mechanism, define

$$\tilde{c}(w) = w - R^{-1}(w)$$

and $\tilde{\Pi}(w) = U(\tilde{c}(w))$. Since $w^{se} < \bar{w}$ by Lemma 3, consumption $\tilde{c}(w^{se})$ does not satisfy...
Thus, there exists $K > 0$ such that
\[ \Pi^*(w^{se}) + K < \Pi(w^{se}). \]

Define
\[ \tilde{f}(w) = \frac{\delta}{1 - \delta} \left( \Pi(w^{se}) - \Pi(w) \right) \]
and
\[ \tilde{p}(w) = w^{se} - R^{-1}(w). \]

Consider $w_0 < w^{se}$ such that $\Pi^*(w_0) > \Pi(w_0)$, and suppose that there exists an equilibrium in which $(w_t)_{t=0}^\infty$ is increasing on the equilibrium path. We claim that $p_t \leq \tilde{p}(w_0)$ and $f_t \leq \tilde{f}(w_0)$ in every $t \geq 0$. Indeed,
\[ f_t \leq \frac{\delta}{1 - \delta} \left( \Pi^*(w_{t+1}) - \Pi(w_{t+1}) \right) \leq \frac{\delta}{1 - \delta} \left( \Pi^*(w^{se}) - \Pi(w_0) \right) = \tilde{f}(w_0), \]
and
\[ p_t = w_t - R^{-1}(w_{t+1}) \leq w^{se} - R^{-1}(w_0) = \tilde{p}(w_0), \]
where the inequalities hold because (i) $w_t, w_{t+1} \leq w^{se}$ by Lemma 2, and (ii) $w_{t+1} \geq w_0$ by our presumption that $(w_t)_{t=0}^\infty$ is increasing.

Since $c_t \leq p_t + f_t$, the household’s payoff satisfies
\[ \Pi^*(w_0) \leq U \left( \tilde{p}(w_0) + \tilde{f}(w_0) \right) = H(w_0). \]

The function $H(w_0)$ is continuous and decreasing in $w_0$, with $H(w^{se}) = \Pi(w^{se})$. Since $\Pi(w^{se}) + K < \Pi(w^{se})$, there exists $w^{tr} < w^{se}$ such that for any $w_0 \in (w^{tr}, w^{se})$,
\[ H(w_0) < \Pi(w_0). \]
Therefore, for any \( w_0 \in (w^{tr}, w^{se}) \), if \( w_0 \in \mathcal{W} \), then \( (w_t)_{t=0}^\infty \) must be strictly decreasing, with \( \lim_{t \to \infty} w_t \leq w^{tr} \).

By definition of \( w^{se} \), there exists \( w_0 \in \mathcal{W} \cap (w^{tr}, w^{se}) \) such that \( \Pi^*(w_0) > \hat{\Pi}(w_0) \). Since \( \Pi^*(\cdot) \) and \( \hat{\Pi}(\cdot) \) are increasing, with \( \hat{\Pi}(\cdot) \) continuous, we conclude that \( \Pi^*(w) > \hat{\Pi}(w) \) on a neighborhood around \( w_0 \). So \( \mathcal{W} \cap [w^{tr}, w^{se}] \) has a positive measure. ■

A.1 Proof of Lemma 4

We break the proof of this lemma into four steps.

A.1.1 Step 1: Locally Bounding the Slope of \( \Pi^*(\cdot) \) from Below

We claim that for any \( w \in [0, w^{se}) \), there exists \( \epsilon_w > 0 \) such that for any \( \epsilon \in (0, \epsilon_w) \),

\[
\Pi^*(w + \epsilon) - \Pi^*(w) > (1 - \delta)\epsilon.
\]

First, suppose \( \hat{\Pi}(w) \geq \Pi_c(w) \), and let \( \{w_t, c_t\}_{t=0}^\infty \) be the wealth and consumption sequences if the household enters the city. The proof of Lemma 3 implies that for any \( w_0 < w^{se} \), \( R(w_0 - \bar{c}) < w_0 \). Proposition 1 says that \( \{w_t\}_{t=0}^\infty \) is increasing, so \( c_0 < \bar{c} \). Hence, there exists \( \epsilon_w > 0 \) such that \( U'(c_0 + \epsilon_w) > 1 \). Since \( \hat{U}'(c) \geq U'(c) \) for all \( c > 0 \), \( \hat{U}'(c_0 + \epsilon_w) > 1 \).

For any \( \epsilon < \epsilon_w \), if \( w_0 = w + \epsilon \), then the household can enter the city and choose \( \hat{c}_0 = c_0 + \epsilon \), with \( \hat{c}_t = c_t \) in all \( t \geq 0 \). We can bound \( \Pi^*(w + \epsilon) \) from below by the payoff from this strategy,

\[
\Pi^*(w + \epsilon) \geq (1 - \delta) \left( \hat{U}(c_0 + \epsilon) - \hat{U}(c_0) \right) + \hat{\Pi}(w) > (1 - \delta)\epsilon + \hat{\Pi}(w) = (1 - \delta)\epsilon + \Pi^*(w).
\]

We conclude that \( \Pi^*(w + \epsilon) - \Pi^*(w) > (1 - \delta)\epsilon \), as desired.

Now, suppose \( \hat{\Pi}(w) < \Pi_c(w) \). Let \( \{w_t, c_t, f_t\}_{t=0}^\infty \) be the wealth, consumption, and favor sequence in a household-optimal equilibrium. There exists \( \tau \geq 0 \) such that \( f_\tau > 0 \) for the first time in period \( \tau \); otherwise, the household could implement the same consumption sequence in the city. Choose \( \epsilon_w > 0 \) to satisfy \( \epsilon_w < \delta^* f_\tau \).
For $\epsilon \in (0, \epsilon_w)$ and initial wealth $w_0 = w + \epsilon$, consider the perturbed strategy such that $\hat{p}_t = p_t$, $\hat{c}_t = c_t$, and $\hat{f}_t = f_t$ in every period except $\tau$. In period $\tau$, $\hat{f}_\tau = f_\tau - \frac{\epsilon}{\delta}$ and $\hat{p}_\tau = p_\tau + \chi$, where $\chi$ is chosen so that $\hat{w}_{t+1} = w_{t+1}$. Then, $\hat{c}_\tau = c_\tau + \chi - \frac{\epsilon}{\delta}$. Based on the proof of Proposition 2, $w_t < w^{se}$ for all $t \leq \tau$. This observation together with $w^{se} \leq \bar{w}$ and Assumption 1, implies that we can choose a sufficiently small $\epsilon_w$ such that the marginal return from capital in every $t < \tau$ is strictly higher than $\frac{1}{\delta}$ even if the initial wealth is $w + \epsilon$ rather than $w$. Hence, $\chi > \frac{\epsilon}{\delta}$.

Under this perturbed strategy, (2) is satisfied in all $t < \tau$ because $f_t = 0$ in these periods; in $t = \tau$ because $\hat{f}_\tau < f_\tau$ and $\hat{w}_{\tau+1} = w_{\tau+1}$; and in $t > \tau$ because play is unchanged after $\tau$. Moreover, $f_\tau - \frac{\epsilon}{\delta} > 0$ because $\epsilon < \epsilon_w$, and $\hat{f}_\tau + \hat{p}_\tau = \hat{c}_\tau$, so this strategy is feasible. Thus, it is an equilibrium. Consequently, $\Pi^*(w + \epsilon)$ is bounded from below by the household’s payoff from this strategy,

$$\Pi^*(w + \epsilon) > (1 - \delta)\frac{\epsilon}{\delta} + \Pi_c(w) = (1 - \delta)\epsilon + \Pi^*(w),$$

as desired.

A.1.2 Step 2: Moving from Local to Global Bound on Slope

Next, we show that for any $0 \leq w < w' < w^{se}$, $\Pi^*(w') - \Pi^*(w) > (1 - \delta)(w' - w)$.

Let

$$z(w) = \sup\{w'' | w < w'' \leq w^{se}, \text{ and } \forall w' \in (w, w''], \Pi^*(w') - \Pi^*(w) > (1 - \delta)(w' - w)\}.$$ 

By Step 1, $z(w) \geq w$ exists. Moreover,

$$\Pi^*(z(w)) - \Pi^*(w) \geq \lim_{\bar{w} \uparrow z(w)} \Pi^*(\bar{w}) - \Pi^*(w) \geq (1 - \delta)(z(w) - w),$$

where the first inequality follows because $\Pi^*(\cdot)$ is increasing, and the second inequality follows
by definition of $z(w)$.

Suppose that $z(w) < w^{se}$. By Step 1, there exists $\epsilon_{z(w)}$ such that for any $\epsilon < \epsilon_{z(w)}$,

$$\Pi^*(z(w) + \epsilon) - \Pi^*(z(w)) > (1 - \delta)\epsilon.$$ 

Hence,

$$\Pi^*(z(w) + \epsilon) - \Pi^*(w) = \Pi^*(z(w) + \epsilon) - \Pi^*(z(w)) + \Pi^*(z(w)) - \Pi^*(w) > (1 - \delta)\epsilon + (1 - \delta)(z(w) - w).$$

This contradicts the definition of $z(w)$, so $z(w) \geq w^{se}$.

For any $w' < w^{se}$, $w' < z(w)$ and so $\Pi^*(w') - \Pi^*(w) > (1 - \delta)(w' - w)$, as desired.

**A.1.3 Step 3: Investment is Increasing in Wealth.**

Consider two wealth levels, $0 \leq w_L < w_H < w^{se}$, and suppose that $\Pi_c(w_L) > \hat{\Pi}(w_L)$ and $\Pi_c(w_H) > \hat{\Pi}(w_H)$. Given any household-optimal equilibria, let $p_H, p_L$ be the respective period-0 payments under $w_H, w_L$. We prove that $w_H - p_H \geq w_L - p_L$. Define $I_k = w_k - p_k$, $k \in \{L, H\}$. Towards a contradiction, suppose that $I_H < I_L$.

We first show that $c_H > c_L + (w_H - w_L)$. Suppose instead that $c_H \leq c_L + (w_H - w_L)$. Since $I_H < I_L$, we have $p_H > p_L + (w_H - w_L)$. But then $f_H < f_L$, since

$$f_H = c_H - p_H < c_H - (p_L + (w_H - w_L)) \leq c_L + (w_H - w_L) - (p_L + (w_H - w_L)) = f_L.$$ 

Consider the following perturbation: $\hat{p}_H = p_L + (w_H - w_L) \in (p_L, p_H)$, $\hat{f}_H = f_H + p_H - \hat{p}_H \geq f_H$, and $\hat{c}_H = c_H$. Under this perturbation, $\hat{I}_H = w_H - \hat{p}_H = I_L$. Thus, to show that the perturbation satisfies (2), we need only show that $\hat{f}_H \leq f_L$. Indeed:

$$\hat{f}_H = f_H + p_H - (p_L + (w_H - w_L)) = c_H - (c_L - f_L) - (w_H - w_L) = f_L + c_H - (c_L + w_H - w_L) \leq f_L,$$

where the final inequality holds because $c_H \leq c_L + (w_H - w_L)$ by assumption. Thus, this
perturbation is also an equilibrium.

We claim that a household with initial wealth $w_H$ strictly prefers this equilibrium to the original equilibrium, which is true so long as

$$(1 - \delta)(U(c_H) - f_H) + \delta \Pi^*(R(I_L)) > (1 - \delta)(U(c_H) - f_H) + \delta \Pi^*(R(I_H))$$

$\iff$

$$(1 - \delta)(f_H - f_H) < \delta(\Pi^*(R(I_L)) - \Pi^*(R(I_H)))$$

$\iff$

$$(1 - \delta)(p_H - \hat{p}_H) < \delta(\Pi^*(R(I_L)) - \Pi^*(R(I_H))).$$

We know that $I_L = I_H + p_H - \hat{p}_H$. Since the household stays in the community, $I_H < I_L < w^{se}$, so $R'(I_H), R'(I_L) > \frac{1}{\delta}$. Thus,

$$R(I_L) - R(I_H) > \frac{1}{\delta}(I_L - I_H) = \frac{1}{\delta}(p_H - \hat{p}_H).$$

By Step 2, $\Pi^*(\cdot)$ increases at rate strictly greater than $(1 - \delta)$, so we conclude

$$\delta(\Pi^*(R(I_L)) - \Pi^*(R(I_H))) > \delta(1 - \delta)\frac{1}{\delta}(p_H - \hat{p}_H),$$

as desired. Thus, if $I_H < I_L$, then $c_H > c_L + (w_H - w_L)$.

We are now ready to prove that $I_H < I_L$ contradicts household optimality. To do so, we consider two perturbations: one at $w_L$ and one at $w_H$. At $w_H$, consider setting

$$\hat{c}_H = c_L + (w_H - w_L) > c_L \geq 0,$$

$$\hat{p}_H = p_L + w_H - w_L \in [p_L, w_H],$$

$$\hat{f}_H = \hat{c}_H - \hat{p}_H = f_L.$$

By construction, $w_H - \hat{p}_H = I_L$. Thus, $\hat{f}_H$ satisfies (2) because $f_L$ does. Moreover, $\hat{p}_H + \hat{f}_H = \hat{c}_H$, so the neighbor is willing to accept. This perturbed strategy is therefore an equilibrium.
For the original equilibrium to be household-optimal, we must therefore have

\[(1 - \delta)(U(c_H) - f_H) + \delta \Pi^*(R(I_H)) \geq (1 - \delta)(\hat{U}(c_H) - \hat{f}_H) + \delta \Pi^*(R(\hat{I}_H)).\]  \hspace{1cm} (3)

At \(w_L\), consider setting

\[\hat{c}_L = c_H - (w_H - w_L) > c_L \geq 0,\]
\[\hat{p}_L = p_H - (w_H - w_L) \in (p_L, w_L),\]
\[\hat{f}_L = \hat{c}_L - \hat{p}_L = f_H.\]

By construction, \(w_L - \hat{p}_L = I_H\). Thus, \(\hat{f}_L\) satisfies (2) because \(f_H\) does. This perturbed strategy is again an equilibrium, so the original equilibrium is household-optimal only if

\[(1 - \delta)(U(c_L) - f_L) + \delta \Pi^*(R(I_L)) \geq (1 - \delta)(\hat{U}(c_L) - \hat{f}_L) + \delta \Pi^*(R(\hat{I}_L)).\]  \hspace{1cm} (4)

Combining (3) and (4) and plugging in definitions, we have

\[U(c_H) - U(c_H - (w_H - w_L)) \geq U(c_L + (w_H - w_L)) - U(c_L).\]

However, \(c_H > c_L + w_H - w_L\) and \(U(\cdot)\) is strictly concave, so this inequality cannot hold. Thus, if \(I_H < I_L\), then at least one of the equilibria at \(w_H\) and \(w_L\) cannot be household-optimal.

A.1.4 Step 4: Establishing Monotonicity

We have shown that investment, \(I(w)\), is increasing in \(w\). Consider a household-optimal equilibrium with \(w_1 \geq w_0\). Then, \(I(w_1) \geq I(w_0)\), so \(w_2 = R(I(w_1)) \geq R(I(w_0)) = w_1\). Thus, \(w_2 \geq w_1\), and \(w_{t+1} \geq w_t\) for all \(t > 1\) by the same argument. Similarly, if \(w_1 \leq w_0\), then \(I(w_1) \leq I(w_0), w_2 \leq w_1,\) and \(w_{t+1} \leq w_t\) in all \(t \geq 0\). We conclude that \((w_t)_{t=0}^{\infty}\) is monotone in any household-optimal equilibrium. □
B Appendix: Other Proofs

B.1 Proof of Proposition 1

Suppose that the household lives in the city. In any period $t$, since future vendors don’t observe $f_t$, the household always chooses $f_t = 0$. Hence, vendor $t$ accepts only if $p_t \geq c_t$. This means that $c_t \in [0, w_t]$ are the feasible consumption levels, so that the household’s equilibrium continuation payoff is at most $\hat{\Pi}(w_t)$ given wealth $w_t$.

The following equilibrium gives the household an equilibrium continuation payoff of $\hat{\Pi}(w_t)$. In period $t$, (i) the household proposes $(c_t, p_t) = (\hat{C}(w_t), \hat{C}(w_t))$; (ii) vendor $t$ accepts if and only if $p_t \geq c_t$. Vendor $t$ has no profitable deviation. This strategy attains $\hat{\Pi}$, so the household has no profitable deviation either.

