Wealth Dynamics in Communities

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Abstract

This paper develops a model to explore how favor exchange influences wealth dynamics. We identify a key obstacle to wealth accumulation: wealth crowds out favor exchange. Therefore, households must choose between growing their wealth and accessing favor exchange. We show that low-wealth households rely on favor exchange at the cost of having tightly limited long-term wealth. As a result, initial wealth disparities persist and can even grow worse. We then explore how communities and policymakers can overcome this obstacle. Using simulations, we show that community benefits and place-based policies can stimulate both saving and favor exchange, and in some cases, can even transform favor exchange into a force that accelerates wealth accumulation.

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1 Introduction

Rising economic tides do not lift all households equally. Even in growing economies, some communities are left behind, with persistently lower wealth than surrounding areas. Such left-behind communities can be found in both rich and developing countries, and in both rural and urban areas.[1]

Faced with limited wealth, members of left-behind communities rely on one another for practical support (e.g., Kranton 1996, Ali and Miller 2016). Community members engage in all kinds of favor exchange, from the trade of food, lodging, and childcare within poor neighborhoods in Milwaukee (Desmond 2012, 2016) to the exchange of rice and kerosene among villagers in India (Jackson et al. 2012). By allowing households to procure goods and services without spending money, in-kind favor exchange frees up resources and therefore has the potential to lead to faster wealth accumulation. Yet in practice, favor exchange does not readily translate into growing wealth. Households instead struggle to "get by" (Warren et al. 2001), even when they have access to high-return savings opportunities like making productive investments or paying off high-interest debts (Ananth et al. 2007, Stegman 2007, Bernheim et al. 2015). Given that favor exchange frees up money that can then be saved, and given the presence of high-return savings opportunities, why doesn't community support translate into growing wealth?

This paper develops a model to study how favor exchange shapes wealth dynamics. While favor exchange could, in principle, encourage saving, it suffers from a commitment problem: households can renege on promised favors and instead use money alone to meet their needs. We show that this commitment problem prevents favor exchange from encouraging saving. Even worse, wealth actually crowds out favor exchange, so that households must keep their wealth artificially low in order to access favor exchange. The result is a persistent wealth gap between left-behind communities and the rest of the economy. We then explore how commu-

[1] Desmond [2012] and Hendrickson et al. [2018] document left-behind communities in rich countries. Hoff and Sen [2006], Jakiela and Ozier [2016], and Munshi and Rosenzweig [2016] document such communities in developing countries.

nities and policymakers can mitigate this tension between favor exchange and saving, and potentially even transform favor exchange into a force that encourages wealth accumulation.

The foundation of our analysis is a model that introduces endogenous wealth dynamics into favor-exchange relationships. A household faces a standard consumption-saving problem, with the twist that it can "purchase" consumption using not only money but also promises of future in-kind favors. However, the household cannot commit to following through on these promises, so they must be credible in the context of an ongoing favor-exchange relationship.

Using this model, we show that wealth undermines the trust that is essential for favor exchange. The reason is that even after losing access to favor exchange, households can use money to buy consumption. Wealthier households therefore have less to lose from not reciprocating favors; namely, they have better outside options to favor exchange. These households therefore have a weaker incentive to return favors. Recognizing this, community members are less willing to do favors for households that are expected to become wealthy in the future. In short, households that accumulate wealth become "too big for their boots" and lose access to community support.

Our main result characterizes how this "too-big-for-their-boots" mechanism constrains wealth accumulation and exacerbates long-term inequality. Equilibrium wealth dynamics are shaped by two forces. First, there is an initial *selection effect:* wealthy households opt out of favor exchange, while low-wealth households rely on it. Second, in response to the "too-big-for-their-boots" mechanism, households that rely on favor exchange engage in sharply constrained saving. Some households are so constrained that they even decrease their wealth over time. Consequently, for households that rely on favor exchange, long-term wealth remains substantially below what it would be without favor exchange. Together, the selection effect and the "too-big-for-their-boots" mechanism imply that initial wealth disparities between households persist and can even grow worse over time.

The "too-big-for-their-boots" mechanism resonates with sociological and ethnographic evidence on favor exchange in left-behind communities. Support for this mechanism dates back at least to Stack [1975]'s classic study of favor exchange in a low-wealth U.S. community. Stack [1975, p43] notes that the *wealthiest* members of that community are most at risk of being excluded from favor exchange:

> As people say, "The poorer you are, the more likely you are to pay back." This criterion often determines which kin and friends are actively recruited into exchange networks.

To explain why wealthier households are excluded, Stack [1975] suggests that everyone knows that these households can more easily leave the community and move to a nearby city. Portes and Sensenbrenner [1993], Briggs [1998], Dominguez and Watkins [2003], and others echo this tension between favor exchange and outside options. Our main result characterizes the dynamic implications of the "too-big-for-their-boots" mechanism for households' saving decisions.

Building on this result, we then explore how community leaders and policymakers can mitigate the "too-big-for-their-boots" mechanism. This mechanism arises because households cannot commit to repaying favors. We first prove that, if a household were able to commit, then favor exchange would unambiguously encourage wealth accumulation instead of discouraging it. This result suggests that easing the commitment problem can transform favor exchange into a force that accelerates wealth accumulation.

Using simulations, we show that communities can unlock this transformative effect by making non-favor-exchange benefits, such as participation in family, social, or religious activities, contingent on repaying past favors. For communities that provide these types of benefits, favor exchange can help at least some low-wealth households catch up to their wealthier peers. We link this analysis to examples of communities that appear to have successfully transformed favor exchange in this way.

Policymakers can similarly mitigate the "too-big-for-their-boots" mechanism using "place-based" policies, which provide benefits that are localized to a particular community (Austin et al. 2018). Using simulations, we show that these policies can relax the commitment problem and lead to higher long-term wealth in the community. While such policies cannot completely eliminate the wealth gap between low-wealth households and their wealthier peers, they can help left-behind communities by narrowing that gap.

In some communities, favor exchange not only augments consumption but also provides insurance against negative shocks. To explore how this insurance role affects wealth dynamics, we enrich our model to incorporate random savings shocks. If shocks are not too large, then we prove that our main result goes through: the "too-big-for-their-boots" mechanism still depresses long-term wealth for households that rely on favor exchange. We then use numerical simulations to explore what happens with large savings shocks. Here, qualitatively different dynamics emerge. Large shocks substantially relax the commitment problem, since even wealthy households are willing to repay substantial favors in order to keep accessing favor exchange following large negative shocks. Thus, similar to community benefits, large shocks can transform favor exchange into a force that accelerates wealth accumulation rather than discouraging it.

A key feature of our model is that favor exchange is in-kind: community members trade consumption goods and services rather than borrowing and lending money. This distinction separates our paper from the literature on informal lending (e.g., Bulow and Rogoff 1989, Ligon et al. 2000). In the final part of the paper, we show that it is exactly this in-kind nature that allows favor exchange to flourish. Indeed, if favors were instead monetary, then favor exchange would be impossible in our setting. The reason is that when a household borrows money, it necessarily gets wealthier and so becomes less reliant on favor exchange in the future. Monetary favor exchange is therefore self-defeating (as in, e.g., Bulow and Rogoff 1989). In contrast, in-kind favor exchange is self-sustaining, because the household can engage in substantial favor exchange without becoming any wealthier.

Related Literature

This paper makes three contributions. First, we build a model that incorporates endogenous wealth dynamics into in-kind favor exchange. Second, we characterize the "too-big-for-their-boots" mechanism and demonstrate how it discourages saving and exacerbates wealth inequality. Third, we identify policies and community characteristics that can mitigate the "too-big-for-their-boots" mechanism and encourage wealth accumulation. We discuss the literature related to the first two contributions here, deferring the discussion of the third contribution to Section 5.

Our modeling approach draws from the literature on community enforcement and favor exchange (e.g., Hauser and Hopenhayn 2008, Jackson et al. 2012, Ambrus et al. 2014, Ali and Miller 2016 and 2020, Miller and Tan 2018, Sugaya and Wolitzky 2021), and the literature on relational contracting (e.g., Levin 2003, Malcomson 2013). Our key departure from these papers is to introduce wealth as an endogenous state variable. This addition gives rise to the wealth dynamics that are at the core of our analysis.