Let $\{c^*_t\}_{t=0}^\infty$ be the consumption sequence in the equilibrium above, given initial wealth $w$. If $w = 0$, then $c^*_t = 0$ in all $t \geq 0$, so $\hat{\Pi}(0) = \hat{U}(0) = 0$ is the unique equilibrium payoff. If $w > 0$, then it must be true that $c^*_t > 0$ in every $t \geq 0$. Suppose otherwise. Let $\tau \geq 0$ be the first period in which $\min\{c^*_\tau, c^*_{\tau+1}\} = 0$ and $\max\{c^*_\tau, c^*_{\tau+1}\} > 0$. If $c^*_\tau > 0$ and $c^*_{\tau+1} = 0$, consider the perturbation $c_\tau = c^*_\tau - \epsilon_1$, $c_{\tau+1} = c^*_{\tau+1} + \epsilon_2$ for some small $\epsilon_1, \epsilon_2 > 0$ such that the wealth $w_{\tau+2}$ stays the same. If $c^*_\tau = 0$ and $c^*_{\tau+1} > 0$, consider the perturbation $c_\tau = c^*_\tau + \epsilon_1$, $c_{\tau+1} = c^*_{\tau+1} - \epsilon_2$ for some small $\epsilon_1, \epsilon_2 > 0$ such that the wealth $w_{\tau+2}$ stays the same. In either case, the perturbation gives a strictly higher payoff, since $\lim_{c \downarrow 0} U'(c) = \infty$.

Next, we show that $\hat{\Pi}(w)$ is the household’s unique equilibrium payoff. At $w = 0$, $\hat{\Pi}(0) = 0$, so the household’s unique equilibrium payoff is indeed $\hat{\Pi}(0)$. For $w > 0$, the household can choose $(c_t, p_t) = ((1 - \epsilon)c^*_t, c^*_t)$ in every $t \geq 0$ for $\epsilon > 0$ small. Vendor $t$ strictly prefers to accept. As $\epsilon \downarrow 0$, the consumption sequence $\{(1 - \epsilon)c^*_t\}_{t=0}^\infty$ gives the household a payoff that converges to $\hat{\Pi}(w)$. So the household must earn at least $\hat{\Pi}(w)$ in any equilibrium.

Turning to properties of $\hat{\Pi}(\cdot)$, we claim that $\hat{\Pi}(\cdot)$ is strictly increasing. Pick $0 \leq w < \tilde{w}$. Let $\{c^*_t\}_{t=0}^\infty$ be the sequence associated with $w$. If the initial wealth is $\tilde{w}$, it is feasible to choose $c_0 = c^*_0 + \tilde{w} - w$ and $c_t = c^*_t$ for $t \geq 1$. Since $\hat{U}(\cdot)$ is strictly increasing, so too is $\hat{\Pi}(\cdot)$. 

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It remains to show that \( \hat{\Pi}(\cdot) \) is continuous for all \( w > 0 \). If \( w > 0 \), then \( \hat{C}(w) > 0 \). For \( \hat{w} \) sufficiently close to \( w \), setting \( c_0 = \hat{C}(w) + (\hat{w} - w) \) and \( c_t = \hat{C}(w_t) \) for \( t \geq 1 \) is feasible. The household’s payoffs converge to \( \hat{\Pi}(w) \) as \( \hat{w} \to w \) under this perturbation, which means that \( \lim_{\hat{w} \uparrow w} \hat{\Pi}(\hat{w}) \geq \hat{\Pi}(w) \) and \( \lim_{\hat{w} \downarrow w} \hat{\Pi}(\hat{w}) \geq \hat{\Pi}(w) \). Since \( \hat{\Pi}(\cdot) \) is increasing, we conclude that \( \hat{\Pi}(\cdot) \) is continuous at every \( w > 0 \).

We now show that \( \hat{\Pi}(\cdot) \) is continuous at \( w = 0 \). Consider \( \lim_{w \downarrow 0} \hat{\Pi}(w) \). Since \( R'(\bar{w}) = \frac{1}{\delta} \), the line tangent to \( R(\cdot) \) at \( \bar{w} \) is \( \hat{R}(w) = R(\bar{w}) + \frac{w - \bar{w}}{\delta} \). Since \( R(\cdot) \) is concave, \( R(w) \leq \hat{R}(w) \) for all \( w \geq 0 \). Therefore, \( \hat{\Pi}(w) \) is bounded from above by the household’s maximum payoff if we replace \( R(\cdot) \) with \( \hat{R}(\cdot) \). For consumption path \( \{c_t\}_{t=0}^\infty \) to be feasible under \( \hat{R}(\cdot) \), it must satisfy

\[
(1 - \delta) \sum_{t=0}^\infty \delta^t c_t \leq (1 - \delta) w_0 + \delta R(\bar{w}) - \bar{w}.
\]

This means that the payoff of a household with initial wealth \( w_0 \) is at most

\[
\hat{U}((1 - \delta) w_0 + \delta R(\bar{w}) - \bar{w}).
\]

Pick any small \( \epsilon > 0 \). There exists \( T < \infty \) and sufficiently small \( w_0 > 0 \) such that

\[
\delta^T \hat{U} ((1 - \delta) R^T(w_0) + \delta R(\bar{w}) - \bar{w}) < \frac{\epsilon}{2},
\]

where \( R^T(w_0) \) denotes the function that applies \( R(\cdot) \) \( T \)-times to \( w_0 \).

Consider a hypothetical setting that is more favorable to the household: we allow the household to both consume and save its wealth until period \( T \), after which it must play the original city game. The household’s payoff from this hypothetical is strictly larger than \( \hat{\Pi}(w_0) \) and is bounded from above by

\[
(1 - \delta) \sum_{t=0}^{T-1} \delta^t (\hat{U}(R^t(w_0))+ \delta^T \hat{U}((1 - \delta) R^T(w_0) + \delta R(\bar{w}) - \bar{w})).
\]
As \( w_0 \downarrow 0 \), \( R^T(w_0) \downarrow 0 \), so \( R^t(w_0) \downarrow 0 \) for any \( t < T \). Thus,

\[
\hat{\Pi}(w_0) \leq (1 - \delta) \sum_{t=0}^{T-1} \delta^t \hat{U}(R^t(w_0)) + \delta^T \hat{U}((1 - \delta)R^T(w_0) + \delta R(\bar{w}) - \bar{w}) < \epsilon.
\]

This is true for any \( \epsilon > 0 \), so \( \lim_{w \downarrow 0} \hat{\Pi}(w) = 0 \).

Finally, consider any equilibrium in the city. If \( w_0 = 0 \), then \( w_t = 0 \) in any \( t \geq 0 \). If \( w_0 > 0 \), then we have shown that \( c_t > 0 \) in every \( t \geq 0 \), so \( w_t > c_t > 0 \). A standard argument (see below) implies the following Euler equation:

\[
\hat{U}'(c_t) = \delta R'(w_t - c_t) \hat{U}'(c_{t+1}). \tag{5}
\]

Together with \( R'(\cdot) \geq \frac{1}{\delta} \) and \( \hat{U}(\cdot) \) strictly concave, (5) implies \( c_t \leq c_{t+1} \), and strictly so if \( w_t - c_t < \bar{w} \).

Next, we argue that \( \hat{C}(\cdot) \) is strictly increasing in \( w \). Let \( \{c_t\}_{t=0}^{\infty} \) and \( \{\bar{c}_t\}_{t=0}^{\infty} \) be the equilibrium consumption sequences for \( w > 0 \) and \( \bar{w} > w \), respectively. Suppose \( c_0 \geq \bar{c}_0 \), and let \( \tau \geq 1 \) be the first period such that \( c_t < \bar{c}_t \), which must exist because \( \hat{\Pi}(\cdot) \) is strictly increasing. Then, \( c_{\tau-1} \geq \bar{c}_{\tau-1} \), \( w_{\tau-1} - c_{\tau-1} < \bar{w}_{\tau-1} - \bar{c}_{\tau-1} \), and \( c_\tau < \bar{c}_\tau \), so at least one of \( (c_{\tau-1}, w_{\tau-1}, c_\tau) \) and \( (\bar{c}_{\tau-1}, \bar{w}_{\tau-1}, \bar{c}_\tau) \) violates (5). Hence, \( \hat{C}(w) \) is strictly increasing in \( w \). Therefore, \( c_{t+1} \geq c_t \) implies \( w_{t+1} \geq w_t \), with strict inequalities if \( w_t < \bar{w} \).

Since \( (w_t)_{t=0}^{\infty} \) is monotone, it converges to some \( w^\infty \in \mathbb{R}_+ \cup \{\infty\} \). Suppose \( w^\infty < R(\bar{w}) \). Since \( w_{t+1} = R(w_t - c_t) \), we must have \( c_t \) and \( w_t - c_t \) converging, with \( \lim_{t \to \infty} (w_t - c_t) < \bar{w} \). But then \( R'(w_t - c_t) \) converges to a number strictly above \( \frac{1}{\delta} \), which implies that (5) is violated as \( t \to \infty \). We conclude that \( \lim_{t \to \infty} w_t \geq R(\bar{w}) \).
Deriving the Euler Equation

Consider a household in the city, and let its optimal consumption and wealth sequence be \( \{c_t^*, w_t^*\}_{t=0}^\infty \). We prove that if \( w_0 > 0 \), then

\[
\dot{U}'(c_t^*) = \delta R'(w_t^* - c_t^*)\dot{U}'(c_{t+1}^*)
\]

in every \( t \geq 0 \).

The proof of Proposition 1 says that \( c_t^* > 0, c_{t+1}^* > 0, \) and \( w_t^* - c_t^* > 0 \). Suppose that \( \dot{U}'(c_t^*) > \delta R'(w_t^* - c_t^*)\dot{U}'(c_{t+1}^*) \). Then, we can perturb \((c_t^*, c_{t+1}^*)\) to \((c_t^* + \epsilon, c_{t+1}^* - \chi(\epsilon))\), where \( \chi(\epsilon) \) is chosen such that \( w_{t+2}^* \) remains the same as before the perturbation. In particular,

\[
R(w_t^* - (c_t^* + \epsilon)) - (c_{t+1}^* - \chi(\epsilon)) = R(w_t^* - c_t^*) - c_{t+1}^*.
\]

Hence, \( \chi'(\epsilon) = R'(w_t^* - (c_t^* + \epsilon)) \).

As \( \epsilon \downarrow 0 \), this perturbation strictly increases the household’s payoff:

\[
\lim_{\epsilon \downarrow 0} \left\{ \dot{U}'(c_t^* + \epsilon) - \delta \dot{U}'(c_{t+1}^* - \chi(\epsilon))\chi'(\epsilon) \right\} = \lim_{\epsilon \downarrow 0} \left\{ \dot{U}'(c_t^* + \epsilon) - \delta \dot{U}'(c_{t+1}^* - \chi(\epsilon))R'(w_t^* - c_t^* - \epsilon) \right\} = \dot{U}'(c_t^*) + \delta R'(w_t^* - c_t^*)\dot{U}'(c_{t+1}^*) > 0.
\]

This contradicts the fact that \((c_t^*, c_{t+1}^*)\) is optimal. Using a similar argument, we can show that \( \dot{U}'(c_t^*) < \delta R'(w_t^* - c_t^*)\dot{U}'(c_{t+1}^*) \) is not possible either. ■

B.2 Proof of Proposition 3

We show that there exists a household-optimal equilibrium such that if \( p_t > 0 \), then \( f_{t'} = 0 \) in all \( t' > t \) so \( \tau \leq t + 1 \).

Define \( \epsilon = \min \{ p_t, \delta^{t'-t} f_{t'} \} \). Consider the following perturbation in periods \( t \) and \( t' \).
(denoted by tildes): in period $t$, $\tilde{p}_t = p_t - \epsilon$, $\tilde{f}_t = f_t + \epsilon$, and $\tilde{c}_t = c_t$; in period $t'$, $\tilde{p}_{t'} = p_{t'} + R(\tilde{w}_{t'-1}) - R(w_{t'-1})$, $\tilde{f}_{t'} = f_{t'} - \frac{\epsilon}{\delta_t'}$, and $\tilde{c}_{t'} = c_{t'} + (\tilde{p}_{t'} - p_{t'}) + (\tilde{f}_{t'} - f_{t'})$. This perturbation is feasible, and neighbors $t$ and $t'$ are willing to accept these requests. Thus, it is also an equilibrium.

Note that $c_t = \tilde{c}_t$ and $\delta_t' \tilde{f}_t + \delta_t' \tilde{f}_{t'} = \delta_t' f_t + \delta_t' f_{t'}$. Thus, this perturbation improves the household’s payoff only if $\tilde{c}_{t'} \geq c_{t'}$, or $\tilde{p}_{t'} + \tilde{f}_{t'} \geq p_{t'} + f_{t'}$. But $\tilde{p}_{t'} - p_{t'} = R(\tilde{w}_{t'-1}) - R(w_{t'-1}) \geq \frac{\epsilon}{\delta_t'}$ because $R'(\cdot) \geq \frac{1}{\delta}$, while $\tilde{f}_{t'} = f_{t'} - \frac{\epsilon}{\delta_t'}$. Therefore, $\tilde{c}_{t'} \geq c_{t'}$, as desired. By similarly perturbing any periods $t < t'$ such that $p_t > 0$ and $f_t > 0$, we can construct a household-optimal equilibrium with the property that if $p_t > 0$, then $f_t = 0$ in all $t' > t$. If $\tau = \infty$ in this equilibrium, then $p_t = 0$ for every $t$ which is clearly suboptimal. Hence, $\tau < \infty$ in this equilibrium and $p_t = 0$ for every $t < \tau - 1$. Since $I_t = w_t$ for every $t < \tau - 1$, the household invests strictly more than in the city with the same wealth level.

We next show that for $t = \tau - 1$, the household also invests strictly more. If $p_{\tau - 1} = 0$, then this is clearly true. If $p_{\tau - 1} > 0$, let $\{c_t\}_{t \geq \tau - 1}$ be the consumption sequence starting from period $\tau - 1$. Optimality implies that

$$U'(c_{\tau - 1}) = \delta R'(w_{\tau - 1} - p_{\tau - 1}) \hat{U}'(c_{\tau}).$$

Now consider a household in the city with wealth $w_{\tau - 1}$. The household can consume $p_{\tau - 1}$ today and continue with the consumption sequence $\{c_t\}_{t \geq \tau}$. Since $p_{\tau - 1} < p_{\tau - 1} + f_{\tau - 1} = c_{\tau - 1}$ and $\hat{U}' \geq U'$,

$$\hat{U}'(p_{\tau - 1}) > \delta R'(w_{\tau - 1} - p_{\tau - 1}) \hat{U}'(c_{\tau}).$$

Hence, the household in the city with wealth $w_{\tau - 1}$ would like to consume strictly more than $p_{\tau - 1}$. Hence, the household would invest strictly less than $w_{\tau - 1} - p_{\tau - 1}$, which is the investment level in the community.
B.3 Proof of Proposition 4

We first claim that if \( w \geq R(\bar{w}) \), then the household leaves the community. This shows that in the community, wealth is at most \( R(\bar{w}) \).

To prove this claim, we give the household a more generous return function \( \hat{R}(w) = (w - \bar{w})/\delta + R(\bar{w}) \). Suppose that the household starts in the community with some initial wealth \( w \geq R(\bar{w}) \). Then the discounted sum of consumptions is at most:

\[
(1 - \delta) \sum_{t=0}^{\infty} \delta^t c_t \leq (1 - \delta)w + \delta R(\bar{w}) - \bar{w} + (1 - \delta) \sum_{t=0}^{\infty} \delta^t s_t.
\]

The neighbor’s payoff at the beginning of period 0 is \(-(1 - \delta) \sum_{t=0}^{\infty} \delta^t s_t\), so it must be the case that \((1 - \delta) \sum_{t=0}^{\infty} \delta^t s_t \leq 0\). Therefore, the discounted sum of consumptions is at most:

\[
(1 - \delta) \sum_{t=0}^{\infty} \delta^t c_t \leq (1 - \delta)w + \delta R(\bar{w}) - \bar{w}.
\]

The best consumption sequence given this constraint is to consume constantly \((1 - \delta)w + \delta R(\bar{w}) - \bar{w}\) in every period. If the household with wealth \( w \) chooses to consume \((1 - \delta)w + \delta R(\bar{w}) - \bar{w}\), then the investment is:

\[
w - ((1 - \delta)w + \delta R(\bar{w}) - \bar{w}) = \delta w + \bar{w} - \delta R(\bar{w}),
\]

which is greater than \( \bar{w} \) if \( w \geq R(\bar{w}) \). Hence, this constant consumption sequence can be sustained in the city under the return function \( R(\cdot) \) for any \( w \geq R(\bar{w}) \). This shows that in the community, the wealth is at most \( R(\bar{w}) \).

Consider an equilibrium such that the household stays in the community. Define \( L_t = (1 - \delta) \sum_{\tau=t}^{\infty} \delta^{\tau-t} s_{\tau} \). We first show that \(-L_t \leq R(\bar{w})\) for any \( t \geq 0 \). This is because:

\[
L_t + R(\bar{w}) = (1 - \delta) \sum_{\tau=t}^{\infty} \delta^{\tau-t}(s_{\tau} + R(\bar{w})) \geq (1 - \delta) \sum_{\tau=t}^{\infty} \delta^{\tau-t}(s_{\tau} + w_t) \\
\geq (1 - \delta) \sum_{\tau=t}^{\infty} \delta^{\tau-t}(s_{\tau} + w_t - I_t) = (1 - \delta) \sum_{\tau=t}^{\infty} \delta^{\tau-t}c_t \geq 0.
\]
Here, the first inequality follows from the previous result that \( w_t \leq R(\bar{w}) \) in the community.

Next, we show that \(-L_t \leq 0\) for any \( t \geq 0\). Suppose not. Let \( k \in (0,1] \) be the smallest number such that:

\[-L_t \leq kR(\bar{w}), \forall t \geq 0.\]

Therefore, there exists a period \( t' \) such that:

\[-L_{t'} > k\delta R(\bar{w}).\]

Consider the following deviation in period \( t' \): the household chooses \( s_{t'} = 0 \) and leaves for the city. In each \( \tau \geq t' \), it chooses the investment \( \tilde{I}_\tau \) given by:

\[\tilde{I}_\tau = I_\tau - s_\tau + \frac{k\delta R(\bar{w}) + L_\tau}{1 - \delta},\]

with the remainder, \( \tilde{c}_\tau \), being consumed. We show that this deviation (i) is feasible, and (ii) strictly improves the household’s payoff.

Note that in each \( \tau \geq t' \),

\[\tilde{I}_\tau - I_\tau = -s_\tau + \frac{k\delta R(\bar{w}) + L_\tau}{1 - \delta} = \frac{k\delta R(\bar{w}) + L_\tau - (1 - \delta)s_\tau}{1 - \delta} = \delta \frac{kR(\bar{w}) + L_{\tau+1}}{1 - \delta} \geq 0, \quad (6)\]

where the inequality holds by the definition of \( k \). Thus, this deviation results in more investment, and so higher wealth, in each period \( \tau \geq t' \).

Next, we show that this deviation is profitable. Using the definition of \( \tilde{I}_\tau \) and the fact that \( \bar{w}_{t'} = w_{t'} \), period-\( t' \) consumption satisfies

\[\tilde{c}_{t'} = \bar{w}_{t'} - \tilde{I}_{t'} = w_{t'} - I_{t'} + s_{t'} - \frac{k\delta R(\bar{w}) + L_{t'}}{1 - \delta} > c_{t'}.\]

Here, the inequality follows from the fact that \( k\delta R(\bar{w}) + L_{t'} < 0 \) by choice of \( t' \). So this deviation results in strictly higher consumption in period \( t' \).
Now consider $\tau > t'$. From (6) and $R'(\cdot) \geq \frac{1}{\delta}$, we have that:

$$\tilde{w}_\tau - w_\tau = R(\tilde{I}_{\tau-1}) - R(I_{\tau-1}) \geq \frac{kR(\bar{w}) + L_\tau}{1-\delta}.$$ 

Therefore,

$$\tilde{c}_\tau = \tilde{w}_\tau - \tilde{I}_\tau \geq w_\tau + \frac{kR(\bar{w}) + L_\tau}{1-\delta} - \left(I_\tau - s_\tau + \frac{kR(\bar{w}) + L_\tau}{1-\delta}\right)$$

$$= c_\tau + kR(\bar{w}) > c_\tau.$$ 

Therefore, this deviation leads to strictly higher consumption in every $\tau \geq t'$. Since $\hat{U} \geq U$, we conclude that this deviation is profitable, which contradicts the presumption that the original strategy was an equilibrium. Thus, it must be that $-L_t \leq 0$ for any $t \geq 0$

Suppose that $s_t > 0$ in some period $t \geq 0$. For the neighbor to be willing to choose $s_t > 0$, it must be that her continuation payoff satisfies $-\sum_{\tau=t+1}^{\infty} \delta^{\tau-(t+1)} s_\tau > 0$, or $-L_{t+1} > 0$. This contradicts $-L_{t+1} \leq 0$. We conclude that $s_t \leq 0$ in all $t \geq 0$, which implies $s_t = 0$ for all $t \geq 0$, proving the proposition. $\blacksquare$
C Online Appendix: Stochastic Investment Returns

C.1 Result and Simulations

In some communities, favor exchange not only augments consumption in normal times but also provides insurance against negative shocks. To explore how this insurance role affects wealth dynamics, this section enriches the model to incorporate stochastic investment returns. We show that a version of our main result continues to hold so long as returns are not too noisy. Using simulations, we also show that extremely noisy returns can transform favor exchange in a way that is similar to community benefits.

We consider a model with stochastic returns, which is identical to the model from Section 2 except that investment returns are given by a distribution rather than a deterministic function. Formally, in every $t \geq 0$, period-$(t + 1)$ wealth, $w_{t+1}$, is drawn from the distribution $F(\cdot | I_t)$, where $I_t = w_t - p_t$ is the investment in period $t$. Wealth is drawn independently across periods conditional on the investment and is publicly observed.

We impose the following assumption on the return distributions $\{F(\cdot | I)\}_{I \geq 0}$.

Assumption 2

1. For every investment $I \geq 0$, the return distribution $F(\cdot | I)$ has support $[R^L(I), R^H(I)] \subseteq \mathbb{R}_+$. Both $R^L(\cdot)$ and $R^H(\cdot)$ are strictly increasing, concave, and continuously differentiable. There exist thresholds $\bar{w}^L, \bar{w}^H > 0$ such that (i) $(R^L)'(w) > \frac{1}{3}$ for $w < \bar{w}^L$ and $(R^L)'(w) = \frac{1}{3}$ for $w \geq \bar{w}^L$; and (ii) $(R^H)'(w) > \frac{1}{3}$ for $w < \bar{w}^H$ and $(R^H)'(w) = \frac{1}{3}$ for $w \geq \bar{w}^H$.

2. Define $c^* > 0$ as the solution to $\hat{U}'(c^*) = 1$. Then, $R^H(\bar{w}^H) - c^* > \bar{w}^H$.

The first part of this assumption requires the bounds on the support of $\{F(\cdot | I)\}_{I \geq 0}$, $R^L(I)$ and $R^H(I)$, to satisfy similar conditions as the return function $R(\cdot)$ in the baseline model, with the sole exception that $R^L(0)$ and $R^H(0)$ need not be zero. The second part is analogous to Assumption 1 and ensures that even if investment returns were given by $R^H(\cdot)$, sufficiently wealthy households would leave the community.
Our main result in this section shows that if returns are not too noisy, then both the selection effect and the “too-big-for-their-boots” mechanism hold.

**Proposition 5** Consider a sequence of return distributions, \( \{F^k(\cdot|I)\}_{I \geq 0} \), such that \( \{F^k(\cdot|I)\}_{I \geq 0} \) satisfies Assumption 2 for every \( k \geq 0 \). Let the corresponding bounds on supports, \( R^{H,k}(\cdot) \) and \( R^{L,k}(\cdot) \), be monotone in \( k \) and converge uniformly to a return function \( R(\cdot) \) as \( k \to \infty \). Suppose that \( R(\cdot) \) satisfies the conditions imposed in Section 2. Then, there exists \( K < \infty \) such that for any \( k \geq K \):

1. **Selection.** There exists a selection threshold \( w^{se,k} > 0 \) and a positive-measure set \( W^k \subseteq \mathbb{R}_+ \) with \( \sup W^k = w^{se,k} \) such that in any household-optimal equilibrium, a household in the community chooses to stay in period \( t \) if \( w_t \subseteq W^k \) and otherwise leaves.

2. **“Too-big-for-their-boots” mechanism.** There exists a wealth level \( w^{tr,k} < w^{se,k} \) such that in any household-optimal equilibrium, if a household chooses to stay in the community and \( w_t \geq w^{tr,k} \), then \( R^{H,k}(I_t) < w^{tr,k} \). The set \( W^k \cap [w^{tr,k}, w^{se,k}] \) has a positive measure.

Moreover, there exist constants \( \bar{w}, \gamma > 0 \) such that \( [0, \bar{w}] \subseteq W^k \) and \( w^{tr,k} < w^{se,k} - \gamma \) for any \( k \geq K \).

We prove Proposition 5 below, in Online Appendix C.2. For sufficiently large \( k \), the return distributions \( \{F^k(\cdot|I)\}_{I \geq 0} \) are not too noisy, in the sense that the bounds on their supports, \( R^{H,k}(\cdot) \) and \( R^{L,k}(\cdot) \), are close to one another. For such \( k \), the selection result says that wealthy households leave the community, while a positive measure of less-wealthy households stay in the current period. This result is the analogue of the selection result in Proposition 2.

The second statement gives a strong version of the “too-big-for-their-boots” mechanism. It says that a relatively wealthy household in the community invests so little that its wealth
drops below $w^{tr,k}$ even under the most favorable return function $R^{H,k}$. In other words, such a household’s wealth decreases with probability 1.

The proof of Proposition 5 follows the structure of the proof of Proposition 2, but stochastic returns make the arguments considerably more complicated. We proceed by bounding equilibrium payoffs from above and below using various deterministic return functions. For example, equilibrium payoffs in the city are bounded from above and below by equilibrium payoffs in the city when returns are given by $R^{H,k}$ and $R^{L,k}$, respectively. As $k \to \infty$, these bounds converge, so we can approximate equilibrium payoffs in the city using properties from the model in Section 2. We can similarly approximate equilibrium payoffs in the community. Given these approximations, we extend the proof of Proposition 2 to stochastic returns, taking care to ensure that the arguments hold uniformly across all wealth levels and for any $k \geq K$.

![Figure 8: Wealth dynamics and investment with moderately large shocks. $F(.)$ has binary support, with $R^{H,I} = 1.15 * R(I)$ and $R^{L,I} = 0.85 * R(I)$ each occurring with probability 1/2. Here, $R(w)$ and all other parameters are the same as those in Figures 1-3. The left panel gives the range of $w_{t+1}$ realizations as a function of $w_t$. The right panel gives investment in the city and the community as a function of wealth.](image)

While Proposition 5 applies to “small” shocks, numerical simulations suggest that it remains true even when shocks are moderately large. For instance, Figure 8 shows that it holds for a binary distribution over returns in which the high return is 35 percent higher than the low return. In this simulation, a household that stays in the community invests so
little that it remains there forever *regardless* of the shocks. Further (unreported) simulations confirm that Proposition 5 holds widely for moderately large shocks.

In contrast, qualitatively different wealth dynamics can emerge when shocks are very large. In Figure 8, the household faces an extremely large binary shock: the high return is 133 percent higher than the low return. With such large shocks, a household that stays in the community invests *strictly more* than it would in the city. This stark transformation occurs because with very large shocks, even a wealthy household has substantial commitment power, since it is willing to repay favors today in order to keep accessing favor exchange following a negative shock. Similar to Section 5, this commitment power turns favor exchange into a force that accelerates wealth accumulation.
C.2 Proof of Proposition 5

C.2.1 Uniform convergence of $\hat{\Pi}^{H,k}(\cdot)$ and $\hat{\Pi}^{L,k}(\cdot)$ to $\hat{\Pi}(\cdot)$.

Let $\hat{\Pi}(\cdot)$ be the equilibrium payoff in the city if the investment is given by $R(\cdot)$. Similarly, $\hat{\Pi}^{H,k}(\cdot)$ is the payoff under the return function $R^{H,k}(\cdot)$ and $\hat{\Pi}^{L,k}(\cdot)$ is the payoff under the return function $R^{L,k}(\cdot)$.

We want to show that $\forall w \geq 0$ and $\forall \epsilon > 0$, there exists $K$ such that $\forall k > K$, $\hat{\Pi}^{H,k}(w) - \hat{\Pi}(w) < \epsilon$ and $\hat{\Pi}(w) - \hat{\Pi}^{H,k}(w) < \epsilon$. This shows pointwise convergence. By the monotonicity of $\hat{\Pi}^{H,k}(\cdot), \hat{\Pi}^{L,k}(\cdot)$ and Dini’s theorem, we obtain uniform convergence.

We begin with the zero wealth $w = 0$. Since $R^{L,k}(0) \leq R(0) = 0$, $\hat{\Pi}^{L,k}(0) = 0$. We now consider the return function $R^{H,k}(\cdot)$ and construct the following more generous return function:

$$\hat{R}^{H,k}(w) = \frac{w - \bar{w}^{H,k}}{\delta} + R^{H,k}(\bar{w}^{H,k}).$$

Given the initial wealth $w$ and the return function $\hat{R}^{H,k}(\cdot)$, the consumption sequence must satisfy the following condition:

$$(1 - \delta) \sum_{t=0}^{\infty} \delta^t c_t = (1 - \delta)w + \delta R^{H,k}(\bar{w}^{H,k}) - \bar{w}^{H,k}. $$

Hence, the equilibrium payoff giving the initial wealth $w$ under $\hat{R}^{H,k}(\cdot)$ is:

$$\hat{U}((1 - \delta)w + \delta R^{H,k}(\bar{w}^{H,k}) - \bar{w}^{H,k}).$$

We now allow the household to both save and consume its entire wealth from period 0 to $T - 1$. From period $T$ onward, we give the household the return function $\hat{R}^{H,k}(\cdot)$. The household’s payoff given the initial wealth $w = 0$ is:

$$(1 - \delta) \sum_{t=0}^{T-1} \delta^t \hat{U}((R^{H,k})^t(0)) + \frac{\delta^T \hat{U}((1 - \delta)(R^{H,k})^T(0) + \delta R^{H,k}(\bar{w}^{H,k}) - \bar{w}^{H,k})}{1 - \delta}.$$ (7)
Fix any $T$, as $k \to \infty$, $(R^{H,k})^T(0) \downarrow 0$, so $(R^{H,k})^t(0) \downarrow 0$ for any $t \leq T$. Moreover, there exists a $T$ such that:
\[
\delta^T \hat{U} \left( \delta R^{H,k}(\bar{w}^{H,k}) - \bar{w}^{H,k} \right) < \frac{\epsilon}{2}.
\]
Therefore, for this value of $T$, there exists $K > 0$ such that for any $k > K$ the payoff (7) is at most $\epsilon$, so the payoff $\hat{\Pi}^{H,k}(0)$ is at most $\epsilon$. This concludes the proof of pointwise convergence at $w = 0$.

We now prove the pointwise convergence at $w > 0$. Let $\{c_t\}_{t=0}^{\infty}$ be the consumption sequence in the city under return function $R(\cdot)$. Recall that $c_0 > 0$ and $c_t$ increases in $t$. For any $\epsilon$, there exists a small enough $\gamma \in (0, c_0)$ such that:
\[
(1 - \delta) \sum_{t=0}^{\infty} \hat{U}(c_t - \gamma) > \hat{\Pi}(w) - \epsilon.
\]
On the other hand, there exists $K > 0$ such that for any $k > K$, $R_{L,k}(w) > R(w) - \gamma$ for every $w \geq 0$. Under such $R_{L,k}(\cdot)$, the consumption sequence $\{c_t - \gamma\}_{t=0}^{\infty}$ is feasible, since the household can choose the same investment sequence as under $R(\cdot)$ and consume the rest in each period. Hence, $\hat{\Pi}^{L,k}(w) > \hat{\Pi}(w) - \epsilon$.

Let $\{c_t^{H,k}\}_{t=0}^{\infty}$ be the consumption sequence under the return function $R^{H,k}(\cdot)$. By the next claim, claim 1, there exists a $c > 0$ such that $c_0^{H,k} > c$ for any $k$. By the Euler equation, $c_t^{H,k}$ increases in $t$. Fix any sufficiently small $\epsilon > 0$ and consider the number $\gamma_k > 0$ which solves the following equation:
\[
\hat{U}(c_0^{H,k}) - \hat{U}(c_0^{H,k} - \gamma_k) = \epsilon.
\]
Since $c_t^{H,k}$ increases in $t$, we have:
\[
\hat{U}(c_t^{H,k}) - \hat{U}(c_t^{H,k} - \gamma_k) \leq \epsilon, \forall t \geq 0.
\]
This implies that
\[
\hat{\Pi}^{H,k}(w) - (1 - \delta) \sum_{t=0}^{\infty} \hat{U}(c_t^{H,k} - \gamma_k) \leq \epsilon.
\]
Moreover, since \( c_0^k \geq \varepsilon \) for any sufficiently large \( k \), there exists \( \gamma > 0 \) such that \( \gamma_k \geq \gamma \) for any such \( k \).

By the convergence of \( R^{H,k}(\cdot) \) to \( R(\cdot) \), there exists \( K > 0 \) such that for any \( k > K \), we have
\[
R(w) > R^{H,k}(w) - \gamma, \ \forall w \geq 0.
\]
This implies that \( \{c_t^{H,k} - \gamma_k\}_{t=0}^{\infty} \) is feasible under the return function \( R(\cdot) \), so \( \hat{\Pi}(w) > \hat{\Pi}^{H,k} - \epsilon \) for any \( k > K \).

**Claim 1:** Fix the initial wealth \( w_0 > 0 \). There exists \( \varepsilon > 0 \) such that \( c_0^{H,k} > \varepsilon \) for any \( k \geq 0 \).