In our model, the "too-big-for-their-boots" mechanism arises because market exchange acts as an endogenous outside option to favor exchange. Thus, our analysis builds on papers that study the role of outside options in relationships (e.g., Baker et al. 1994, Kovrijnykh 2013), as well as papers that study how favor and market exchange interact (e.g., Kranton 1996, Banerjee and Newman 1998, Gagnon and Goyal 2017, Banerjee et al. 2020, Jackson and Xing 2020). Unlike those papers, the outside option in our setting evolves based on the household's saving decisions. This dynamic feedback is what leads to the "too-big-for-their-boots" mechanism. More distantly related are papers that study other types of distortions in communities, such as those that arise from signaling (e.g., Austen-Smith and Fryer 2005) or hold-up (e.g., Hoff and Sen 2006).

Our main result shows how persistent wealth inequality arises from the combination of a selection effect and the "too-big-for-their-boots" mechanism. The selection effect says that wealthy households opt out of favor exchange. By identifying how the value of favor exchange varies with wealth, this effect relates to papers that consider the costs and benefits of being part of a close-knit community (e.g., Kranton 1996, Banerjee and Newman 1998) and the role of wealth in migration decisions (e.g., Munshi and Rosenzweig 2016). On its own, however, the selection effect is silent about how favor exchange influences saving decisions *within* communities. The "too-big-for-their-boots" mechanism addresses this gap by identifying a hidden cost of saving, which is that it undermines favor exchange.

Our analysis provides an explanation for the well-documented fact that poverty can persist even in the presence of high-return savings opportunities (e.g., Ananth et al. 2007, Karlan et al. 2019). In contrast to other explanations of under-saving, our mechanism does not rely on fixed costs of investment (e.g., Nelson 1956, Advani 2019), monopolistic credit markets (e.g., Mookherjee and Ray 2002, Liu and Roth 2020), or behavioral preferences (e.g., Banerjee and Mullainathan 2010, Bernheim et al. 2015). Thus, while we share Advani [2019]'s emphasis on favor exchange and Liu and Roth [2020]'s focus on outside options, our mechanism leads to persistently low wealth even in the absence of frictions like fixed-cost investments (in contrast to Advani 2019) or monopolistic credit markets (in contrast to Liu and Roth 2020). By connecting a household's saving decisions to its ability to engage in favor exchange, our mechanism leads to novel empirical predictions and policy prescriptions.

2 Model

A long-lived **household** ("it") has initial wealth $w_0 \geqslant 0$ and discount factor $\delta \in (0,1)$. The household starts in a community. At the beginning of each period $t \in \{0, 1, ...\}$, if the household still lives in the community, it can choose to either stay or move to a city. Once it moves to the city, it remains there forever.

If the household is in the community in period t, then it plays the following **community game** with a short-lived **neighbor** t ("she"), who is another member of the community:

1. The household requests a consumption level $c_t \geqslant 0$ and offers a payment $p_t \geqslant 0$ in

exchange. The payment cannot exceed the household's wealth, $p_t \leqslant w_t$.

2. Neighbor t either accepts or rejects this exchange, $d_t \in \{1, 0\}$. If she accepts ($d_t = 1$), then she receives p_t and incurs the cost of providing c_t. If she rejects ($d_t = 0$), then no trade occurs.

3. The household decides how much of a favor, $f_t \geqslant 0$, to perform for neighbor t. The household incurs the cost of providing f_t.

4. The household invests its remaining wealth, $w_t - p_t d_t$, to generate w_{t+1}. Let $R(\cdot)$ denote the return on investment, so that $w_{t+1} = R(w_t - p_t d_t)$.

Let $U(\cdot)$ be the household's consumption utility in the community. The household's period-t payoff in the community is $\pi_t = U(c_t d_t) - f_t$. Neighbor t's payoff is $(p_t - c_t)\, d_t + f_t$. The community is tight-knit and so all actions are observed by all neighbors.

We assume that consumption utility $U(\cdot)$ and investment returns $R(\cdot)$ are strictly increasing, with $U''(\cdot)$ and $R'(\cdot)$ continuous, $U(0) = R(0) = 0$, $U(\cdot)$ strictly concave, $R(\cdot)$ concave, $\lim_{c \downarrow 0} U'(c) = \infty$, and $\lim_{c \to \infty} U'(c) = 0$. We say that investment *generates positive returns* at investment level w if $R'(w) \geqslant \frac{1}{\delta}$. We assume that $R'(w) > \frac{1}{\delta}$ for $w < \bar{w}$ and $R'(w) = \frac{1}{\delta}$ for $w \geqslant \bar{w}$, so that investment generates positive returns at all investment levels and strictly so below a threshold $\bar{w} > 0$. Investment returns are deterministic in this model; Section 6 considers a setting with stochastic returns.

If the household has moved to the city by period t, then it plays the **city game** with a short-lived **vendor** t ("she"), who has the same actions and payoff as neighbor t. The city game is identical to the community game in all but two ways. First, each vendor observes only her own interaction with the household, so that interactions are anonymous in the city. Second, the household's marginal utility of consumption is weakly higher in the city. Formally, the household's period-t payoff in the city is $\pi_t = \hat{U}(c_t d_t) - f_t$, where $\hat{U}(\cdot)$ satisfies the same regularity conditions as $U(\cdot)$, with $\hat{U}'(c) \geqslant U'(c)$ for all $c > 0$.

The household's continuation payoff at the beginning of period t is

$$\Pi_t = (1-\delta)\sum_{s=t}^{\infty}\delta^{s-t}\pi_s.$$

We characterize household-optimal equilibria, which are the perfect Bayesian equilibria that maximize the household's *ex ante* expected payoff. Without loss of generality, we assume that the household leaves the community if it is indifferent between staying and leaving.

The following assumption ensures that in equilibrium, households that stay in the community have access to strictly positive-return investments.

Assumption 1 *Define $\bar{c} > 0$ as the solution to $U'(\bar{c}) = 1$. Then, $R(\bar{w} - \bar{c}) > \bar{w}$.*

In the context of Stack [1975], the household and neighbors are members of a low-income Midwestern community called "the Flats." Members regularly exchange food, clothing, childcare, lodging, and other goods and services (c_t). To compensate one another, households can pay with money (p_t) and "pay" with future favors (f_t). For example, the recipient of childcare ($c_t > 0$) can reciprocate with future childcare ($f_t > 0$). These favors are in-kind, in the sense that they involve directly trading goods and services. Households accumulate wealth ($R(\cdot)$) by repaying high-interest debt or making other investments. The Flats is a tight-knit community and "everyone knows who is working, when welfare checks arrive, and when additional resources are available" (p. 37), as well as who has reneged on promised favors. Households in the Flats can move to a nearby city, Chicago, which harbors greater opportunities ($\hat{U}' \geqslant U'$) but separates them from their favor-exchange network.

While we assume that moving to the city is irreversible, this assumption is not essential for our results; in online appendix C.1, we prove that our main findings are robust to allowing the household to return to the community after leaving. Neither is it essential for the household's cost of providing f_t to be linear; in online appendix C.2, we prove a similar result if the cost of providing f_t is convex. As a special case, the cost of providing f_t could be $U^{-1}(f_t)$, in which case neighbors value the favor in the same way as the household values consumption.

We model the city as an alternative location to the community, but our main result would hold without change if we eliminated the city and instead assumed that any deviation was punished by reversion to a Markov perfect equilibrium in the community. The reason is that even within the community, wealth crowds out favor exchange, since wealthier households are better able to meet their needs using money alone. Given this equivalence, we include the city in our analysis in order to match our applications, which typically include mobility across locations, and to derive empirical and policy implications. In applications that do not have mobility across locations, one can interpret "leaving for the city" as opting out of favor exchange in the community.

3 Life in the City

We begin by characterizing wealth dynamics in the city. The household faces a standard consumption-saving problem. It takes full advantage of investment opportunities and accumulates wealth.

Interactions are anonymous in the city, so $f_t = 0$ in equilibrium. Vendor t is therefore willing to accept an offer only if the payment covers her cost (i.e., $p_t \geqslant c_t$), and strictly prefers to do so if $p_t > c_t$. Consequently, every equilibrium entails $p_t = c_t$ in every $t \geqslant 0$, so that $w_{t+1} = R(w_t - c_t)$. For a household with wealth w, the resulting optimal payoff and consumption are given, respectively, by:

$$\hat{\Pi}(w) = \max_{c\in[0,w]} \Big((1-\delta)\hat{U}(c) + \delta\hat{\Pi}\left(R\left(w-c\right)\right)\Big) \text{ and}$$
$$\hat{C}(w) \in \arg\max_{c\in[0,w]} \Big((1-\delta)\hat{U}(c) + \delta\hat{\Pi}\left(R\left(w-c\right)\right)\Big).$$

Our first result shows that in any equilibrium of the city, the household consumes $\hat{C}(w_t)$ and its payoff is $\hat{\Pi}(w_t)$. Moreover, both consumption and wealth increase over time, with long-term wealth above $R(\bar{w})$.