**Proof of claim 1:** We proceed by bounding period-\( t \) consumption from above as a function of \( c_0^{H,k} \). By (Euler), equilibrium consumption satisfies
\[
\hat{U}'\left(c_t^{H,k}\right) = \delta(R^{H,k})'(w_t^{H,k} - c_t^{H,k}) \hat{U}'\left(c_{t+1}^{H,k}\right).
\]
Since \( w_t^{H,k} - c_t^{H,k} \) is increasing in \( t \) and \( (R^{H,k})'(\cdot) \) is decreasing,
\[
\hat{U}'\left(c_t^{H,k}\right) \leq \delta(R^{H,k})'(w_0 - c_0^{H,k}) \hat{U}'\left(c_{t+1}^{H,k}\right).
\]
Moreover,
\[
(R^{H,k})'(1) \leq \frac{R^{H,k}(I) - R^{H,k}(0)}{I} \leq \frac{R^{H,0}(I)}{I}.
\]
where the first inequality follows because $R^{H,k}$ is concave, and the second inequality follows because $R^{H,k}(0) \geq 0$ and $R^{H,k}(I)$ is decreasing in $k$. Thus, in equilibrium,

$$\hat{U}'(c_{t}^{H,k}) \leq \frac{\delta R^{H,0}(w_{0} - c_{0}^{H,k})}{w_{0} - c_{0}^{H,k}} \hat{U}'(c_{t+1}^{H,k}).$$

(8)

Let $\tilde{c}(c_{t}^{H,k}; w_{0}, c_{0}^{H,k})$ be the largest $c_{t+1}^{H,k}$ that satisfies (8), given $(c_{t}^{H,k}, w_{0}, c_{0}^{H,k})$. Since $\hat{U}'(\cdot)$ is decreasing, $\tilde{c}$ satisfies this inequality with equality. Notice that for a fixed $(w_{0}, c_{0}^{H,k})$, $\tilde{c}(\cdot; w_{0}, c_{0}^{H,k})$ is continuous, with $
abla_{c_{t}} \tilde{c}(c; w_{0}, c_{0}^{H,k}) = 0$. Thus, for any $t \geq 1$, we have that $\lim_{c_{t} \downarrow 0} \nabla_{c_{t}} \tilde{c}(w_{0}, c_{0}^{H,k}) = 0$, where $\nabla_{c_{t}} \tilde{c}$ is defined inductively by $\nabla_{c_{t}} \tilde{c} = \tilde{c}(c_{0}^{H,k}; w_{0}, c_{0}^{H,k})$ and $\nabla_{c_{t}} \tilde{c}(w_{0}, c_{0}^{H,k}) = \tilde{c}(\nabla_{c_{t}} \tilde{c}^{-1}(w_{0}, c_{0}^{H,k}); w_{0}, c_{0}^{H,k})$ for $t > 1$.

For any $t \geq 1$, we claim that $\nabla_{c_{t}} \tilde{c}(w_{0}, c_{0}^{H,k}) \geq c_{t}^{H,k}$. We prove this inductively: for $t = 1$, $\nabla_{c_{1}} \tilde{c}(w_{0}, c_{0}^{H,k}) \geq c_{1}^{H,k}$ because $c_{1}^{H,k}$ satisfies (8) and $c_{1}^{H,k}$ is the largest consumption that satisfies this inequality. If $\nabla_{c_{t-1}} \tilde{c}(w_{0}, c_{0}^{H,k}) \geq c_{t-1}^{H,k}$, then (suppressing arguments):

$$\hat{U}'(c_{t}^{H,k}) \geq \frac{w_{0} - c_{0}^{H,k}}{\delta R^{H,0}(w_{0} - c_{0}^{H,k})} \hat{U}'(c_{t-1}^{H,k}) \geq \frac{w_{0} - c_{0}^{H,k}}{\delta R^{H,0}(w_{0} - c_{0}^{H,k})} \hat{U}'(\nabla_{c_{t-1}} \tilde{c}^{-1}) = \hat{U}'(\nabla_{c_{t}} \tilde{c}),$$

where the first inequality is a rewriting of (8), the second inequality follows from $\nabla_{c_{t-1}} \tilde{c} \geq c_{t-1}^{H,k}$ and $\hat{U}'(\cdot)$ decreasing, and the final equality holds because $\nabla_{c_{t}} \tilde{c} = \tilde{c}(\nabla_{c_{t}} \tilde{c}^{-1}; w_{0}, c_{0}^{H,k})$. We conclude that $\nabla_{c_{t}} \tilde{c} \geq c_{t}^{H,k}$, proving the claim.

Define $\bar{c}(w_{0}, c_{0}^{H,k}) = c_{t}^{H,k}$. We provide an upper bound on $\hat{H}^{H,k}(w)$ using the following bounds on consumption: (i) for $t \in \{0, ..., T - 1\}$, the household consumes $\nabla_{c_{t}} \tilde{c}$, which is greater than $c_{t}^{H,k}$; (ii) during these periods, the household saves its entire wealth using the return function $R^{H,0}(\cdot)$ which is greater than $R^{H,k}(\cdot)$, so that its wealth at the beginning of period $T$ is $(R^{H,0})^{T}(w_{0})$; and (iii) from period $T$ onward, the return function is given by the more generous function $\bar{R}^{H,0}(w) = \frac{w - \bar{w}^{H,0}}{\delta} + R^{H,0}(\bar{w}^{H,0})$. Thus, for any $T \geq 0$,

$$\hat{H}^{H,k}(w_{0}) \leq (1 - \delta) \sum_{t=0}^{T-1} \delta^{t} \hat{U}(\nabla_{c_{t}} \tilde{c}(w_{0}, c_{0}^{H,k})) + \delta^{T} \hat{U}((1 - \delta)(R^{H,0})^{T}(w_{0}) + \delta R^{H,0}(\bar{w}^{H,0}) - \bar{w}^{H,0}).$$

(9)
Since $R^{H,0}(w) \leq \hat{R}^{H,0}(w)$,

$$(R^{H,0})^T(w) \leq (\hat{R}^{H,0})^T(w) = \frac{w}{\delta T} + \frac{\delta (1 - \delta T)}{(1 - \delta)\delta T} \left( R^{H,0}(\bar{w}^{H,0}) - \frac{\bar{w}^{H,0}}{\delta} \right).$$

Consequently,

$$\lim_{T \to \infty} \delta T \hat{U}\left((1 - \delta)(R^{H,0})^T(w_0) + \delta R^{H,0}(\bar{w}^{H,0}) - \bar{w}^{H,0}\right) \leq \lim_{T \to \infty} \delta T \hat{U}\left((1 - \delta)\frac{w}{\delta T} + \frac{\delta (1-\delta T)}{\delta T} \left( R^{H,0}(\bar{w}^{H,0}) - \frac{\bar{w}^{H,0}}{\delta} \right) + \delta R^{H,0}(\bar{w}^{H,0}) - \bar{w}^{H,0}\right) = 0,$$

where the equality follows from L’Hopital’s rule and the assumption that $\hat{U}'(c) \to 0$ as $c \to \infty$. Thus, the final term in (9) goes to zero as $T \to \infty$.

We are now prepared to demonstrate that the lower bound $c > 0$ exists. Since $R^{H,k}(\cdot) \geq R(\cdot)$, $\hat{\Pi}^{H,k}(w_0) \geq \hat{\Pi}(w_0)$. Choose $\epsilon \in (0, \hat{\Pi}(w_0))$. There exists $T$ independent of $k$ such that the final term in (9) is at most $\epsilon/2$. For this $T$, there exists $\xi > 0$ independent of $k$ such that the first term in (9) is at most $\epsilon/2$ if $c^{H,k}_0 < \xi$. This shows that for any $k$, $c^{H,k}_0 > \xi$. Otherwise, there exists $k$ such that $\hat{\Pi}^{H,k}(w_0) < \epsilon < \hat{\Pi}(w_0)$, leading to a contradiction. ■

C.2.2 Selection

For any $k$, consider the game with stochastic return $F^k$ and let $\mathcal{W}^k$ be the collection of wealth levels such that the household stays in the community if its wealth is in $\mathcal{W}^k$ and leaves the community otherwise. Let $w^{se,k} = \sup \mathcal{W}^k$ denote the selection margin. We want to show that when $k$ is large enough:

1. $\mathcal{W}^k$ includes $[0, w]$ for some wealth level $w > 0$ which is independent of $k$;

2. $w^{se,k} \leq \bar{w}^{H,k} + c^*$ which is smaller than $R^{H,k}(\bar{w}^{H,k})$.

We first show that $\mathcal{W}^k$ includes $[0, w]$ for some fixed $w > 0$. Recall that $\Pi^*(0) = \max_c \{U(c) - c | c \leq \frac{\delta}{1-\delta} (U(c) - c)\} > 0$ and that $\hat{\Pi}(w)$ is a continuous function with
\[ \hat{\Pi}(0) = 0. \] Fix a small \( \gamma > 0. \) Let \( w = \max\{w | \hat{\Pi}(w') \leq \Pi^*(0) - 2\gamma, \text{ for every } w' \in [0, w] \}. \]

We next show that there exists \( K \) such that for every \( k \geq K, [0, w] \subseteq \mathcal{W}^k. \)

Denote by \( \Pi^{*, k}(w) \) the household-optimal equilibrium payoff under the stochastic return \( F^k(\cdot). \) Then

\[
\Pi^{*, k}(w) \geq \max_c \left\{ U(c) - c | c \leq \frac{\delta}{1 - \delta} \left( U(c) - c - \hat{\Pi}^{H,k}(R^{H,k}(0)) \right) \right\},
\]

since the household can always consume all wealth in each period and rely entirely on favor exchange. Since \( \hat{\Pi}^{H,k} \) is continuous, with \( \hat{\Pi}^{H,k} \) and \( R^{H,k} \) converging uniformly to \( \hat{\Pi} \) and \( R, \) respectively, we have \( \lim_{k \to \infty} \hat{\Pi}^{H,k}(R^{H,k}(0)) = \hat{\Pi}(R(0)) = 0. \) Therefore, there exists \( K_1 \) such that if \( k \geq K_1, \Pi^{*, k}(w) > \Pi^*(0) - \gamma. \) Meanwhile, there exists \( K_2 \) such that if \( k \geq K_2, \hat{\Pi}^{H,k}(w) - \hat{\Pi}(w) < \gamma \) for every \( w \geq 0. \) Therefore, if \( k \geq \max\{K_1, K_2\}, \) for any \( w \in [0, w]: \)

\[
\Pi^{*, k}(w) > \Pi^*(0) - \gamma \geq \hat{\Pi}(w) + \gamma > \hat{\Pi}^{H,k}(w),
\]

where the second inequality follows from the definition of \( w. \) This shows that \([0, w] \subseteq \mathcal{W}^k. \)

We now show that when \( k \) is large enough, \( w^{se, k} \leq w^{H,k} + c^*. \) For any \( k, \) we define an auxiliary game which differs from the original game with the stochastic return \( F^k \) in two ways:

1. the investment return is given by the deterministic function \( R^{H,k}(\cdot) \) instead of the stochastic return \( F^k(\cdot) \) in the original game;

2. in any period \( t, \) if the household stays in the community, he can commit to \( f_t. \) In other words, the household faces no dynamic enforcement constraint.

Let \( \tilde{\Pi}^{H,k}(\cdot) \) be the household’s optimal equilibrium payoff in this auxiliary game if it initially lives in the community. Then, \( \tilde{\Pi}^{H,k}(w) = \max\{\tilde{\Pi}^{H,k}(w), \Pi_c^{H,k}(w)\}, \) where \( \Pi_c^{H,k}(w) \) is given by:

\[
\Pi_c^{H,k}(w) = \max_{I \in [0, w], f \geq 0} \left\{ (1 - \delta) \left( U(w + f - I) - I \right) + \delta \tilde{\Pi}^{H,k}(R^{H,k}(I)) \right\}. \tag{10}
\]
It follows that $\Pi^H,c_k(w)$ is an upper bound on the household’s payoff if it optimally stays in the community in the original game. On the other hand, $\hat{\Pi}^L,c_k(w)$ is a lower bound on the household’s payoff if it leaves for the city in the original game. If $\Pi^H,c_k(w) < \hat{\Pi}^L,c_k(w)$, then the household strictly prefers to leave for the city at the wealth level $w$ in the original game.

We now show that this is the case for any $w \geq \bar{w}^{H,k} + c^*$.

**Claim 2:** In the auxiliary game, given a wealth level $w_t \geq 0$, if the solution to (10) is such that $w_t + f_t - c_t > \bar{c}$, then $f_t = 0$ and $\hat{\Pi}^{H,k}(R^{H,k}(I_t)) = \hat{\Pi}^{H,k}(R^{H,k}(I_t))$.

**Proof of Claim 2:** We will show that $w_t + f_t - I_t > \bar{c}$ implies that $f_t = 0$ for all $t' \geq t$. This implies that the optimal continuation payoff at the beginning of $(t + 1)$ is $\hat{\Pi}^{H,k}(R^{H,k}(I_t))$.

The first step is to show that $w_t + f_t - I_t > \bar{c}$ implies that $f_t = 0$. Suppose otherwise. Then we could decrease $f_t$ by a small amount which will increase the household’s period-$t$ payoff at rate $-U'(w_t + f_t - I_t) + 1 > 0$, where the inequality follows because $U''(w_t + f_t - I_t) < U''(\bar{c}) = 1$.

Now suppose that $f_t = 0$ and that there exists $t' > t$ such that $f_{t'} > 0$. Consider the following perturbation: (i) in period $t$, increase $I_t$ by $\epsilon > 0$ and decrease $c_t$ by the same amount; (ii) in period $t'$ decrease $f_{t'}$ and $I_{t'}$ by $\frac{\epsilon}{\delta^{t'-t}}$. We claim that this perturbation is feasible for small $\epsilon$. Indeed, $w_t - I_t > \bar{c}$, so we can increase $I_t$ by $\epsilon$. Moreover, this perturbation increases $w_{t'}$ by at least $\frac{\epsilon}{\delta^{t'-t}}$ because $(R^{H,k})'(w) \geq \frac{1}{\delta}$, so decreasing $I_{t'}$ by $\frac{\epsilon}{\delta^{t'-t}}$ is feasible. As $\epsilon \to 0$, this perturbation increases the household’s payoff at a rate which is at least:

$$-U'(w_t - I_t) + \delta^{t'-t} \frac{1}{\delta^{t'-t}} > 0,$$

where the inequality follows because $U''(w_t - I_t) < 1$.

We conclude that if the solution to (10) is such that $w_t + f_t - c_t > \bar{c}$, then $f_{t'}$ for all $t' \geq t$, which implies the claim.
Claim 3: Suppose that \( w_0 \geq \bar{w}^{H,k} + c^* \). Then:

\[
\Pi^{H,k}_{c}(w_0) < \hat{\Pi}^{H,k}(w_0) - (1 - \delta) \left( \hat{U}(\bar{c}) - U(\bar{c}) \right).
\]

(11)

Proof of Claim 3: It suffices to show that the solution to (10) entails \( w_0 + f_0 - I_0 > \bar{c} \). If so, the previous claim says that \( f_0 = 0 \) and the household’s continuation payoff \( \hat{\Pi}^{H,k}(R^{H,k}(I_0)) \) is equal to its payoff from the city \( \hat{\Pi}^{H,k}(R^{H,k}(I_0)) \). If the household instead leaves at the beginning of period 0, it could replicate the entire consumption sequence in the city. Its payoff from doing so is

\[
(1 - \delta)\hat{U}(c_0) + \delta \hat{\Pi}^{H,k}(R^{H,k}(I_0)),
\]

which exceeds \( \Pi^{H,k}_{c}(w_0) \) by \((1 - \delta)(\hat{U}(c_0) - U(c_0)) > (1 - \delta)(\hat{U}(\bar{c}) - U(\bar{c})) \), and is weakly smaller than \( \hat{\Pi}^{H,k}(w_0) \). The inequality (11) then follows. For the rest of the proof, we will show that the solution to (10) entails \( w_0 + f_0 - I_0 > \bar{c} \).

Suppose, towards contradiction, that the solution to (10) given the wealth level \( w_0 \geq \bar{w}^{H,k} + c^* \) entails \( w_0 + f_0 - I_0 \leq \bar{c} \). Then, \( I_0 \geq w_0 - \bar{c} \geq \bar{w}^{H,k} + c^* - \bar{c} > \bar{w}^{H,k} \). So, \( w_1 = R^{H,k}(I_0) > R^{H,k}(\bar{w}^{H,k}) > \bar{w}^{H,k} + c^* \). We now consider the auxiliary game starting from period 1. Either the household leaves the community in \( t = 1 \) or it stays. In either case, it must be unable to improve its payoff \( \Pi^{H,k}_{c}(w_0) \) by decreasing \( I_0 \) slightly and increasing \( c_1 \) by an amount equal to the resulting decrease in \( w_1 \), which implies that:

\[
U'(c_0) \leq \delta(R^{H,k})'(I_0)\hat{U}'(c_1)
\]

if the household leaves the community in \( t = 1 \), or

\[
U'(c_0) \leq \delta(R^{H,k})'(I_0)U'(c_1)
\]

if the household stays in \( t = 1 \).

Suppose that the household leaves the community \( t = 1 \). We first argue that \( c_1 > c^* \).
If $c_1 \leq c^*$, then $I_1 = w_1 - c_1 > \bar{w}^{H,k}$, so $(R^{H,k})'(I_1) = \frac{1}{\delta}$. This implies that $c_2 = c_1$ by the Euler equation and that $I_2 = w_2 - c_2 > R^{H,k}(\bar{w}^{H,k}) - c^* > \bar{w}^{H,k}$. Iterating this argument, we can show that if $c_1 \leq c^*$ and $I_1 > \bar{w}^{H,k}$, then $c_t$ is constant for all $t \geq 1$, which contradicts the fact that $w_1 > R^{H,k}(\bar{w}^{H,k}) > \bar{w}^{H,k} + c^*$. Therefore, $c_1 > c^*$. But then, (12) implies that $U'(c_0) \leq \delta(R^{H,k})'(I_0)\hat{U}'(c_1) < 1$, where the last inequality follows from $(R^{H,k})'(I_0) = 1$ and $\hat{U}'(c_1) < \hat{U}'(c^*) = 1$. This contradicts the presumption that $U'(c_0) \geq 1$. Therefore, the household cannot leave the community in $t = 1$ if $c_0 \leq \bar{c}$ and $w_1 > R^{H,k}(\bar{w}^{H,k})$.

Suppose instead that the household stays in the community in $t = 1$. Then, (13) implies that $U'(c_0) \leq U'(c_1)$ since $(R^{H,k})'(I_0) = \frac{1}{\delta}$. Therefore, $c_1 \leq c_0 \leq \bar{c}$ and $w_2 > R^{H,k}(\bar{w}^{H,k})$. Consequently, the argument in the previous paragraph implies that the household must stay in the community in $t = 2$ as well, with $c_2 \leq \bar{c}$ and $w_3 > R^{H,k}(\bar{w}^{H,k})$. We can iterate this argument to conclude that the household must stay in the community in every $t \geq 1$, with $c_t \leq \bar{c}$ in every $t \geq 1$. But this cannot be optimal, since the household can instead leave in $t = 1$ and receive a payoff strictly greater than $\hat{U}(c^*) > U(\bar{c})$. So the household cannot stay in the community in $t = 1$.

We have shown a contradiction to either the household leaving the community in $t = 1$ or staying. We conclude that $w_0 + f_0 - I_0 > \bar{c}$ in any household-optimal equilibrium, which proves Claim 3. ■

Completing the proof of Selection: Finally, we turn to the original game. Choose $K$ such that for any $k \geq K$, $\hat{\Pi}^{H,k}(w) - \hat{\Pi}^{L,k}(w) \leq (1 - \delta)(\hat{U}(\bar{c}) - U(\bar{c}))$ for any $w \geq 0$. For any such $k$, fix $w \geq \bar{w}^{H,k} + c^*$, and suppose that the household optimally remains in the community. As we noted previously, its payoff is then bounded from above by $\Pi^H_{c,k}(w)$, which by Claim 3, satisfies:

$$\Pi^H_{c,k}(w) < \hat{\Pi}^{H,k}(w) - (1 - \delta)(\hat{U}(\bar{c}) - U(\bar{c})) < \hat{\Pi}^{L,k}(w).$$
But $\hat{\Pi}^{L,k}(w)$ bounds the household’s payoff in the city from below. Therefore, it cannot be optimal for the household to remain in the community. We conclude that for any $k \geq K$ and $w \geq \bar{w}^{H,k} + c^*$, the household optimally leaves the community at wealth level $w$. ■

C.2.3 The “Too Big for their Boots” Mechanism

We want to prove the following claim. There exists $\gamma > 0$ and $K > 0$, such that for any $k > K$ in the game with stochastic return $F^k$, if $w_t \in [w^{se,k} - \gamma, w^{se,k}]$ and the household chooses to stay in the community in period $t$, then $R^{H,k}(I_t) < w^{se,k} - \gamma$.

We first explain how $\gamma$ is chosen. We have shown in the selection proof that there exists $K_0$ such that for any $k > K_0$, $w^{se,k} \geq \underline{w}$. Define the function $\Delta(\epsilon)$ as follows:

$$\Delta(\epsilon) = \max_{w \in [\underline{w}, \infty)} \left( \hat{\Pi}(w + \epsilon) - \hat{\Pi}(w - 2\epsilon) \right),$$

with $\lim_{\epsilon \to 0} \Delta(\epsilon) = 0$. Consider the following expression as a function of $w \in [w^{se,k} - \epsilon, w^{se,k}]$:

$$\hat{U} \left( w - R^{-1}(w^{se,k} + \epsilon) \right) - U \left( w - R^{-1}(w^{se,k} - 2\epsilon) + \frac{\delta}{1 - \delta} (\Delta(\epsilon) + 2\epsilon) \right). \quad (14)$$

As $\epsilon \to 0$, the value of (14) converges to

$$\hat{U} \left( w^{se,k} - R^{-1}(w^{se,k}) \right) - U \left( w^{se,k} - R^{-1}(w^{se,k}) \right). \quad (15)$$

Since $(w - R^{-1}(w))$ is an increasing function of $w$ and $w^{se,k} \geq \underline{w}$ for any $k > K_0$, the value of (15) is at least:

$$\hat{U} \left( w - R^{-1}(\underline{w}) \right) - U \left( w - R^{-1}(\underline{w}) \right) > 0.$$  

Hence, there exists $\gamma \in (0, \underline{w})$ such that for any $k > K_0$, (14) is greater than $\frac{\delta}{1 - \delta} (\Delta(\gamma) + 2\gamma)$ for every $w \in [w^{se,k} - \gamma, w^{se,k}]$.

Given this $\gamma > 0$, there exists $K_1 > 0$ such that for every $k > K_1$, $R^{H,k}(w) - R^{L,k}(w)$, $\hat{\Pi}^{H,k}(w) - \hat{\Pi}(w)$, and $\hat{\Pi}(w) - \hat{\Pi}^{L,k}(w)$ are all smaller than $\gamma$ for every $w \geq 0$. For the rest of
the proof, we will consider only \( k > \max\{K_0, K_1\} \).

Suppose that \( w_t \in [w^{se,k} \gamma, w^{se,k}] \) and the household chooses to stay in the community in period \( t \). Towards contradiction, suppose that \( R^{H,k}(I_t) > w^{se,k} - \gamma \). This implies that \( R^{L,k}(I_t) > w^{se,k} - 2\gamma \), so the wealth at the beginning of period \( (t+1) \) is at least \( (w^{se,k} - 2\gamma) \). On the other hand, the household will never choose \( I_t \) such that \( R^{L,k}(I_t) > w^{se,k} \), since in that case \( f_t = 0 \) and the household could profitably deviate by leaving for the city at the beginning of period \( t \). Hence, \( R^{L,k}(I_t) \leq w^{se,k} \) and \( R^{H,k}(I_t) \leq w^{se,k} + \gamma \), so the wealth at the beginning of period \( (t+1) \) is at most \( (w^{se,k} + \gamma) \). Then a necessary condition for the dynamic enforcement constraint is

\[
f_t \leq \frac{\delta}{1 - \delta} \left( \hat{\Pi}^{H,k}(w^{se,k} + \gamma) - \hat{\Pi}^{L,k}(w^{se,k} - 2\gamma) \right) . \tag{16}
\]

A necessary condition for (16) is that:

\[
f_t \leq \frac{\delta}{1 - \delta} \left( \hat{\Pi}(w^{se,k} + \gamma) - \hat{\Pi}(w^{se,k} - 2\gamma) + 2\gamma \right) \leq \frac{\delta}{1 - \delta} (\Delta(\gamma) + 2\gamma) . \tag{17}
\]

Here, the first inequality follows from the fact that \( \hat{\Pi}^{H,k}(w^{se,k} + \gamma) \leq \hat{\Pi}(w^{se,k} + \gamma) \) and that \( \hat{\Pi}^{L,k}(w^{se,k} - 2\gamma) > \hat{\Pi}(w^{se,k} - 2\gamma) - \gamma \), and the second inequality follows from the definition of \( \Delta(\gamma) \). Now we can bound the household’s payoff at the beginning of period \( t \):

\[
\Pi^{*,(k)}(w_t) \leq (1 - \delta)U(w_t - I_t + f_t) + \delta \Pi^{*,(k)}(w^{se,k} + \gamma) \leq (1 - \delta)U (w_t - (R^{H,k})^{-1}(w^{se,k} - \gamma) + \frac{\delta}{1 - \delta} (\Delta(\gamma) + 2\gamma)) + \delta \hat{\Pi}^{H,k}(w^{se,k} + \gamma) . \tag{18}
\]

Here, the first inequality plugs in the definition of \( c_t \) and uses the fact that on \( w_{t+1} \leq w^{se,k} + \gamma \); and the second inequality plugs in the lower bound on \( I_t \) implied by \( R^{H,k}(I_t) \geq w^{se,k} - \gamma \), the upper bound on \( f_t \) given by (17), and \( \Pi^{*,(k)}(w^{se,k} + \gamma) \leq \hat{\Pi}^{H,k}(w^{se,k} + \gamma) \).

Now, consider the following deviation: the household leaves the community in period \( t \), invests \( I_t \), and then continues to play according to the optimal equilibrium in the city. This
deviation results in a payoff of at least:

\[(1 - \delta)\hat{U}(w_t - I_t) + \delta \hat{\Pi}^k(R^{L,k}(I_t)) \geq (1 - \delta)\hat{U}(w_t) + \delta \hat{\Pi}^L_k(w_{se,k} - 2\gamma) \geq (1 - \delta)\hat{U}(w - (R^{L,k})^{-1}(w_{se,k})) + \delta \hat{\Pi}^{L,k}(w_{se,k} - 2\gamma),\]

where \(\hat{\Pi}^k(\cdot)\) is a household’s payoff in the city under the stochastic return \(F^k\). The first inequality uses \(\hat{\Pi}^k \geq \hat{\Pi}^{L,k}\) and \(R^{L,k}(I_t) \geq R^{H,k}(I_t) - \gamma \geq w_{se,k} - 2\gamma\). The second inequality uses the fact that \(I_t \leq (R^{L,k})^{-1}(w_{se,k})\).

Comparing the bounds in (18) and (19), this deviation is profitable so long as

\[(1 - \delta)\hat{U}(w_t - (R^{L,k})^{-1}(w_{se,k})) + \delta \hat{\Pi}^{L,k}(w_{se,k} - 2\gamma) > (1 - \delta)U(w_t - (R^{H,k})^{-1}(w_{se,k} - \gamma) + \delta \frac{1}{1 - \delta}(\Delta(\gamma) + 2\gamma)) + \delta \hat{\Pi}^{H,k}(w_{se,k} + \gamma)\]

or, equivalently,

\[\hat{U}(w_t - (R^{L,k})^{-1}(w_{se,k})) - U(w_t - (R^{H,k})^{-1}(w_{se,k} - \gamma) + \frac{\delta}{1 - \delta}(\Delta(\gamma) + 2\gamma)) > \frac{\delta}{1 - \delta}\left(\hat{\Pi}^{H,k}(w_{se,k} + \gamma) - \hat{\Pi}^{L,k}(w_{se,k} - 2\gamma)\right).\]

The definition \(\Delta(\cdot)\) implies that \(\hat{\Pi}^{H,k}(w_{se,k} + \gamma) - \hat{\Pi}^{L,k}(w_{se,k} - 2\gamma) \leq \Delta(\gamma) + 2\gamma\). Moreover, since \(R^{H,k}(w) - R^{L,k}(w) < \gamma\), \((R^{L,k})^{-1}(w) < (R^{H,k})^{-1}(w + \gamma)\) for any \(w \geq 0\). Thus,

\[\hat{U}(w_t - (R^{L,k})^{-1}(w_{se,k})) - U(w_t - (R^{H,k})^{-1}(w_{se,k} - \gamma) + \frac{\delta}{1 - \delta}(\Delta(\gamma) + 2\gamma)) > \hat{U}(w_t - (R^{H,k})^{-1}(w_{se,k} + \gamma)) - U(w_t - (R^{L,k})^{-1}(w_{se,k} - 2\gamma) + \frac{\delta}{1 - \delta}(\Delta(\gamma) + 2\gamma)) \geq (20)\]

\[\hat{U}(w_t - R^{-1}(w_{se,k} + \gamma)) - U(w_t - R^{-1}(w_{se,k} - 2\gamma) + \frac{\delta}{1 - \delta}(\Delta(\gamma) + 2\gamma)).\]

Note that the last line of (20) is the same as (14) with \(w = w_t\), which is greater than \(\frac{\delta}{1 - \delta}(\Delta(\gamma) + 2\gamma)\). Hence, the deviation is indeed profitable.
D Online Appendix: Other Extensions

D.1 Reversible Exit

This appendix shows that underinvestment occurs even if the household can return to the community after leaving for the city. Formally, we modify the game in Section 2 so that at the start of every period while the household is in the city, it can return to the community. If it does, then it plays the community game until it again chooses to leave for the city. Payoffs and information structures are the same as in Section 2, and so neighbors observe all of the household’s interactions with neighbors, while vendors observe only their own interactions.

We impose a slightly stronger version of Assumption 1 and also assume that the household’s marginal utility of consumption is strictly higher in the city.

Assumption 3 Define $\bar{c}_m$ as the solution to $\hat{U}'(\bar{c}_m) = 1$, and let $\hat{w}_m$ satisfy $R(\hat{w}_m - \bar{c}_m) = \hat{w}_m$. Then, $R'(\hat{w}_m) > \frac{1}{\delta}$. Moreover, for every $c > 0$, $\hat{U}'(c) > U'(c)$.

Under this assumption, we can prove that underinvestment occurs even if exit is reversible.

Proposition 6 Impose Assumption 3. There exists a $w^{**} < \bar{w}$, a $w^{cc} < w^{**}$, and a positive-measure set $\mathcal{W} \subseteq [0, w^{**}]$ with $\sup \mathcal{W} = w^{**}$ such that (i) if $w_0 \notin \mathcal{W}$, the household permanently exits the community, with $\lim_{t \to \infty} w_t > \bar{w}$, and (ii) if $w_0 \in \mathcal{W}$, the household is in the community for an infinite number of periods.

If $w_0 \in \mathcal{W}$, then for every $t \geq 0$, $w_t < w^{**}$. Moreover, $w_{t+1} < w_t$ whenever $w_t \in (w^{cc}, w^{**}) \cap \mathcal{W}$.

D.1.1 Proof of Proposition 6

Much like the proof of Proposition 2, we break this proof into a sequence of lemmas. We begin by showing that the household’s worst equilibrium payoff equals $\hat{\Pi}(\cdot)$, its worst equilibrium payoff from the game in Section 2.
Lemma 5 For any initial wealth $w \geq 0$, the household’s worst equilibrium payoff is $\hat{\Pi}(\cdot)$.

Proof of Lemma 5: This proof is similar to the proof of Proposition 1. It is an equilibrium for $f_t = 0$ in every $t \geq 0$, in which case it is optimal for the household to permanently leave the community. In the city, vendor $t$ accepts only if $p_t \geq c_t$. Therefore, $\hat{\Pi}(\cdot)$ gives the maximum equilibrium payoff if the household permanently leaves the community. But as in the proof of Proposition 1, the household cannot earn less than $\hat{\Pi}(\cdot)$, because vendor $t$ must accept whenever $p_t > c_t$. ■

Now, we turn to the household’s maximum equilibrium payoff. Define $\Pi^{**}(w)$ as the maximum equilibrium payoff with initial wealth $w$. Define $\Pi^*_c(\cdot)$ identically to $\Pi_c(\cdot)$, except that $\Pi^*(\cdot)$ is replaced by $\Pi^{**}(\cdot)$. Define

$$
\Pi^*_m(w) = \max_{0 \leq c \leq w} \left\{ (1 - \delta)\hat{U}(c) + \delta \Pi^{**}(R(w - c)) \right\}
$$

as the household’s maximum equilibrium payoff if it chooses the city in the current period. The key difference between this model and the baseline model is that $\Pi^*_m(\cdot)$ might entail the household returning to the community to take advantage of relational contracts in the future. Therefore, $\Pi^*_m(\cdot) \geq \hat{\Pi}(\cdot)$, since the latter entails staying in the city forever.