Proposition 1 *Both* $\hat{\Pi}(\cdot)$ *and* $\hat{C}(\cdot)$ *are strictly increasing, with* $\hat{\Pi}(\cdot)$ *continuous. In any equilibrium of the city,* $\Pi_t = \hat{\Pi}(w_t)$ *and* $c_t = \hat{C}(w_t)$ *in any* $t \geqslant 0$*, with* $(w_t)_{t=0}^{\infty}$ *increasing and*

$$\lim_{t\to\infty} w_t \geq R(\bar{w})$$

on the equilibrium path.

The proof of Proposition 1 is routine and relegated to online appendix B. Since $R'(w) > \frac{1}{\delta}$ for $w < \bar{w}$, the standard Euler equation,

$$\hat{U}'(\hat{C}(w_t)) = \delta R'(w_t - \hat{C}(w_t))\hat{U}'(\hat{C}(w_{t+1})),\ \forall\, t, \tag{Euler}$$

implies that the household's long-term wealth is at least $R(\bar{w})$ in the city. Figure 1 simulates equilibrium outcomes in the city.[2]

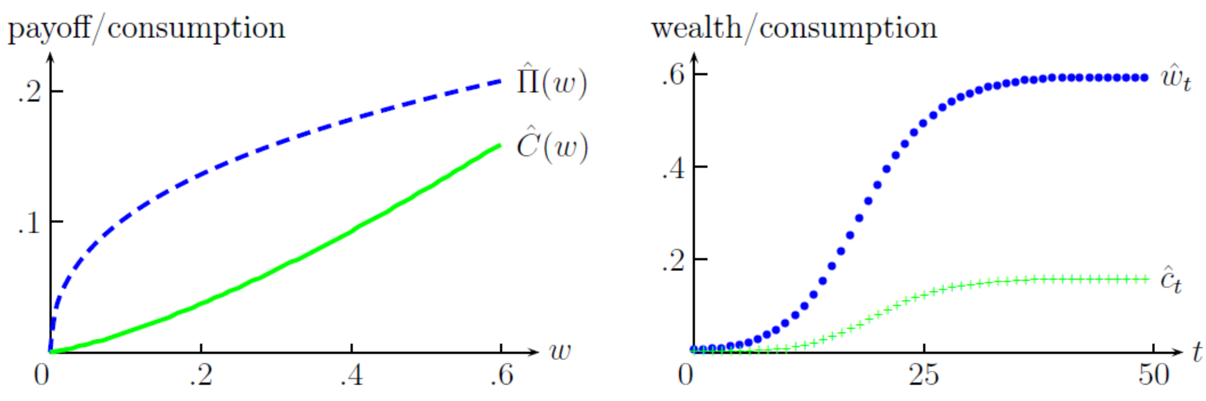

Figure 1: Left panel: the household's equilibrium payoff and consumption in the city as a function of w. Right panel: consumption and wealth over time in the city, starting at $w_0 = 0.006$.

[2]Parameters in Figures 1 to 3 are $\delta = \frac{8}{10}, U(c) = \frac{\sqrt{c}}{2}, \hat{U}(c) = \frac{13\sqrt{c}}{25}$, and

$$R(w) = \begin{cases} 3\left(\sqrt{w+1} - 1\right) & w \leqslant \frac{11}{25}; \\ \frac{5w}{4} + \frac{1}{20} & \text{otherwise.} \end{cases}$$

4 Wealth Dynamics in the Community

This section characterizes household-optimal equilibria for a household starting in the community. Section 4.1 presents our main result, which shows that households that rely on favor exchange have sharply limited long-term wealth. Section 4.2 discusses empirical implications of this result. Section 4.3 gives the proof.

4.1 Underinvestment and Persistent Inequality

Our main result identifies two reasons why wealth in the community remains substantially below wealth in the city. First, there is a *selection effect:* sufficiently wealthy households leave the community, whereas poorer households stay. Second, the "too-big-for-their-boots" mechanism constitutes an extra cost of investment in the community, resulting in sharply limited long-term wealth for households that stay.

To understand this result, note that the community's sole advantage over the city is that favor exchange can augment consumption in the community but not in the city. In particular, neighbors are able to observe and punish a household that reneges on $f_t > 0$. Consequently, the household can credibly promise $f_t > 0$ to repay neighbor t for providing consumption. Given a credibly promised favor $f_t > 0$, neighbor t is willing to accept any request that satisfies $c_t \leq p_t + f_t$.

The opportunity to engage in favor exchange is most attractive to low-wealth households, which would otherwise have low consumption and a high marginal utility of consumption. Conversely, wealthy households already consume a lot, so their marginal utility from further increasing c_t is low. Thus, wealthy households leave the community while low-wealth households stay, giving us our selection effect.

To understand the "too-big-for-their-boots" mechanism, consider a household with wealth w_t that stays in the community, consumes $c_t = p_t + f_t$, and invests $I_t = w_t - p_t$. Since the household can renege on f_t, leave the community, and earn $\hat{\Pi}(R(I_t))$ in the city, the *maximum* favor that can be sustained in equilibrium depends on the investment I_t. Denote a household's maximum continuation payoff if it is in the community and has wealth $R(I_t)$ by $\Pi^*(R(I_t))$. Then, f_t must satisfy the following **dynamic enforcement constraint**:

$$f_t \leqslant \bar{f}(I_t) = \frac{\delta}{1-\delta}(\Pi^*(R(I_t)) - \hat{\Pi}(R(I_t))). \tag{DE}$$

This constraint ensures that the household prefers to do favor f_t and earn continuation payoff $\Pi^*(R(I_t))$, rather than reneging on f_t and earning punishment payoff $\hat{\Pi}(R(I_t))$.

If (DE) binds, then changing I_t affects the size of the favor, which affects period-t consumption because $c_t = p_t + f_t$. The "too-big-for-their-boots" mechanism holds when $\bar{f}(I_t)$ is decreasing in I_t, so that investment crowds out favor exchange. When this occurs, the standard cost-benefit trade-off captured by the Euler equation, (Euler), is augmented by an extra cost: the favor f_t, and so consumption c_t, decrease as investment increases. Consequently, the household optimally invests less than the investment that would satisfy (Euler). This is the sense in which a household in the community underinvests.

This intuition elides a key complication: $\bar{f}(\cdot)$ depends on both $\Pi^*(\cdot)$ and $\hat{\Pi}(\cdot)$, which in turn depend on the household's future decisions about consumption, favor exchange, and investment. Wealth affects all of these decisions, rendering a full characterization of household-optimal equilibria intractable.

Our main result, Proposition 2, focuses on the selection effect and the "too-big-for-their-boots" mechanism. *Selection* is summarized by a wealth threshold, $w^{se} < \bar{w}$, and a set $\mathcal{W} \subseteq [0, w^{se}]$. The household stays forever if $w_0 \in \mathcal{W}$ and otherwise leaves the community immediately. The *"too-big-for-their-boots" mechanism* is summarized by a wealth level, $w^{tr} < w^{se}$, such that the long-term wealth of a household that stays in the community is below w^{tr}. Since $w^{tr} < w^{se} < R(\bar{w})$, long-term wealth in the community is substantially below long-term wealth in the city. Moreover, a household that stays with wealth $w_0 \in (w^{tr}, w^{se})$ has strictly declining wealth over time. This is the sense in which wealth inequality persists and can grow worse over time.

Proposition 2 *Impose Assumption 1. There exist wealth levels w^{tr} and w^{se} satisfying $0 \leq w^{tr} < w^{se} < \bar{w}$, and a positive-measure set $\mathcal{W} \subseteq [0, w^{se}]$ with $\sup \mathcal{W} = w^{se}$, such that in any household-optimal equilibrium:*

1. ***Selection.*** *The household stays in the community forever if $w_0 \in \mathcal{W}$, and otherwise leaves in period 0.*

2. ***"Too-big-for-their-boots" mechanism.*** *If the household stays in the community, then $(w_t)_{t=0}^{\infty}$ is monotone, with*

$$\lim_{t \to \infty} w_t \leqslant w^{tr}.$$

   Moreover, $\mathcal{W} \cap [w^{tr}, w^{se}]$ has a positive measure.