Lemma 6 Both $\Pi^*_c(\cdot)$ and $\Pi^*_m(\cdot)$ are strictly increasing. For all $w \geq 0$,

$$
\Pi^{**}(w) = \max \{ \Pi^*_c(w), \Pi^*_m(w) \}.
$$

Proof of Lemma 6: As in Lemma 1, conditional on choosing the community in period 0, the household’s maximum equilibrium payoff equals $\Pi^*_c(w_0)$. If the household instead chooses the city in period 0, then $f_0 = 0$ in any equilibrium. Thus, the household optimally sets $p_t = c_t$, so its maximum equilibrium continuation payoff equals $\Pi^{**}(R(w - c))$. We conclude that $\Pi^*_m(w)$ is the household’s maximum equilibrium payoff conditional on choosing the city.
It then immediately follows that $\Pi^*(w) = \max \{\Pi_c^*(w), \Pi_m^*(w)\}$. Both $\Pi_c^*(\cdot)$ and $\Pi_m^*(\cdot)$ are strictly increasing by inspection. ■

Apart from some details of the proof, the next result is similar to Lemma 2.

**Lemma 7** If $\Pi^*(w_0) > 1\Pi(w_0)$, then $\Pi^*(w_t) > 1\Pi(w_t)$ in all $t \geq 0$ of any household-optimal equilibrium.

**Proof of Lemma 7:** Suppose not, and let $\tau > 0$ be the first period such that $\Pi^*(w_\tau) = 1\Pi(w_\tau)$. In period $\tau - 1$, inequality (2) in Appendix A implies that $f_t = 0$ if the household stays in the community. Therefore, it is optimal for the household to leave the community in $\tau - 1$. But then it is optimal for the household to permanently leave the community in $\tau - 1$, since $\Pi^*(w_\tau) = 1\Pi(w_\tau)$. So $\Pi^*(w_{\tau-1}) = 1\Pi(w_{\tau-1})$, contradicting the definition of $\tau$. ■

**Lemma 8** Suppose that $\Pi^*(w_0) > 1\Pi(w_0)$. Then in every $t \geq 0$, (i) $\hat{U}'(c_t) \geq 1$, and (ii) there exists $\tau > t$ such that the household stays in the community in period $\tau$.

**Proof of Lemma 8:** Towards contradiction, suppose that there exists $t \geq 0$ such that $\hat{U}'(c_t) < 1$, so that a fortiori, $U'(c_t) < 1$. If $f_t > 0$, then we can decrease $f_t$ and $c_t$ by the same $\epsilon > 0$. This perturbation is also an equilibrium, and increases the household’s period-t payoff at rate $1 - \hat{U}'(c_t) > 0$ as $\epsilon \to 0$. Thus, $f_t = 0$, which implies that $p_t = c_t > 0$.

By Lemma 7, $\Pi^*(w_{t+1}) > 1\Pi(w_{t+1})$. Therefore, there exists a $\tau > t$ such that $f_{\tau} > 0$, since otherwise the household could do no better than exiting the city permanently. Let $\tau$ be the first period after $t$ such that $f_{\tau} > 0$. Note that the household must be in the community in period $\tau$.

Consider the following perturbation: decrease $p_t$ and $c_t$ by $\epsilon > 0$, and increase $p_{\tau}$ and decrease $f_{\tau}$ by $\chi(\epsilon)$, where $\chi(\epsilon)$ is chosen so that $w_{\tau+1}$ remains constant. Then, $\chi(\epsilon) \geq \frac{\epsilon}{3\tau-\tau}$ because $R'(\cdot) \geq \frac{1}{3}$. As $\epsilon \to 0$, $\chi(\epsilon) \to 0$. Hence, this perturbation is feasible for small enough $\epsilon > 0$. It is an equilibrium, since (2) is trivially satisfied in all $t' \in [t, \tau - 1]$ because $f_{t'} = 0$.
in those periods. This perturbation changes the household’s period-
$t$ continuation payoff at rate no less than

\[-(1 - \delta)U'(c_t) + \delta^{\tau-t}(1 - \delta)\frac{1}{\delta^{\tau-t}} > 0\]

as $\epsilon \to 0$. Thus, the original equilibrium could not have been household-optimal. ■

Next, we show that the household stays in the community for sufficiently low initial wealth levels.

**Lemma 9** The set

$$W = \left\{ w \mid \Pi^*(w) > \hat{\Pi}(w) \right\}$$

has positive measure, with

$$w^* = \sup \left\{ w \mid \Pi^*(w) > \hat{\Pi}(w) \right\} < \bar{w}.$$ 

**Proof of Lemma 9**: The proof that $\left\{ w \mid \Pi^*(w) > \hat{\Pi}(w) \right\}$ has positive measure is identical to the proof in Lemma 3, since the same constructions work at $w = 0$, $\Pi^*(\cdot)$ is increasing, and $\hat{\Pi}(\cdot)$ is continuous. If $w_0 \in W$, then $c_t \leq \bar{c}_m$ in every $t$ by Lemma 8, so $\Pi^*(w_0) \leq \hat{U}(\bar{c}_m)$. If $w_0$ is such that $R(w_0 - \bar{c}_m) > w_0$, then $\Pi^*(w_0) \geq \hat{\Pi}(w_0) > \hat{U}(\bar{c}_m)$. This implies that $R(w_0 - \bar{c}_m) \leq w_0$ for any $w_0 \in W$. Hence, by Assumption 3, $w^* \leq \bar{w}_m < \bar{w}$. ■

We are now in a position to prove Proposition 6. So far, the argument has hewn closely to the proof of Proposition 2. The rest of the proof marks a more substantial departure.

Lemma 9 shows that a positive-measure set $W$ exists such that $\Pi^*(w_0) > \hat{\Pi}(w_0)$ for all $w_0 \in W$ and $\sup W < \bar{w}$. For any $w_0 \notin W$, $\Pi^*(w_0) = \hat{\Pi}(w_0)$ and so the household permanently exits the community and has a long-term wealth strictly above $\bar{w}$. Lemma 7 implies that for any $w_0 \in W$, we can construct an infinite sequence of periods such that the household remains in the community for each period in that sequence. This proves the first part of Proposition 6.

Suppose $w_0 \in W$. Then, Lemma 7 and the definition of $w^*$ immediately imply that
$w_t < w^{**}$ in every $t \geq 0$. It remains to identify a $w^{cc} < w^{**}$ such that if $w_t \in W \cap (w^{cc}, w^{**})$, then $w_{t+1} < w_t$. Since $\Pi^{**}(\cdot)$ is increasing, (2) holds only if

$$f_t \leq \Delta(w_t) = \frac{\delta}{1 - \delta}(\Pi^{**}(w^{**}) - \hat{\Pi}(w_t)).$$

Proposition 1 implies that $\Delta(w_t)$ is strictly decreasing and continuous. Continuity implies that $\lim_{w \uparrow w^{**}} \hat{\Pi}(w) = \hat{\Pi}(w^{**})$. If $\Pi^{**}(w^{**}) > \hat{\Pi}(w^{**})$, then $\Pi^{**}(\cdot)$ increasing and $\hat{\Pi}(\cdot)$ continuous imply $\Pi^{**}(w) > \hat{\Pi}(w)$ just above $w^{**}$, contradicting the definition of $w^{**}$. Therefore, $\Pi^{**}(w^{**}) = \hat{\Pi}(w^{**})$ and $\Delta(w^{**}) = 0$.

For any $w \in [R^{-1}(w^{**}), w^{**}]$, define

$$G(w) = \hat{U}(w - R^{-1}(w^{**})) - (U(w^{**} - R^{-1}(w) + \Delta(w)) + \Delta(w)).$$

Then, $G(\cdot)$ is strictly increasing, continuous, and strictly crosses 0 from below. Define $w^{cc} \in (R^{-1}(w^{**}), w^{**})$ as the unique wealth such that $G(w^{cc}) = 0$.

Consider a household with $w_t > w^{cc}$ such that $\Pi^{**}(w_t) = \Pi^*_t(w_t) > \hat{\Pi}(w_t)$, with corresponding household-optimal choices $(c_t, p_t, f_t)$. Towards contradiction, suppose $w_{t+1} > w^{cc}$. Then, this household can exit permanently, choose $\hat{c}_t = \hat{p}_t = p_t = c_t - f_t$ and $\hat{f}_t = 0$, where $p_t \geq 0$ because $w^{cc} \geq R^{-1}(w^{**})$. This deviation leaves $w_{t+1}$ unchanged and results in continuation payoff $\hat{\Pi}(w_{t+1})$.

We argue that this deviation is profitable:

$$(1 - \delta)\hat{U}(\hat{c}_t) + \delta\hat{\Pi}(w_{t+1}) \geq (1 - \delta)\hat{U}(c_t - f_t) + \delta\hat{\Pi}(w^{cc})$$

$$> (1 - \delta)(U(c_t) + \Delta(w^{cc})) + \delta\hat{\Pi}(w^{cc})$$

$$= (1 - \delta)(U(c_t) + \Delta(w^{cc})) + \delta\Pi^{**}(w^{**}) - (1 - \delta)\Delta(w^{cc})$$

$$= (1 - \delta)U(c_t) + \delta\Pi^{**}(w_{t+1})$$

$$\geq (1 - \delta)U(c_t) + \delta\Pi^{**}(w_{t+1})$$

$$= \Pi^{**}(w_t).$$
Here, the first line follows from \( \hat{c}_t = c_t - f_t \) and \( w_{t+1} \geq w_{\text{cc}} \); the second line is proven below; the third line follows from the definition of \( \Delta(w_{\text{cc}}) \); and the fifth line because, by Lemma 7, \( w_{t+1} \leq w^* \). The fourth and sixth lines are algebra.

The second line follows because for any \( w_t > w_{\text{cc}} \), \( \hat{U}(c_t - f_t) > U(c_t) + \Delta(w_{\text{cc}}) \). To see this, note

\[
w^* \geq w_{t+1} = R(w_t - p_t) > R(w_{\text{cc}} - p_t).
\]

Hence, \( p_t > w_{\text{cc}} - R^{-1}(w^*) \). Similarly,

\[
w_{\text{cc}} < w_{t+1} = R(w_t - p_t) \leq R(w^* - p_t),
\]

so \( p_t < w^* - R^{-1}(w^*) \). Therefore,

\[
\hat{U}(c_t - f_t) = \hat{U}(p_t) > \hat{U}(w_{\text{cc}} - R^{-1}(w^*)) \\
\geq U(w^* - R^{-1}(w_{\text{cc}}) + \Delta(w_{\text{cc}})) + \Delta(w_{\text{cc}}) \\
> U(p_t + \Delta(w_t)) + \Delta(w_{\text{cc}}) \\
\geq U(p_t + \Delta(w_t)) + \Delta(w_{\text{cc}}) \\
\geq U(c_t) + \Delta(w_{\text{cc}}).
\]

Here, the first and third lines follow from \( w^* - R^{-1}(w_{\text{cc}}) > p_t > w_{\text{cc}} - R^{-1}(w^*) \); the second line from \( G(w_{\text{cc}}) = 0 \); the fourth line from the fact that \( w_{\text{cc}} < w_t \) and \( \Delta(\cdot) \) strictly decreasing; and the last line from \( p_t + \Delta(w_t) \geq p_t + f_t = c_t \). The household therefore has a profitable deviation if \( w_{t+1} > w_{\text{cc}} \), so \( w_{t+1} \leq w_{\text{cc}} \) whenever \( w_t \in (w_{\text{cc}}, w^*) \).

Define the function

\[
F(w) = (1 - \delta)\hat{U}(w^* - R^{-1}(w)) + \delta\hat{\Pi}(w^*) - \hat{\Pi}(w)
\]
on $w \in [0, w^{**}]$. Then $F(\cdot)$ is strictly decreasing and continuous, with

$$F(0) = (1 - \delta)\hat{U}(w^{**}) + \delta\hat{\Pi}(w^{**}) > 0.$$ 

At $w_0 = w^{**}$, it is feasible for the household to permanently leave the community and consume $c_t = w^{**} - R^{-1}(w^{**})$ in each $t \geq 0$. However, doing so violates (Euler), since $R'(w^{**}) > \frac{1}{1 - \delta}$. Therefore, this consumption path must be dominated by some other feasible consumption path once the household permanently leaves the community, which implies

$$\hat{U}(w^{**} - R^{-1}(w^{**})) < \hat{\Pi}(w^{**}).$$

Consequently,

$$F(w^{**}) = (1 - \delta)\hat{U}(w^{**} - R^{-1}(w^{**})) - (1 - \delta)\hat{\Pi}(w^{**}) < 0.$$ 

We conclude that there exists a unique $w^{cc2} \in (0, w^{**})$ such that $F(w^{cc2}) = 0$.

Suppose the household with $w_t \in \mathcal{W}$ and $w_t > w^{cc2}$ chooses to live in the city in $t$. Towards contradiction, suppose that $w_{t+1} \geq w_t$. Therefore, the household’s payoff satisfies

$$\Pi^{**}(w_t) \leq (1 - \delta)\hat{U}(w^{**} - R^{-1}(w_t)) + \delta\Pi^{**}(w^{**})$$

$$= (1 - \delta)\hat{U}(w^{**} - R^{-1}(w_t)) + \delta\hat{\Pi}(w^{**})$$

$$< \hat{\Pi}(w_t).$$

Here, the first inequality follows because $f_t = 0$, so that $p_t = c_t = w_t - R^{-1}(w_{t+1}) \leq w^{**} - R^{-1}(w_t)$; the equality follows because $\Pi^{**}(w^{**}) = \hat{\Pi}(w^{**})$; and the final, strict inequality follows because $w_t > w^{cc}$ and so $F(w_t) < 0$. But $\Pi^{**}(\cdot) \geq \hat{\Pi}(\cdot)$, proving a contradiction. So $w_{t+1} < w_t$.

Set $w^{cc} = \max\{w^{cc1}, w^{cc2}\}$. We have shown that $w_{t+1} < w_t$ whenever $w_t > w^{cc}$. This completes the proof. $\blacksquare$
D.2 Non-Linear Favors

D.2.1 Model and Statement of Result

In this appendix, we show that a (slightly weaker version of) Proposition 2 holds if the household’s cost of providing \( f_t \) is convex.

Consider the game with non-linear favors, which is identical to the game in Section 2 except that the household’s stage-game payoff is

\[
\pi_t = \begin{cases} 
U(c_t d_t) - k(f_t) & \text{in the community} \\
\hat{U}(c_t, d_t) - k(f_t) & \text{in the city.}
\end{cases}
\]

Assume that \( k(\cdot) \) is strictly increasing, strictly convex, twice continuously differentiable, and satisfies \( k'(0) = 0 \), \( k(0) = 0 \), and \( \lim_{f \to \infty} k'(f) = \infty \). We assume \( \hat{U}'(\cdot) > U'(\cdot) \). A special case of this game has \( k = U^{-1} \), which corresponds to a setting in which the neighbors value \( f_t \) according to the same utility function as the household’s consumption.

Define \( f^*(c) \) as the unique solution to \( U'(c) = k'(f^*(c)) \) and \( c^* \) as the solution to \( U(c^*) = \hat{U}(c^* - f^*(c^*)) \). Note that \( c^* \) exists and is unique because \( \hat{U}'(c) > U'(c), f^*(\cdot) \) is decreasing, and \( \lim_{c \to \infty} f^*(c) = 0 \). We impose the following assumption.

**Assumption 4** Define \( \tilde{c}(w) \) as the unique solution to \( R(w - \tilde{c}(w)) = w \). Assume that

\[
\hat{U}(\tilde{c}(w) - f^*(\tilde{c}(w))) - U(\tilde{c}(w)) > \frac{\delta}{1 - \frac{\epsilon}{\delta}} U(c^*).
\]

Suppose that the household has wealth \( \bar{w} \), which is the wealth level at which strictly positive-return investments are exhausted. Assumption 4 ensures that this household’s value from moving to the city and consuming \( \tilde{c}(\bar{w}) - f^*(\tilde{c}(\bar{w})) \) forever after is substantially greater than its utility from staying in the community and consuming \( \tilde{c}(\bar{w}) \) forever. Note that \( \tilde{c}(w) \) is increasing because \( R'(\cdot) \geq \frac{1}{\delta} > 1 \).

We now prove our main result for the game with non-linear favors.
Proposition 7 Impose Assumption 4 in the game with non-linear favors. Then, there exists a $w^{se} < \bar{w}$, a $w^{tr} \in [0,w^{se})$, and a positive-measure set $\mathcal{W} \subseteq [0,w^{se})$, such that in any household-optimal equilibrium,

1. **Selection:** The household stays in the community forever if $w_0 \in \mathcal{W}$ and otherwise leaves immediately.

2. "**Too-big-for-their-boots:**" If $w_0 \in \mathcal{W}$ and $w_t \geq w^{tr}$ on the equilibrium path, then $w_{t+1} < w_t$.

D.2.2 Proof of Proposition 7

We focus only on those parts of the proof that differ substantially from the proof of Proposition 2.

The proof of Proposition 1 goes through without change, since $f_t = 0$ in every period of any equilibrium in the city. Similarly, once we substitute the cost function $k(f)$ into the household’s payoff, the proofs of Lemmas 1 and 2 go through without change.

The next step of the proof, which shows that wealthy households leave the community and poorer households stay, requires a new argument.

**Lemma 10** The set $\{ w : \Pi^*(w) > \hat{\Pi}(w) \}$ has positive measure. Moreover, $w^{se} = \sup \{ w : \Pi^*(w) > \hat{\Pi}(w) \}$ satisfies $0 < w^{se} < \bar{w}$.

**Proof of Lemma 10:** The argument that $\Pi^*(0) > \hat{\Pi}(0) = 0$ goes through essentially without change. Thus, we need to show only that $w^{se} < \infty$.

**Step 1:** In any $t \geq 0$ of any household-optimal equilibrium, $f_t \leq f^*(c_t)$. If the household is in the city, $f_t = 0 \leq f^*(c_t)$. If it is in the community, then suppose that $f_t > f^*(c_t)$. Consider perturbing the equilibrium by decreasing $c_t$ and $f_t$ by $\epsilon > 0$. This perturbation is feasible as $\epsilon \to 0$, and moreover, increases the household’s stage-game payoff.
at rate \( k'(f_t) - U'(c_t) > k'(f^*(c_t)) - U'(c_t) = 0 \). Thus, the original equilibrium was not household-optimal. This proves Step 1.

**Step 2:** For any \( w \geq 0 \), \( \Pi^*(w) - \hat{\Pi}(w) \leq U(c^*) \). If \( \Pi^*(w) = \hat{\Pi}(w) \), then the result is immediate. Suppose that \( \Pi^*(w) > \hat{\Pi}(w) \). By Lemma 2, a household with \( w_0 = w \) stays in the community in any \( t \geq 0 \) of any household-optimal equilibrium.

Fixing such an equilibrium, we can derive a lower bound on \( \hat{\Pi}(w) \) using the following perturbation: the household leaves the community, chooses \( \tilde{p}_t = p_t \) in each \( t \geq 0 \), and request consumption \( \tilde{c}_t = c_t - f_t \geq 0 \). This strategy is feasible, and the resulting payoff satisfies

\[
\sum_{t=0}^{\infty} \delta^t (1 - \delta) \tilde{U}(c_t - f_t) = \sum_{t|c_t \geq c^*} \delta^t (1 - \delta) \tilde{U}(c_t - f_t) + \sum_{t|c_t < c^*} \delta^t (1 - \delta) \tilde{U}(c_t - f^*(c_t)),
\]

where the inequality follows because \( f_t \leq f^*(c_t) \) and \( c_t - f_t \geq 0 \). Therefore,

\[
\Pi^*(w) - \hat{\Pi}(w) \leq \sum_{t=0}^{\infty} \delta^t (1 - \delta) \left( U(c_t) - \tilde{U}(c_t - f_t) \right) \leq \sum_{t|c_t \geq c^*} \delta^t (1 - \delta) \left( U(c_t) - \tilde{U}(c_t - f^*(c_t)) \right) + \sum_{t|c_t < c^*} \delta^t (1 - \delta) U(c_t) \leq U(c^*),
\]

where the first two inequalities follow from our lower bound on \( \hat{\Pi}(w) \), and the final inequality holds because \( U(c_t) \leq \tilde{U}(c_t - f^*(c_t)) \) for all \( c_t \geq c^* \), and \( U(c_t) \leq U(c^*) \) for all \( c_t < c^* \). This proves Step 2.

**Step 3:** There exists a \( w^{se} \) such that for all \( w \geq w^{se} \), \( \Pi^*(w) = \hat{\Pi}(w) \). Fix a wealth level such that \( \Pi^*(w) > \hat{\Pi}(w) \) and \( \hat{\Pi}(w) > U(c^*) \), and consider a household-optimal equilibrium with \( w_0 = w \). By Lemma 2, the household must stay in the community forever. Therefore, there exists \( t \geq 0 \) such that \( U(c_t) \geq \hat{\Pi}(w) \), so \( c_t > c^* \).

In this period \( t \), we must have \( \Pi^*(w_{t+1}) > \hat{\Pi}(w_{t+1}) \). Since the household can always leave
the community, consume \( c_t - f^*(c_t) \), and earn \( \hat{\Pi}(w_{t+1}) \), \( \Pi^*(w_t) > \hat{\Pi}(w_t) \) holds only if
\[
(1 - \delta)U(c_t) + \delta \Pi^*(w_{t+1}) > (1 - \delta)\hat{U}(c_t - f^*(c_t)) + \delta \hat{\Pi}(w_{t+1})
\]
or
\[
\frac{\delta}{1 - \delta} \left( \Pi^*(w_{t+1}) - \hat{\Pi}(w_{t+1}) \right) > \hat{U}(c_t - f^*(c_t)) - U(c_t).
\]
By Step 2, this inequality holds only if
\[
\frac{\delta}{1 - \delta} U(c^*) > \hat{U}(c_t - f^*(c_t)) - U(c_t).
\] (21)

Now, choose \( w^{se} \) such that \( \hat{U}(\tilde{c}(w^{se}) - f^*(\tilde{c}(w^{se}))) - U(\tilde{c}(w^{se})) = \frac{\delta}{1 - \delta} U(c^*) \); note that \( w^{se} \) exists and satisfies \( w^{se} < \tilde{w} \) by Assumption 4. For any \( w \geq w^{se} \), (21) cannot hold, which means that in any household-optimal equilibrium, there exists a \( t \geq 0 \) such that \( \Pi^*(w_t) = \hat{\Pi}(w_t) \). Lemma 2 then implies that for all \( w \geq w^{se} \), \( \Pi^*(w_0) = \hat{\Pi}(w_0) \). We conclude that \( w^{se} < \tilde{w} \), as desired. ■

We can now complete the proof of Proposition 7. As in the proof of Proposition 2, selection follows from Lemmas 2 and 10. The argument for “too-big-for-their-boots” is similar to the argument in Proposition 6, with one change. We must substitute \( k(f_t) \) into the household’s payoff. With this change, we can construct a similar \( w^{cel} \) as in Proposition 6 so \( w_{t+1} < w_t \) whenever \( w_t > w^{cel} \). We can therefore take \( w^{tr} = w^{cel} \) to prove Proposition 7. ■

D.3 A Long-Lived Neighbor

The model with a long-lived neighbor is identical to the baseline model, except that the sequence of short-lived neighbors in the community are replaced by a single, long-lived neighbor. If the household leaves for the city in period \( t \), then this neighbor earns 0 in that and every subsequent period. Otherwise, the neighbor’s stage-game payoff is identical to
the stage-game payoff of the short-lived neighbors. The long-lived neighbor has the same
discount factor as the household, $\delta$. In the city, the vendors remain short-lived players.

We focus on pure-strategy equilibria. We want to prove the following result.

**Proposition 8** In the model with a long-lived neighbor, household-optimal equilibrium pay-
offs are identical to those in the baseline model. Moreover, there exists a household-optimal
equilibrium in which actions, and hence wealth dynamics, are the same as in a household-
optimal equilibrium of the baseline model.

**Proof of Proposition 8**

Denote the household’s and neighbor’s continuation payoff in period $t$ by $\Pi_t$ and $V_t$, respec-
tively. Note that once the household leaves the community, its continuation payoff is $\hat{\Pi}(w_t)$,
just like in the baseline model. We prove this proposition with the following sequence of
lemmas.

**Lemma 11** Consider an equilibrium such that $\Pi_0 > \hat{\Pi}(w_0)$. Then, $\Pi_t > \hat{\Pi}(w_t)$ for all $t \geq 0$
on the equilibrium path.

**Proof of Lemma 11:** Towards contradiction, suppose that $\Pi_0 > \hat{\Pi}(w_0)$ and there exists
a $t > 0$ such that $\Pi_t = \hat{\Pi}(w_t)$. Note that $\hat{\Pi}(w_t)$ is the household’s minimum equilibrium
continuation payoff. Without loss, let $t > 0$ be the first such period. In period $t-1$, $f_t = 0$,
because that household earns $\hat{\Pi}(w_t)$ both on the equilibrium path and following a deviation.
Consequently, either $d_{t-1} = 0$ or $p_{t-1} \geq c_{t-1}$. As in the proof of Lemma 2, the household
can therefore leave for the city, make the same monetary payment, and receive the same
consumption. Since $\hat{U} \geq U$, the household’s payoff weakly increases if it deviates in this
way. Moreover, its payoff from this deviation is weakly below its equilibrium payoff from the
city. Thus, $\Pi_{t-1} = \hat{\Pi}(w_{t-1})$, contradicting the assumption that $t > 0$ is the first period such
that $\Pi_t = \hat{\Pi}(w_t)$. ■
Now, with abuse of notation, let $\Pi^*(V|w)$ be the household’s maximum equilibrium payoff, given that the neighbor earns continuation payoff $V$. Suppose $w$ is such that $\Pi^*(V|w) > \hat{\Pi}(w)$. Then, $\Pi^*(V|w)$ solves the constrained maximization problem

$$
\Pi^*(V|w) = \max_{\{c_t, p_t, f_t\}} \left\{ \sum_{s=0}^{\infty} \delta^s (1 - \delta) (U(c_s) - f_s) \right\},
$$

subject to the following constraints:

1. The household’s participation constraint to (i) stay in the community, and (ii) make the equilibrium request:

$$
\forall t, \sum_{s=t}^{\infty} \delta^{s-t} (1 - \delta) (U(c_s) - f_s) \geq \hat{\Pi}(w_t).
$$

   (22)

2. The neighbor’s incentive-compatibility constraint to accept the request is subsumed in the promise-keeping constraint:

$$
\sum_{s=0}^{\infty} \delta^s (1 - \delta) (f_s + p_s - c_s) = V
$$

   (23)

$$
\forall t > 0, \sum_{s=t}^{\infty} \delta^{s-t} (1 - \delta) (f_s + p_s - c_s) \geq 0.
$$

   (24)

3. The household’s dynamic enforcement constraint:

$$
\forall t \geq 0, f_t \leq \sum_{s=t+1}^{\infty} \delta^{s-t} (U(c_s) - p_s - f_s) - \delta \hat{\Pi}(w_{t+1}).
$$

   (25)

4. Feasibility constraint for $p_t$,

$$
\forall t \geq 0, \ 0 \leq p_t \leq w_t.
$$

(26)
5. Law of motion for $p_t$,

$$\forall t \geq 0, \quad w_{t+1} = R(w_t - p_t). \tag{27}$$

Our next few lemmas derive characteristics of this constrained maximization problem. (While we could write this maximization problem recursively, the proofs are easier with a non-recursive formulation.)

**Lemma 12** Suppose $V$ and $w$ are such that $\Pi^*(V|w) > \hat{\Pi}(w)$. Then, in every equilibrium that attains household and neighbor payoffs $\Pi^*(V|w)$ and $V$, respectively, it is the case that $c_0 \leq \bar{c}$.

**Proof of Lemma 12:** Towards contradiction, suppose that there exists such an equilibrium with $c_0 > \bar{c}$.

We first claim that $f_t = 0$ in all $t \geq 0$. Suppose not, and define $T \geq 0$ as the first period such that $f_T > 0$. If $T = 0$, then consider perturbing the equilibrium by decreasing both $c_0$ and $f_0$ by a small amount, $\epsilon > 0$. This perturbation strictly increases the household’s payoff by $1 - U''(c_0) > 0$, where the strict inequality follows from $c_0 > \bar{c}$. It also gives the neighbor the same payoffs in each $t \geq 0$, so it continues to satisfy all constraints. Thus, the original equilibrium couldn’t have attained the maximum household payoff $\Pi^*(V|w)$.

The previous argument shows that $T > 0$. There are then two possibilities. First, (24) might be slack in all periods $t \in \{1, \ldots, T\}$. In that case, perturb the equilibrium by decreasing $c_0$ by a small $\epsilon > 0$ and decreasing $f_t$ by $\frac{\epsilon}{\bar{r}_t}$. Such a deviation continues to satisfy (23), weakly relaxes (25) in all periods, and increases the household’s payoff at rate $1 - U'(c_0) > 0$. For $\epsilon > 0$ sufficiently small, it continues to satisfy (24) in all periods, because these constraints were slack in all $t \in \{1, \ldots, T\}$. We conclude that the original equilibrium couldn’t have attained $\Pi^*(V|w)$, a contradiction.

The second possibility is that (24) binds in some period $t \in \{1, \ldots, T\}$. In that case, let
\( t > 0 \) be the first such period. Then, we can write the neighbor’s payoff in period \( t = 0 \) as

\[
\sum_{s=0}^{t-1} \delta^s (1 - \delta) (f_s + p_s - c_s) + \delta^t \sum_{s=0}^{\infty} \delta^{s-t} (1 - \delta) (f_s + p_s - c_s) = \\
\sum_{s=0}^{t-1} \delta^s (1 - \delta) (f_s + p_s - c_s) = \sum_{s=0}^{t-1} \delta^s (1 - \delta) (p_s - c_s) \geq 0 \quad (28)
\]

Here, the first equality follows because (24) binds in \( t \in \{1, \ldots, T\} \); the second equality holds because \( f_s = 0 \) in all \( s < t \), because \( t \leq T \); and the inequality holds by (23), since \( V \geq 0 \). Since \( c_0 > \bar{c} > 0 \), (28) implies that there exists some \( \hat{t} \in \{0, \ldots, t-1\} \) such that \( p_\hat{t} > 0 \).

Consider the following perturbation: decrease \( c_0 \) by \( \epsilon > 0 \), decrease \( p_\hat{t} > 0 \) by \( \frac{\epsilon}{\delta} \), decrease \( f_T \) by \( \chi(\epsilon) \), and increase \( p_T \) by \( \chi(\epsilon) \), where \( \chi(\epsilon) \) is chosen so that \( w_{T+1} \) remains constant. Note that \( \chi(\epsilon) \to 0 \) as \( \epsilon \to 0 \), and since \( R'(\cdot) \geq \frac{1}{\delta}, \chi(\epsilon) \geq \frac{\epsilon}{\delta T} \). Therefore, this perturbation increases the household’s payoff at rate \( \delta' \chi(\epsilon) - U'(c_0) \geq 1 - U'(c_0) > 0 \). The neighbor’s period-0 payoff increases by \( \epsilon \), its period-\( \hat{t} \) payoff decreases by \( \frac{\epsilon}{\delta T} \), and its period-\( T \) payoff stays constant. Thus, the neighbor’s continuation payoff in period 0 is constant, so (23) is satisfied. The neighbor’s continuation payoff in each of periods \( s \in \{1, \ldots, \hat{t}\} \) decreases; however, \( \hat{t} < t \), so (24) was slack in these periods and still holds for \( \epsilon > 0 \) sufficiently close to 0. In all \( s > \hat{t} \), the neighbor’s continuation payoff is unchanged and (24) holds. The dynamic enforcement constraint, (25), is unchanged in all periods \( s > T \) and is relaxed in period \( s = T \). In periods \( s < T \), (25) is tightened, because \( \hat{\Pi}(\cdot) \) is increasing and the perturbation increases \( w_{s+1} \) in these periods. However, Lemma 11 implies that \( \Pi^*(V|w_{s+1}) > \hat{\Pi}(w_{s+1}) \), and Proposition 1 says that \( \hat{\Pi}(\cdot) \) is continuous. Thus, (25) is slack in periods \( s < T \), because \( f_s = 0 \) in all such periods. It therefore remains satisfied for \( \epsilon > 0 \) sufficiently close to 0.

We have found a perturbed equilibrium that results in a strictly higher household payoff and gives the neighbor a payoff of \( V \), which means that the original equilibrium does not attain \( \Pi^*(V|w_0) \). A contradiction. We conclude that in any equilibrium attaining payoffs \( \Pi^*(V|w) \) and \( V \), if \( c_0 > \bar{c} \), then \( f_t = 0 \) in all \( t \geq 0 \).

The final step of this argument is to show that if \( f_t = 0 \) in all \( t \geq 0 \), then \( \Pi^*(V|w) \leq \hat{\Pi}(w) \).
We do so by showing that \( p_t \geq c_t \) in all \( t \geq 0 \) of any equilibrium with \( f_t = 0 \) in all \( t \geq 0 \).

The proof of this result is very similar to the proof of Proposition 4. In particular, suppose \( f_t = 0 \) in all \( t \geq 0 \), and define \( s_t = c_t - p_t \). Then, we can reformulate the household’s request as a tuple \((s_t, c_t)\), subject to the constraints \( s_t \in [-w_t, c_t] \) and \( c_t \geq 0 \). Under this reformulation, the household invests \( w_t + d_t(s_t - c_t) \). The household’s period-\( t \) payoff in the community is \( U(d_t c_t) \), while the neighbor’s period-\( t \) payoff is \(-s_t\).