Section 4.3 gives the proof of this result. We have already argued that sufficiently wealthy households leave the community, giving us the selection threshold w^{se}, while some poorer households stay, giving us the set $\mathcal{W}$. Moreover, any household that stays in $t = 0$ stays forever. The reason is that a household that leaves in period $t > 0$ cannot engage in favor exchange during period $(t-1)$, so it might as well leave in period $(t-1)$. Iterating this argument, such a household might as well leave in period 0.

To understand why long-term wealth in the community is below w^{tr}, consider a household that stays with wealth *just below* w^{se}. Such a household is close to indifferent between leaving and staying. Since staying implies that wealth remains below w^{se}, which is lower than long-term wealth in the city, this household optimally stays only if it engages in significant favor exchange in some period; that is, only if $f_t \gg 0$ in some t. If the household's wealth always remains near w^{se}, then the right-hand side of the dynamic enforcement constraint, (DE), is close to zero, so $f_t \approx 0$ in every $t \geq 0$. Therefore, for $f_t \gg 0$ to be sustainable in equilibrium, the household must underinvest so severely that its wealth decreases. We conclude that long-term wealth in the community is below some level, w^{tr}, which is below the selection threshold w^{se} and so sharply below long-term wealth in the city. The proof of Proposition 2 strengthens this result by showing that $(w_t)_{t=0}^{\infty}$ is monotone in the community and that a positive measure of households, $\mathcal{W} \cap [w^{tr}, w^{se}]$, stay in the community. These households have decreasing wealth.

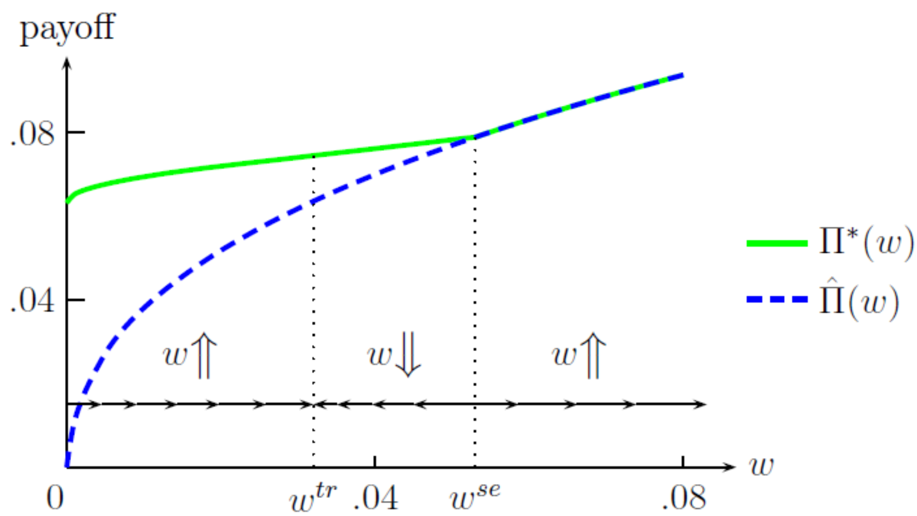

Figure 2: Household-optimal equilibrium payoffs and wealth dynamics.

Figure 2 summarizes Proposition 2. In this simulation, the household moves to the city if $w_0 \geqslant w^{se}$ and otherwise stays in the community. Among those that stay, households with $w_0 < w^{tr}$ grow their wealth, but only up to w^{tr}. Those with $w_0 \in (w^{tr}, w^{se})$ have declining wealth over time.

4.2 Empirical Implications

An immediate implication of Proposition 2 is that one-time transfers do not necessarily improve long-term wealth. In Figure 2, consider transferring money to a household with initial wealth $w_0 < w^{se}$. If the household's post-transfer wealth is still below w^{se}, then this extra money is consumed rather than saved, and long-term wealth remains below w^{tr}. This implication resonates with Karlan et al. [2019], who study what happens when indebted individuals receive one-time debt relief. Consistent with Figure 2, they find that recipients tend to quickly fall back into debt. In contrast, a transfer that is large enough to bring wealth above w^{se} induces further investment, but only by spurring the household to opt out of favor exchange.

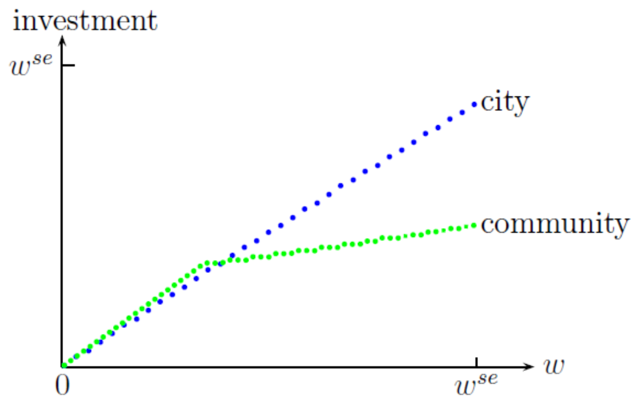

Figure 3: Investment in the city versus in the community.

We use numerical simulations to explore further implications of our model. Figure 3 identifies a key non-monotonicity in how favor exchange affects wealth accumulation. This figure plots investment as a function of the household's current wealth both in the city (the blue sparsely-dotted curve) and in the community (the green densely-dotted curve). As expected, the wealthier households in the community underinvest relative to the city; however, the poorest households invest strictly *more* than they would in the city. The reason is that the outside option for the poorest households is so low that the dynamic enforcement constraint, (DE), does not bind. These households optimally invest all of their wealth and rely on favor exchange to meet their needs. Of course, these households eventually reach wealth levels where (DE) binds, at which point the "too-big-for-their-boots" mechanism kicks in and limits their long-term wealth. Nevertheless, this simulation result shows that favor exchange has the potential to free up resources and thereby accelerate wealth accumulation. We explore this idea further in Section 5.

Our model also sheds light on how changes in more prosperous areas can spill over onto left-behind communities. For instance, productivity has diverged across localities in the United States over the past 20 years, with the most productive metropolitan areas growing even more productive relative to the rest of the country (Parilla and Muro 2017).

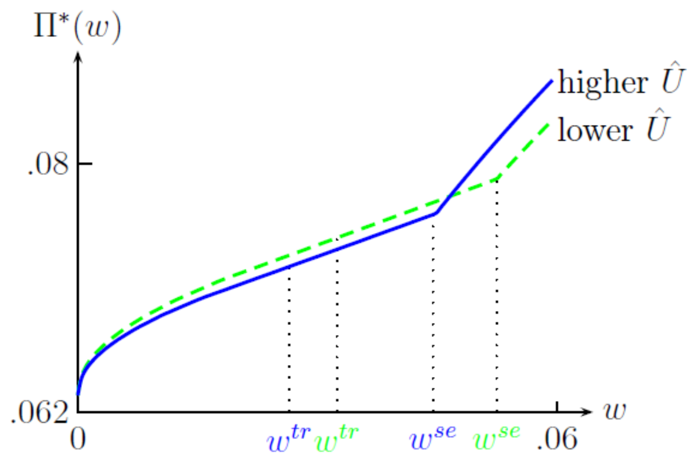

Figure 4: Simulated comparative statics with respect to $\hat{U}(\cdot)$.

Figure 4 illustrates how growing productivity in the city, which we model by increasing $\hat{U}$, affects wealth dynamics in the community.[3] While nothing material has changed within the community, a higher $\hat{U}$ translates into a higher outside option and so tightens (DE). Households in the community respond by investing less, leading to lower long-term wealth, w^{tr}.

4.3 The Proof of Proposition 2

Let $\Pi^*(w)$ be the maximum equilibrium payoff of a household with wealth w. Define

$$\begin{aligned} \Pi_c(w) = \quad & \max_{c\geqslant 0, f\geqslant 0} \quad \{(1-\delta)(U(c)-f)+\delta\Pi^*(R(w+f-c))\} \\ & \text{s.t.} \qquad 0 \leqslant c-f \leqslant w \qquad (1) \\ & \qquad\qquad f \leqslant \frac{\delta}{1-\delta}\Big(\Pi^*(R(w+f-c)) - \hat{\Pi}(R(w+f-c))\Big). \qquad (2) \end{aligned}$$

We show that $\Pi_c(w)$ is the household's maximum payoff conditional on staying in the community in the current period. Hence, the household's maximum equilibrium payoff, $\Pi^*(w)$, is the maximum of $\hat{\Pi}(w)$ and $\Pi_c(w)$.

[3]The simulation for "lower $\hat{U}$" uses parameters identical to Figures 1-3. For "higher $\hat{U}$," parameters are the same except that $\hat{U}(c) = \frac{27\sqrt{c}}{50}$.

Lemma 1 *The household's maximum equilibrium payoff is* $\Pi^*(w_0) = \max\left\{\hat{\Pi}(w_0), \Pi_c(w_0)\right\}$*, where* $\Pi_c(\cdot)$ *and* $\Pi^*(\cdot)$ *are strictly increasing.*

Proof of Lemma 1: In any equilibrium, neighbor 0 accepts only if $c_0 \leqslant p_0 + f_0$. The household's continuation payoff is at most $\Pi^*(R(w_0-p_0))$ and at least $\hat{\Pi}(R(w_0-p_0))$. Hence, it is willing to do favor f_0 only if

$$f_0 \leqslant \frac{\delta}{1-\delta}\left(\Pi^*(R(w_0 - p_0)) - \hat{\Pi}(R(w_0 - p_0))\right).$$

Setting $c_0 = p_0 + f_0$ yields $\Pi_c(w_0)$ as an upper bound on the household's payoff from staying.

This bound is tight. For any (c_0, f_0) that satisfies (1) and (2), it is an equilibrium to set $p_0 = c_0 - f_0 \geqslant 0$, play a household-optimal continuation equilibrium on-path, and respond to any deviation with the household leaving and $f_t = 0$ in all future periods.[4] Thus, $\Pi_c(\cdot)$ is the household's maximum equilibrium payoff conditional on staying. It follows that $\Pi^*(w) = \max\{\hat{\Pi}(w), \Pi_c(w)\}$. Since $\Pi_c(\cdot)$ is strictly increasing by inspection and $\hat{\Pi}(\cdot)$ is strictly increasing by Proposition 1, $\Pi^*(\cdot)$ is strictly increasing. ■

The next three lemmas characterize household-optimal equilibria in the community. First, we show that households that stay in the community, stay forever.

Lemma 2 *If* $w_0 \geqslant 0$ *is such that* $\Pi^*(w_0) > \hat{\Pi}(w_0)$*, then in any* $t \geqslant 0$ *of any household-optimal equilibrium,* $\Pi^*(w_t) > \hat{\Pi}(w_t)$ *on the equilibrium path.*

Proof of Lemma 2: Suppose $t > 0$ is the first period in which $\Pi^*(w_t) = \hat{\Pi}(w_t)$, so $\Pi^*(w_{t-1}) = \Pi_c(w_{t-1}) > \hat{\Pi}(w_{t-1})$. Let $\{c_{t-1}, f_{t-1}\}$ achieve $\Pi_c(w_{t-1})$. Since $\Pi^*(w_t) = \hat{\Pi}(w_t)$, (2) implies $f_{t-1} = 0$. Therefore, $\hat{\Pi}(w_{t-1}) \geqslant \Pi_c(w_{t-1})$, since if the household exits in $t-1$, it could choose the same p_{t-1} and c_{t-1} and earn continuation payoff $\hat{\Pi}(w_t) = \Pi^*(w_t)$. This contradicts the presumption that $\Pi_c(w_{t-1}) > \hat{\Pi}(w_{t-1})$. ■

[4] If $f_t = 0$ in all $t \geq 0$, then the household is willing to leave because $U \leq \hat{U}$.

Second, we show that wealthy households leave the community, while poorer households stay.

Lemma 3 *The set* $\mathcal{W} = \left\{ w : \Pi^*(w) > \hat{\Pi}(w) \right\}$ *has positive measure. Moreover,* $w^{se} = \sup \left\{ w : \Pi^*(w) > \hat{\Pi}(w) \right\}$ *satisfies* $0 < w^{se} < \bar{w}$.

Proof of Lemma 3: First, we show that $\Pi^*(0) > \hat{\Pi}(0) = 0$. Because $\lim_{c \downarrow 0} U'(c) = \infty$, there exists a $c > 0$ such that $c \leqslant \delta U(c)$. Suppose that in all $t \geqslant 0$, $f_t = c_t = c$ and $p_t = 0$ on the equilibrium path, so the household's equilibrium payoff is $U(c) - c$. Any deviation is punished by $f_t = 0$ in all future periods and the household immediately exiting. This strategy delivers a strictly positive payoff. It is an equilibrium because $c \leqslant \delta U(c)$ implies (2). Thus, $\Pi^*(0) > 0$. Since $\Pi^*(w)$ is increasing and $\hat{\Pi}(w)$ is continuous, there exists an interval around 0 such that $\Pi^*(w) > \hat{\Pi}(w)$. So $\left\{ w : \Pi^*(w) > \hat{\Pi}(w) \right\}$ has positive measure.

Next, we show that $w^{se} < \bar{w}$. Let $\bar{c}$ satisfy $U'(\bar{c}) = 1$, and let w_0 be such that $\Pi^*(w_0) > \hat{\Pi}(w_0)$. By Lemma 2, $\Pi^*(w_t) > \hat{\Pi}(w_t)$ in any $t \geqslant 0$ of any household-optimal equilibrium. Suppose that $c_t > \bar{c}$ in period $t \geqslant 0$. If $f_t > 0$, then we can perturb the equilibrium by decreasing c_t and f_t by $\epsilon > 0$, which increases the household's payoff at rate $1 - U'(c_t) > 0$ as $\epsilon \to 0$. So, $f_t = 0$.

Let $\tau > t$ be the first period after t such that $f_\tau > 0$. Consider decreasing p_t and c_t by $\epsilon > 0$, increasing p_τ by $\chi(\epsilon)$, and decreasing f_τ by $\chi(\epsilon)$, where $\chi(\epsilon)$ is chosen so that $w_{\tau+1}$ remains constant. Then, $\chi(\epsilon) \geqslant \frac{\epsilon}{\delta^{\tau-t}}$ because $R'(\cdot) \geqslant \frac{1}{\delta}$. As $\epsilon \to 0$, this perturbation increases the household's payoff by at least $\delta^{\tau-t}\frac{1}{\delta^{\tau-t}} - U'(c_t) > 0$. It is an equilibrium because $f_s = 0$ for all $s \in [t, \tau - 1]$, so (2) still holds in these periods.

The above argument implies that if $c_t > \bar{c}$, then $f_\tau = 0$ for all $\tau \geqslant t$. But then $\Pi^*(w_t) \leqslant \hat{\Pi}(w_t)$, contradicting Lemma 2. Therefore, if $\Pi^*(w_0) > \hat{\Pi}(w_0)$, then $c_t \leqslant \bar{c}$ in every $t \geq 0$ and so $\Pi^*(w_0) \leqslant U(\bar{c})$. Since $R(\bar{w} - \bar{c}) > \bar{w}$ by Assumption 1, it follows that $\Pi^*(\bar{w}) \geqslant \hat{\Pi}(\bar{w}) > \hat{U}(\bar{c}) \geqslant U(\bar{c})$. By the definition of w^{se}, there exists a sequence of initial wealth levels in $\mathcal{W}$ which are arbitrarily close to w^{se} such that the household strictly prefers to stay in the community with those initial wealth levels. If $w^{se} \geqslant \bar{w}$, the equilibrium payoffs at those initial wealth levels would be strictly above $U(\bar{c})$ due to the continuity of $\hat{\Pi}(w)$. This leads to a contradiction, so $w^{se} < \bar{w}$. ■

Finally, we show that household-optimal equilibria exhibit monotone wealth dynamics.

Lemma 4 *In any household-optimal equilibrium, $(w_t)_{t=0}^{\infty}$ is monotone.*

The proof of Lemma 4 is in appendix A. The key step of this proof shows that household-optimal investment, $w_t - p_t$, increases in w_t. Thus, if $w_1 \geqslant w_0$, then $w_2 = R(w_1 - p_1) \geqslant R(w_0 - p_0) = w_1$ and so on, and similarly if $w_1 \leqslant w_0$.