This reformulation is almost identical to the model with monetary favors in Section 6 except for two differences. First, \( d_t \) multiplies \( c_t \) in both investment and the household’s payoff. This change is immaterial, because \( d_t = 1 \) in all \( t \geq 0 \) on the equilibrium path, and the household’s payoff following any deviation is given by \( \hat{\Pi}(w_{t+1}) \) in both models. Second, \( s_t \) is bounded from above by \( c_t \) in this model. This bound makes staying in the community less attractive. Since Proposition 4 implies that the household earns no more than \( \hat{\Pi}(w_t) \) in the model with monetary favors, and since staying in the community with \( f_t \geq 0 \) in all \( t \geq 0 \) is worse than staying in the community in that model, we conclude that \( \Pi^*(V|w) \leq \hat{\Pi}(w) \), as desired.

We have shown that if \( c_0 > \bar{c} \) in an equilibrium, then \( \Pi^*(V|w_0) \leq \hat{\Pi}(w_0) \). Hence, if \( \Pi^*(V|w_0) > \hat{\Pi}(w_0) \), then every equilibrium must satisfy \( c_0 \leq \bar{c} \). ■

**Lemma 13** For any \( V \geq 0 \) such that \( \Pi^*(V|w) > \hat{\Pi}(w) \), \( \Pi^*(0|w) - \Pi^*(V|w) \geq V \).

**Proof of Lemma 13:** First, note that if \( \Pi^*(V|w) > \hat{\Pi}(w) \), then for any \( \epsilon > 0 \), \( \Pi^*(V - \epsilon|w) > \Pi^*(V|w) \). To see this, note that one way to give the neighbor a payoff of \( V - \epsilon \) is to perturb an equilibrium with payoffs \( \Pi^*(V|w) \) and \( V \) by increasing \( c_0 \) by \( \epsilon \frac{1}{\gamma - \delta} \). This alternative is also an equilibrium, and it gives the household a payoff of \( \Pi^*(V|w) + (1 - \delta)(U(c_0 + \epsilon \frac{1}{\gamma - \delta}) - U(c_0)) > \Pi^*(V|w) \), as desired.

Towards contradiction, suppose that \( \Pi^*(0|w) - \Pi^*(V|w) < V \). Then, there exists some \( \epsilon > 0 \) such that \( \Pi^*(0|w) - \Pi^*(V|w) \leq (V - \epsilon) \). For any \( K > 0 \), we can write the difference
\[ \Pi^*(0|w) - \Pi^*(V|w) \] as a telescoping series:

\[ \Pi^*(0|w) - \Pi^*(V|w) = \sum_{k=0}^{K-1} \left( \Pi^* \left( \frac{k}{K} V|w \right) - \Pi^* \left( \frac{k+1}{K} V|w \right) \right). \]

Since \( \Pi^*(0|w) - \Pi^*(V|w) \leq (V - \epsilon) \), then there must exist some \( k \in \{0, ..., K\} \) such that

\[ \Pi^* \left( \frac{k}{K} V|w \right) - \Pi^* \left( \frac{k+1}{K} V|w \right) \leq \frac{1}{K} (V + \epsilon). \]

Now, choose \( K \) large enough that

\[ U \left( \bar{c} + \frac{1}{K} \frac{1}{1 - \delta} V \right) - U (\bar{c}) > \frac{1}{K} \frac{1}{1 - \delta} (V - \epsilon). \] (29)

Such a \( K \) exists, because \( U(\cdot) \) is continuously differentiable, \( U'(\bar{c}) = 1 \), and \( U \left( \bar{c} + \frac{1}{K} \frac{1}{1 - \delta} V \right) - U(\bar{c}) = \int_{\bar{c}}^{\bar{c} + \frac{1}{K} \frac{1}{1 - \delta} V} U'(x)dx \). For such a \( K \), consider an equilibrium that delivers payoffs \( \Pi^* \left( \frac{k+1}{K} V|w \right) \) and \( \frac{k+1}{K} V \) to the household and neighbor, respectively. Perturb this equilibrium by increasing \( c_0 \) by \( \frac{1}{K} \frac{1}{1 - \delta} V \). As we have already argued, this perturbed strategy is also an equilibrium that gives the neighbor a payoff of \( \frac{k}{K} V \). Letting \( c_0 \) be period-0 consumption in the original equilibrium, the household’s payoff from this perturbed equilibrium can be written \( (1 - \delta) \left( U \left( c_0 + \frac{1}{K} \frac{1}{1 - \delta} V \right) - U(c_0) \right) + \Pi^* \left( \frac{k+1}{K} V|w \right) \).

The household’s payoff from this perturbed equilibrium cannot be larger than \( \Pi^* \left( \frac{k}{K} V \right) \), so

\[ \Pi^* \left( \frac{k}{K} V \right) - \Pi^* \left( \frac{k+1}{K} V|w \right) \geq (1 - \delta) \left( U \left( c_0 + \frac{1}{K} \frac{1}{1 - \delta} V \right) - U(c_0) \right). \]

Lemma 12 says that \( c_0 \leq \bar{c} \), so (29) and the concavity of \( U(\cdot) \) imply

\[ \Pi^* \left( \frac{k}{K} V \right) - \Pi^* \left( \frac{k+1}{K} V|w \right) > \frac{1}{K} (V - \epsilon). \]

This holds for all \( k \in \{0, ..., K - 1\} \), contradicting the requirement that there exists some \( k \) such that \( \Pi^* \left( \frac{k}{K} V|w \right) - \Pi^* \left( \frac{k+1}{K} V|w \right) \leq \frac{1}{K} (V - \epsilon) \). This proves the claim. ■

We can now complete the proof of Proposition 8. It is easy to see that \( V_0 = 0 \) in any
household-optimal equilibrium. Consider an equilibrium that gives payoffs \( \Pi^*(0|w_0) \) and 0 to the household and neighbor, respectively. If \( \Pi^*(0|w_0) = \bar{\Pi}(w_0) \), then the household can attain this payoff by leaving for the city in \( t = 0 \).

Suppose that \( \Pi^*(0|w_0) > \bar{\Pi}(w_0) \). Consider the following perturbation, where perturbed variables are marked by tildes and original variables do not have tildes: set \( \tilde{V}_1 = 0 \) and \( \tilde{f}_0 = f_0 + \frac{\delta}{1-\delta}V_1 \), all other period-0 actions are unchanged, and the continuation equilibrium gives payoffs \( \Pi^*(0|w_1) \) and 0 to the household and neighbor, respectively. The neighbor’s payoff from this perturbed strategy is

\[
(1 - \delta) \left( p_0 + f_0 + \frac{\delta}{1-\delta}V_1 - c_0 \right) = (1 - \delta)(p_0 + f_0 - c_0) + \delta V_1,
\]

which is identical to her payoff from the original equilibrium. Thus, this strategy satisfies (23) and (24). The household’s payoff from this perturbed strategy is

\[
(1 - \delta) \left( U(c_0) - f_0 - \frac{\delta}{1-\delta}V_1 \right) + \delta \Pi^*(0|w_1) =
\]

\[
(1 - \delta)(U(c_0) - f_0) + \delta \left( \Pi^*(0|w_1) + V_1 \right) \geq (1 - \delta)(U(c_0) - f_0) + \delta \Pi^*(V_1|w_1),
\]

where the inequality holds by Lemma 13. Thus, the household earns a weakly higher payoff, so (22) is satisfied. Finally, the perturbed strategy satisfies (25), because

\[
\tilde{f}_0 = f_0 + \frac{\delta}{1-\delta}V_1 \leq \frac{\delta}{1-\delta} \left( \Pi^*(V_1|w_1) - \bar{\Pi}(w_1) \right) + \frac{\delta}{1-\delta} \tilde{V}_1 \leq \frac{\delta}{1-\delta} \left( \Pi^*(0|w_1) - \bar{\Pi}(w_1) \right),
\]

where the first inequality holds because \( f_0 \) satisfies (25) in the original equilibrium, and the second inequality holds because \( \Pi^*(V_1|w_1) + \tilde{V}_1 \leq \Pi^*(0|w_1) \) by Lemma 13. We conclude that the perturbed strategy is an equilibrium that gives the household a weakly higher payoff. It is therefore a household-optimal equilibrium.

We can perform a similar perturbation in every period \( t > 0 \) such that \( V_t > 0 \). The resulting equilibrium is household-optimal and satisfies \( V_t = 0 \) in all \( t \geq 0 \). Since \( V_t = 0 \) in
all $t \geq 0$, (24) can be written $p_t + f_t - c_t \geq 0$, which is the same as the incentive constraint in the model with short-lived neighbors. Consequently, this equilibrium is also an equilibrium of the model with short-lived neighbors. It is a household-optimal equilibrium of that mode, because any equilibrium of the model with short-lived neighbors is also an equilibrium of the model with a long-lived neighbor. We conclude that there exists a household-optimal equilibrium of the model with a long-lived neighbor that is also a household-optimal equilibrium of the model with short-lived neighbors. The characterization in Proposition 2 holds for any such equilibrium, proving Proposition 8. ■

D.4 Wealth Dynamics Without a City

D.4.1 Model Variant 1: Removing the City, Imposing Reversion to Markov Perfect Equilibrium

This appendix considers the model without the city. It proves that there is a unique Markov Perfect Equilibrium (MPE) in this model, with a payoff equal to the unique equilibrium payoff in the city, $\hat{\Pi}$, for the case $\hat{U} = U$. This confirms that our main result would still hold if we eliminated the city and instead assumed that any deviation was punished by reversion to MPE in the community.

The model without a city is identical to the model in Section 2, except that the household cannot choose to leave the community.

**Proposition 9** Consider the model without a city. For any initial wealth $w_0 \geq 0$, there is a unique MPE payoff, which equals $\hat{\Pi}(w_0)$ for the case $\hat{U} = U$.

**Proof of Proposition 9:** Note that $f_t$ is not a payoff-relevant variable in periods $t + 1, t + 2, \ldots$. Thus, by definition of MPE, the household's continuation payoff from period $t + 1$ onward cannot depend on $f_t$ in equilibrium. Any MPE must therefore have $f_t = 0$ in all $t \geq 0$. 

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Since $f_t = 0$ in all $t \geq 0$, it is a best response for neighbor $t$ to accept iff $p_t \geq c_t$, and this best response is unique if $p_t > c_t$. Consequently, either $d_t = 0$ or $p_t = c_t$ in any MPE. Note that any MPE with $d_t = 0$ is payoff-equivalent to an MPE in which $d_t = 1$ and $c_t = p_t = 0$. Consequently, in each period $t \geq 0$, the household chooses a consumption level $c_t \in [0, w_t]$, sets $p_t = c_t$, and invests $w_t - c_t$. This is exactly the problem that defines $\hat{\Pi}(w_t)$ and $\hat{C}(w_t)$ for the case with $\hat{U} = U$. We conclude that there is a unique MPE consumption and equilibrium payoff in each $t \geq 0$, equal to the unique consumption and equilibrium payoff in the city with $\hat{U} = U$. ■

Consider household-optimal equilibria in the model without a city and impose the constraint that deviations are punished by reversion to Markov Perfect Equilibrium. The household’s payoff following a deviation is therefore $\hat{\Pi}(w_t)$ for the special case $\hat{U} = U$. On the equilibrium path, neighbor $t$ is willing to accept a request if $c_t \leq p_t + f_t$, just as in the baseline model. The household is willing to pay a favor $f_t$ so long as the dynamic enforcement constraint, (DE), holds. We conclude that the problem of finding a household-optimal equilibrium is identical to the baseline model, so Proposition 2 holds by the same proof.

D.4.2 Model Variant 2: Dispensing with the Assumption of Reversion to Markov Perfect Equilibrium

The previous part of this appendix shows how our wealth dynamics hold even if we eliminate the city from our model. However, this analysis imposes the assumption that deviations are punished by reversion to Markov Perfect Equilibrium, even though MPE are typically not the harshest available punishment. In this section, we develop an alternative model that allows us to do away with this assumption of reversion to MPE.

Consider the following alternate model without a city. A single, long-lived household interacts with a sequence of short-lived neighbors, indexed by $\{0, 1, \ldots\}$. The household starts the game with wealth $w_0 \geq 0$. In each period $t \in \{0, 1, \ldots\}$, the household interacts with neighbor $t$. They play the following game:
1. The household chooses an amount of consumption to purchase on the market, 
   \( p_t \in [0, w_t] \).

2. Neighbor \( t \) chooses the size of a favor, \( x_t \geq 0 \), to do for the household.

3. The household chooses a reciprocal favor to do for the neighbor, \( f_t \geq 0 \).

4. The household invests its remaining wealth, generating wealth in the next period according to 
   \( w_{t+1} = R(w_t - p_t) \).

All of the actions in this game are publicly observed. Neighbor \( t \)'s payoff is \( f_t - x_t \). Defining \( c_t \equiv p_t + x_t \), the household's period-\( t \) payoff is \( \pi_t = U(c_t) - f_t \). The household has a discount factor \( \delta \in [0, 1) \), with a normalized discounted continuation payoff equal to 
   \[ \Pi_t = \sum_{\nu = 1}^{\infty} \delta^\nu (1 - \delta) \pi_\nu \] in each period \( t \geq 0 \).

We prove that our main result, Proposition 2, holds in this model. The first step in doing so is to characterize the household's lowest equilibrium payoff.

**Lemma 14** In any equilibrium of the alternate model without a city and any \( t \geq 0 \), \( \Pi_t \geq \hat{\Pi}(w_t) \) for the special case \( \hat{U}(\cdot) = U(\cdot) \). Moreover, there exists an equilibrium that gives the household a payoff of \( \hat{\Pi}(w_t) \).

**Proof of Lemma 14:** Consider the following strategy for the household: in each period \( t \geq 0 \), \( f_t = 0 \) and \( p_t \) equals consumption from the unique equilibrium outcome in the city of the baseline model. By construction, this strategy is feasible, and it delivers a stage-game payoff of no less than \( U(p_t) \) in each period \( t \geq 0 \), which is equal to the stage-game payoff from the city. By following this strategy, the household can guarantee a payoff of at least \( \hat{\Pi}(w_t) \).

To construct an equilibrium in which the household earns \( \hat{\Pi}(w_t) \), set \( f_t = x_t = 0 \) for all histories in all \( t \geq 0 \). Neighbor \( t \) has no profitable deviation from this strategy. Given that \( x_t = 0 \) at all histories, the household has no profitable deviation from \( f_t = 0 \). The
household’s optimal consumption then equals $\hat{C}(w)$ in the baseline model for the special case $\hat{U} = U$, resulting in equilibrium payoff $\hat{\Pi}(w_t)$. ■

We can use Lemma 14 to show that household-optimal equilibria solve the same constrained maximization problem as household-optimal equilibria in the baseline model.

Lemma 15 Set $x_t = f_t$. Then, the household-optimal equilibrium solves $\Pi^*(w_t) = \max\{\hat{\Pi}(w_t), \Pi_c(w_t)\}$, where $\hat{\Pi}$ is calculated for the special case $\hat{U} = U$, and

$$\Pi_c(w) = \max_{c \geq 0, f \geq 0} \{(1 - \delta)(U(c) - f) + \delta \Pi^*(R(w + f - c))\}$$

s.t. \[0 \leq c - f \leq w \quad (30)\]

\[f \leq \frac{\delta}{1 - \delta} \left( \Pi^*(R(w + f - c)) - \hat{\Pi}(R(w + f - c)) \right). \quad (31)\]

Proof of Lemma 15: In any equilibrium, neighbor 0 accepts only if $x_t \leq f_t$. The household’s continuation payoff is at most $\Pi^*(R(w_0 - p_0)) = \Pi^*(R(w_0 - c_0 + f_0))$ and, by Lemma 14, it is at least $\hat{\Pi}(R(w_0 - c_0 + f_0))$. Hence, the household is willing to choose $f_0$ only if it satisfies (31). Setting $x_t = f_t$ yields $\Pi_c(w_0)$ as an upper bound on the household’s payoff from staying.

This bound is tight whenever $\Pi_c(w_0) \geq \hat{\Pi}(w_0)$. For any $(c_0, f_0)$ that satisfies (30) and (31), it is an equilibrium to set $p_0 = c_0 - f_0 \geq 0$ and $x_0 = f_0$, play a household-optimal continuation equilibrium on-path, and respond to any deviation with the worst possible continuation payoff, $\hat{\Pi}(R(w_0 + f_0 - c_0))$. The household is willing to choose $f_0$ by (31); neighbor 0 is willing to choose $x_0 = f_0$ because he earns 0 from doing so and no more than 0 from a deviation; and the principal is willing to choose $p_0$ because $\Pi_c(w_0) \geq \hat{\Pi}(w_0)$. Thus, $\Pi_c(w_0)$ is the household’s maximum equilibrium payoff. It follows that $\Pi^*(w) = \max\{\hat{\Pi}(w), \Pi_c(w)\}$, as desired. ■

Since household-optimal equilibria in this model and the baseline model solve the same constrained maximization problem, the resulting payoffs and wealth levels, $\{w_t\}_{t=0}^\infty$, where
\[ w_{t+1} = R(w_t + f_t - c_t), \] are identical. We conclude that Proposition 2 applies to this model.

■