We can now prove Proposition 2. **Selection** is implied by Lemma 2 and Lemma 3. For the **"too-big-for-their-boots" mechanism,** define

$$\tilde{c}(w) = w - R^{-1}(w)$$

and $\tilde{\Pi}(w) = U(\tilde{c}(w))$. Since $w^{se} < \bar{w}$ by Lemma 3, consumption $\tilde{c}(w^{se})$ does not satisfy (Euler). Thus, there exists $K > 0$ such that

$$\tilde{\Pi}(w^{se}) + K < \hat{\Pi}(w^{se}).$$

Define

$$\tilde{f}(w) = \frac{\delta}{1-\delta}\left(\hat{\Pi}(w^{se}) - \hat{\Pi}(w)\right)$$

and

$$\tilde{p}(w) = w^{se} - R^{-1}(w).$$

Consider $w_0 < w^{se}$ such that $\Pi^*(w_0) > \hat{\Pi}(w_0)$, and suppose that there exists an equilibrium in which $(w_t)_{t=0}^{\infty}$ is increasing on the equilibrium path. We claim that $p_t \leqslant \tilde{p}(w_0)$ and

$f_t \leqslant \tilde{f}(w_0)$ in every $t \geq 0$. Indeed,

$$f_t \leqslant \frac{\delta}{1-\delta}\left(\Pi^*(w_{t+1}) - \hat{\Pi}(w_{t+1})\right) \leqslant \frac{\delta}{1-\delta}\left(\Pi^*(w^{se}) - \hat{\Pi}(w_0)\right) = \tilde{f}(w_0),$$

and

$$p_t = w_t - R^{-1}(w_{t+1}) \leqslant w^{se} - R^{-1}(w_0) = \tilde{p}(w_0),$$

where the inequalities hold because (i) $w_t, w_{t+1} \leqslant w^{se}$ by Lemma 2, and (ii) $w_{t+1} \geq w_0$ by our presumption that $(w_t)_{t=0}^{\infty}$ is increasing.

Since $c_t \leqslant p_t + f_t$, the household's payoff satisfies

$$\Pi^*(w_0) \leqslant U\left(\tilde{p}(w_0) + \tilde{f}(w_0)\right) = H(w_0).$$

The function $H(w_0)$ is continuous and decreasing in w_0, with $H(w^{se}) = \tilde{\Pi}(w^{se})$. Since $\tilde{\Pi}(w^{se}) + K < \hat{\Pi}(w^{se})$, there exists $w^{tr} < w^{se}$ such that for any $w_0 \in (w^{tr}, w^{se})$,

$$H(w_0) < \hat{\Pi}(w_0).$$

Therefore, for any $w_0 \in (w^{tr}, w^{se})$, if $w_0 \in \mathcal{W}$, then $(w_t)_{t=0}^{\infty}$ must be strictly decreasing, with $\lim_{t\to\infty} w_t \leqslant w^{tr}$.

By definition of w^{se}, there exists $w_0 \in \mathcal{W} \cap (w^{tr}, w^{se})$ such that $\Pi^*(w_0) > \hat{\Pi}(w_0)$. Since $\Pi^*(\cdot)$ and $\hat{\Pi}(\cdot)$ are increasing, with $\hat{\Pi}(\cdot)$ continuous, we conclude that $\Pi^*(w) > \hat{\Pi}(w)$ on a neighborhood around w_0. So $\mathcal{W} \cap [w^{tr}, w^{se}]$ has a positive measure. ■

5 Transforming Favor Exchange

While favor exchange increases consumption in left-behind communities, the accompanying "too-big-for-their-boots" mechanism is a serious obstacle to wealth accumulation. In this section, we explore how communities and policymakers can mitigate this obstacle, or even transform favor exchange into a force that accelerates wealth accumulation. Section 5.1 proves that with commitment, favor exchange leads to *faster* wealth accumulation than would occur in the city. Informed by this result, Section 5.2 explores practical ways to relax the household's commitment problem.

5.1 With Commitment, Favor Exchange Accelerates Wealth Accumulation

Define the **model with commitment** identically to the baseline model in Section 2, except that the household can commit to following through on promised favors. Formally, if the household is still in the community at the start of period $t \geq 0$, then it chooses a promised favor, $f_t^* \geq 0$, at the same time as it chooses the payment, p_t, and the consumption request, c_t. The promised favor is observed by all neighbors. If neighbor t accepts the request, then the household is committed to following through on its promised favor: $f_t = f_t^*$. If neighbor t rejects the request, then $f_t = 0$. We assume that favor exchange is not possible in the city, so as in Section 3, the household earns continuation payoff $\hat{\Pi}(w_t)$ if it leaves the community at the start of period t.

Recall that $\hat{C}(w_t)$ is the household's equilibrium consumption in the city, so that investment in the city equals $w_t - \hat{C}(w_t)$. We say that favor exchange *accelerates wealth accumulation* if a household that stays in the community invests strictly more than it would in the city. Our next result shows that favor exchange unambiguously accelerates wealth accumulation in the model with commitment.

Proposition 3 *Consider a household in the community with initial wealth $w_0 > 0$. In the model with commitment, there exists a household-optimal equilibrium in which the household leaves at the start of period $\tau < \infty$. For any $t < \tau$, the household invests more than it would in the city by choosing $I_t = w_t - p_t > w_t - \hat{C}(w_t)$. Moreover, for any $t < \tau - 1$, the household invests its entire wealth by choosing $p_t = 0$, so that $I_t = w_t$.*

The proof of Proposition 3 is in Appendix A. Favor exchange allows the household to meet its needs with favors and invest the money it would have otherwise used for consumption. Indeed, the household optimally invests its entire wealth in every period until it is about to leave the community. To see why, consider the problem of maximizing the household's payoff while in the community, subject to holding fixed its continuation payoff upon leaving, $\hat{\Pi}(w_\tau)$, and its total discounted cost of favors, $\sum_{t=0}^{\tau-1} \delta^t f_t$. Holding this "budget" of favors fixed, the household can decrease f_t at rate 1 in order to increase f_{t+1} at rate $\frac{1}{\delta}$. Holding w_τ, and hence $\hat{\Pi}(w_\tau)$, fixed, the household can decrease payment p_t at rate 1 in order to increase p_{t+1} at rate $R'(w_t - p_t)$. Since $R'(w_t - p_t) \geq \frac{1}{\delta}$, the household gets higher returns from delaying monetary payments relative to delaying favors. It therefore relies entirely on favor exchange in early periods and entirely on monetary payments in later periods. Once it starts relying on monetary payments alone, the household optimally leaves the community.

5.2 Ways to Increase Long-term Wealth

Proposition 3 implies that relaxing the household's commitment problem can mitigate the tension between favor exchange and investment. In this section, we show that both community leaders and policymakers have tools to accomplish this goal, although they do so in different ways.

Community Benefits: Thus far in our model, the only reason for the household to repay favors is to access future favors. In reality, households sometimes repay favors to avoid losing access to other benefits that communities provide (Ambrus et al. 2014). They might do so to maintain warm relationships with their friends, to keep their family happy, or to remain a respected member of their social or religious community.

These types of community benefits have two properties that help transform favor exchange. First, these benefits can be conditioned on the household's behavior, and in particular, they can be withheld if the household reneges on a promised favor. Second, a household values these benefits even after it no longer relies on favor exchange. We therefore treat these benefits as being available in both the community and the city if and only if the household has not reneged.[5]

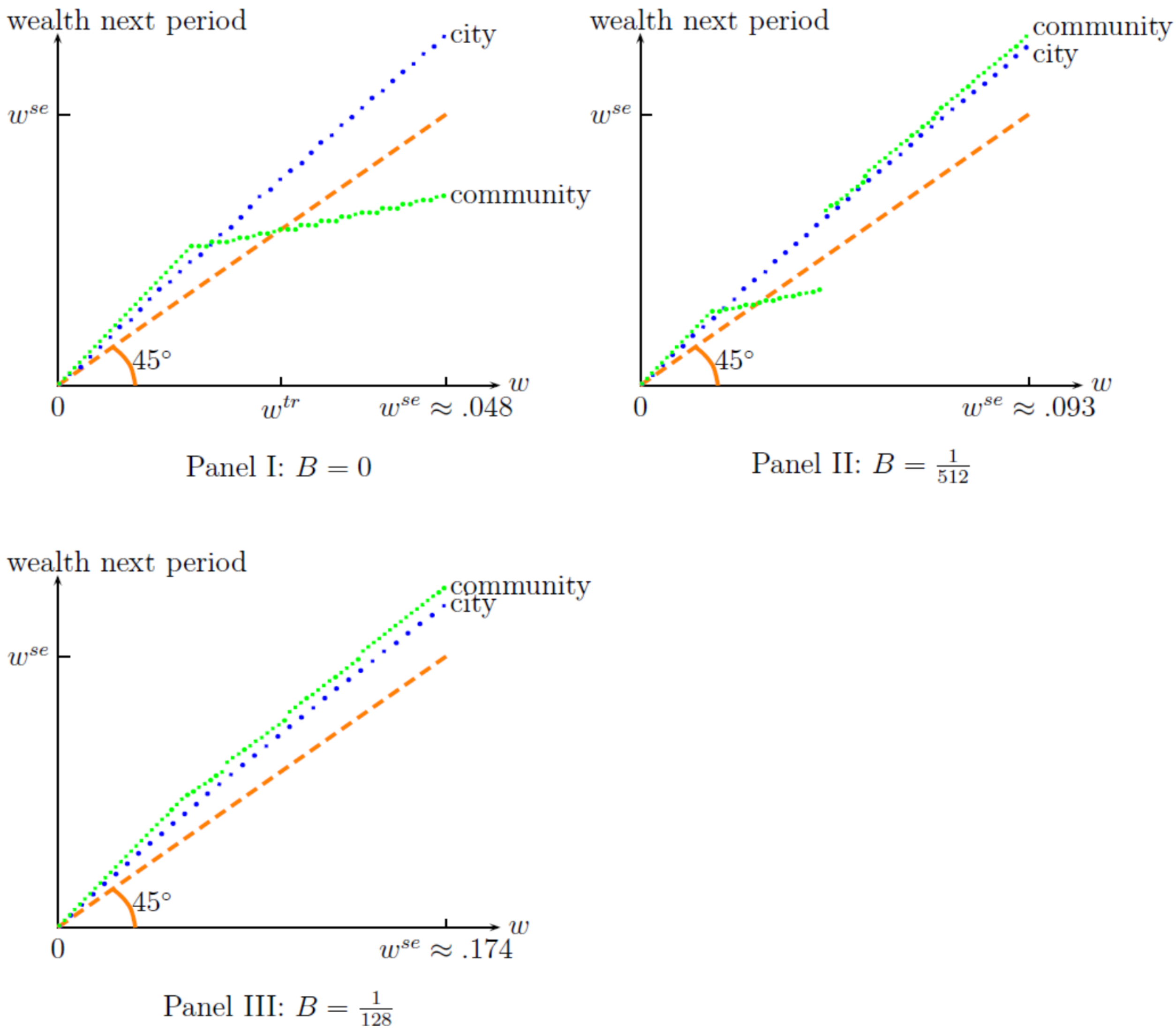

Figure 5: Wealth dynamics for various levels of community benefits. The x-axis is the household's current wealth. The orange dashed line is the 45-degree line, corresponding to current wealth. Wealth is increasing in the community if the green densely-dotted line is above the orange dashed line and decreasing otherwise. Investment is higher than it would be in the city if the green densely-dotted line is above the blue sparsely-dotted line.

Formally, we model community benefits as providing a per-period additive utility benefit,

[5]As discussed in the final paragraph of Section 2, "moving to the city" can be interpreted as staying in the community but no longer engaging in favor exchange, rather than moving to a different location. Under this interpretation, these benefits include benefits that are available only within the community.

$B > 0$, so long as the household has not deviated. These benefits are lost following a deviation, so the household's punishment payoff remains the same as in the baseline model. These benefits therefore relax the commitment problem by adding B to the right-hand side of the dynamic enforcement constraint, (DE).

Figure 5 presents simulated equilibrium wealth dynamics for various levels of B.[6] Each figure gives wealth in period $t+1$ as a function of wealth in period t, so that the household's wealth is increasing in the community whenever the green densely-dotted line is above the orange dashed line and decreasing otherwise. The right-hand boundary of these figures corresponds to the selection threshold w^{se}.

Panel I sets $B = 0$. The "too-big-for-their-boots" mechanism is apparent from the fact that $w_{t+1} < w_t$ whenever $w_t \in (w^{tr}, w^{se})$. As we increase B in Panel II, new wealth dynamics emerge: wealthier households in the community now accumulate wealth until they eventually leave. Even when they are about to leave, these households can engage in some favor exchange, because they prefer repaying small favors to losing B. This feature is what allows households to smoothly transition from the community to the city. As we will show, this feature is a key difference between community benefits and place-based policies.

Panel II shows that, as in Proposition 3, wealthier households in the community invest some of the money that is freed up by favor exchange and so grow their wealth *strictly faster* than they would in the city. In contrast, while low-wealth households also benefit from $B > 0$, they still suffer from the "too-big-for-their-boots" mechanism and so still have substantially lower long-term wealth than in the city.

Panel III shows that large enough community benefits can completely transform favor exchange, so that the household invests more than it would in the city for *any* $w_t \in (0, w^{se})$. In this case, for any initial wealth w_0, the household has the same long-term wealth as it would in the city. Large enough community benefits can therefore turn favor exchange into a force that helps left-behind communities catch up.

[6]These simulations use parameters identical to Figures 1-3, with values of B given in Figure 5.

In practice, community benefits are likely to be large when members rely heavily on the community for social relationships. Indeed, Portes and Sensenbrenner [1993] suggest that members of the Dominican community in New York were able to accumulate wealth because the community provided a valuable safe haven, a place of shared language and culture that was free from discrimination. In describing the economic success of the Chinese immigrant community in New York, Zhou [1992] highlights that members received similarly large benefits from the community. Religious organizations can also be a source of community benefits. Coleman [1988] notes that for the New York community of Jewish diamond merchants, the threat of social ostracism from synagogues and other religious activities supported business relationships.

Place-Based Policies: Policymakers also have tools that can relax the household's commitment problem. However, unlike community benefits, public policies typically cannot condition on a household's behavior in its favor-exchange relationships. The closest that a policy can come is to condition on whether the household locates in the community or the city. The following simulations show that such "place-based" policies can lead to higher long-term wealth in the community, albeit without the transformative effects of community benefits.

Place-based policies include local infrastructure improvements, job subsidies, and other policies that provide localized benefits; see Austin et al. [2018] for a detailed discussion. We model these policies as providing a per-period additive utility benefit, $B > 0$, *so long as the household stays in the community.* Unlike community benefits, place-based policies cannot condition on whether or not the household has deviated. Instead, a household loses the benefit B if and only if it leaves the community.

Figure 6 simulates the equilibrium effects of a place-based policy,[7] where we assume that players play a Markov perfect equilibrium following a deviation.[8] This policy increases w^{se}

[7] This figure uses the same parameters as Figures 1 to 3, with $B = \frac{1}{250}$.

[8] Place-based policies might induce a household to stay in the community following a deviation. Thus, in contrast to our baseline model, Markov perfect equilibria do not necessarily min-max the household. This

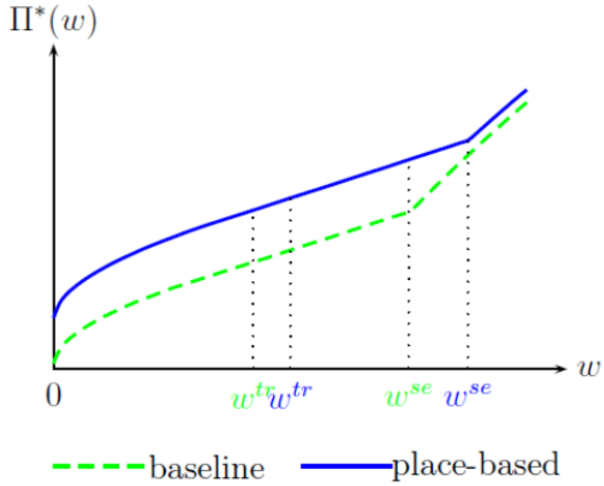

Figure 6: Household-optimal equilibrium payoffs with a place-based policy (blue solid line) versus without the policy (green dashed line).

by inducing the household to stay for a wider range of wealth levels. It also relaxes the commitment problem and so increases long-term wealth in the community, w^{tr}. However, unlike community benefits, place-based policies do not induce households to smoothly transition from the community to the city; instead, households that choose to stay in the community stay forever, with long-term wealth substantially below that in the city. This difference arises because unlike community benefits, place-based policies cannot induce a household that was already planning to leave to repay favors. Nevertheless, because place-based policies relax the commitment problem, they have a special role to play in decreasing the wealth gap between left-behind communities and the rest of the economy.

Place-based policies are most effective if the household leaves the community following a deviation. In particular, the household must prefer to leave rather than staying and continuing to benefit from the policy. Therefore, place-based policies tend to have a large effect when the utility gain from moving to the city, $\hat{U} - U$, is large relative to the policy benefit, B, since then a deviating household is likely to actually move to the city.

simulation therefore gives an *upper* bound on punishment payoffs and so a *lower* bound on household-optimal equilibrium payoffs.

6 Stochastic Investment Returns

In some communities, favor exchange not only augments consumption in normal times but also provides insurance against negative shocks. To explore how this insurance role affects wealth dynamics, this section enriches the model to incorporate stochastic investment returns. We show that a version of our main result continues to hold so long as returns are not too noisy. Using simulations, we also show that extremely noisy returns can transform favor exchange in a way that is similar to community benefits.

We consider a **model with stochastic returns**, which is identical to the model from Section 2 except that investment returns are given by a distribution rather than a deterministic function. Formally, in every $t \geqslant 0$, period-$(t+1)$ wealth, w_{t+1}, is drawn from the distribution $F(\cdot|I_t)$, where $I_t = w_t - p_t$ is the investment in period t. Wealth is drawn independently across periods conditional on the investment and is publicly observed.

We impose the following assumption on the return distributions $\{F(\cdot|I)\}_{I \geqslant 0}$.

Assumption 2 1. *For every investment $I \geqslant 0$, the return distribution $F(\cdot|I)$ has support $[R^L(I), R^H(I)] \subseteq \mathbb{R}_+$. Both $R^L(\cdot)$ and $R^H(\cdot)$ are strictly increasing, concave, and continuously differentiable. There exist thresholds $\bar{w}^L, \bar{w}^H > 0$ such that (i) $(R^L)'(w) > \frac{1}{\delta}$ for $w < \bar{w}^L$ and $(R^L)'(w) = \frac{1}{\delta}$ for $w \geqslant \bar{w}^L$; and (ii) $(R^H)'(w) > \frac{1}{\delta}$ for $w < \bar{w}^H$ and $(R^H)'(w) = \frac{1}{\delta}$ for $w \geqslant \bar{w}^H$.*

2. *Define $c^* > 0$ as the solution to $\hat{U}'(c^*) = 1$. Then, $R^H(\bar{w}^H) - c^* > \bar{w}^H$.*

The first part of this assumption requires the bounds on the support of $\{F(\cdot|I)\}_{I \geqslant 0}$, $R^L(I)$ and $R^H(I)$, to satisfy similar conditions as the return function $R(\cdot)$ in the baseline model, with the sole exception that $R^L(0)$ and $R^H(0)$ need not be zero. The second part is analogous to Assumption 1 and ensures that even if investment returns were given by $R^H(\cdot)$, sufficiently wealthy households would leave the community.

Our main result in this section shows that if returns are not too noisy, then both the selection effect and the "too-big-for-their-boots" mechanism hold.

Proposition 4 *Consider a sequence of return distributions,* $\{\{F^k(\cdot|I)\}_{I\geqslant 0}\}_{k=0}^{\infty}$*, such that* $\{F^k(\cdot|I)\}_{I\geqslant 0}$ *satisfies Assumption 2 for every* $k \geqslant 0$*. Let the corresponding bounds on supports,* $R^{H,k}(\cdot)$ *and* $R^{L,k}(\cdot)$*, be monotone in* k *and converge uniformly to a return function* $R(\cdot)$ *as* $k \to \infty$*. Suppose that* $R(\cdot)$ *satisfies the conditions imposed in Section 2. Then, there exists* $K < \infty$ *such that for any* $k \geq K$*:*

1. ***Selection.*** *There exists a selection threshold* $w^{se,k} > 0$ *and a positive-measure set* $\mathcal{W}^k \subseteq \mathbb{R}_+$ *with* $\sup \mathcal{W}^k = w^{se,k} < \bar{w}^{H,k} + c^*$ *such that in any household-optimal equilibrium, a household in the community chooses to stay in period* t *if* $w_t \subseteq \mathcal{W}^k$ *and otherwise leaves.*

2. ***"Too-big-for-their-boots" mechanism.*** *There exists a wealth level* $w^{tr,k} < w^{se,k}$ *such that in any household-optimal equilibrium, if a household chooses to stay in the community and* $w_t \geq w^{tr,k}$*, then* $R^{H,k}(I_t) < w^{tr,k}$*. The set* $\mathcal{W}^k \cap [w^{tr,k}, w^{se,k}]$ *has a positive measure.*

Moreover, there exist constants $\underline{w}, \gamma > 0$ *such that* $[0, \underline{w}] \subseteq \mathcal{W}^k$ *and* $w^{tr,k} < w^{se,k} - \gamma$ *for any* $k \geq K$*.*

We prove Proposition 4 in online appendix B. For sufficiently large k, the return distributions $\{F^k(\cdot|I)\}_{I\geqslant 0}$ are not too noisy, in the sense that the bounds on their supports, $R^{H,k}(\cdot)$ and $R^{L,k}(\cdot)$, are close to one another. For such k, the selection result says that wealthy households leave the community, while a positive measure of less-wealthy households stay in the current period. This result is the analogue of the selection result in Proposition 2.

The second statement gives a strong version of the "too-big-for-their-boots" mechanism. It says that a relatively wealthy household in the community invests so little that its wealth drops below $w^{tr,k}$ even under the most favorable return function $R^{H,k}(\cdot)$. In other words, such a household's wealth decreases *with probability 1.*

The proof of Proposition 4 follows the structure of the proof of Proposition 2, but stochastic returns make the arguments considerably more complicated. We proceed by bounding equilibrium payoffs from above and below using various deterministic return functions. For example, equilibrium payoffs in the city are bounded from above and below by equilibrium payoffs in the city when returns are given by $R^{H,k}(\cdot)$ and $R^{L,k}(\cdot)$, respectively. As $k \to \infty$, these bounds converge, so we can approximate equilibrium payoffs in the city using properties from the model in Section 2. We can similarly approximate equilibrium payoffs in the community. Given these approximations, we extend the proof of Proposition 2 to stochastic returns, taking care to ensure that the arguments hold uniformly across all wealth levels and for any $k \geq K$.

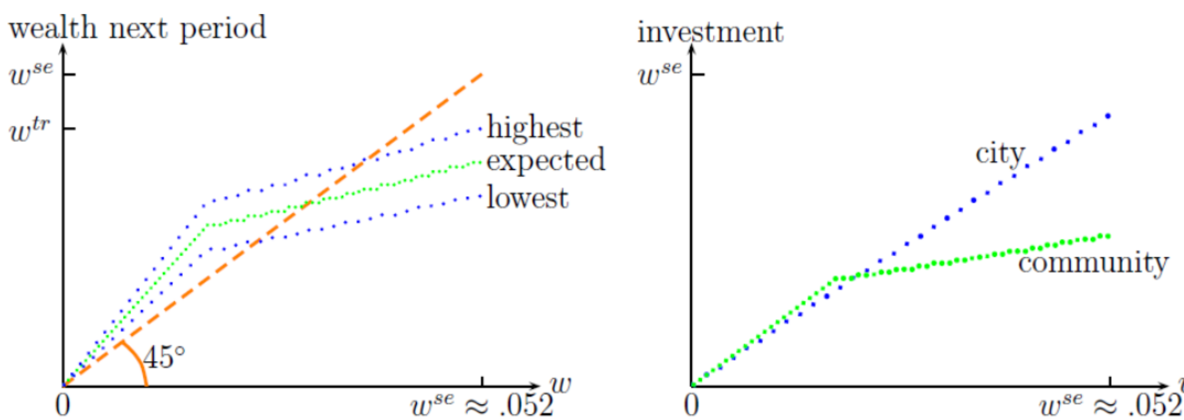

Figure 7: Wealth dynamics and investment with moderately large shocks. $F(\cdot)$ has binary support, with $R^H(I) = 1.15 * R(I)$ and $R^L(I) = 0.85 * R(I)$ each occurring with probability 1/2. Here, $R(w)$ is the return function from Figures 1-3. The left panel gives the range of w_{t+1} realizations as a function of w_t. The right panel gives investment in the city and the community as a function of wealth.

While Proposition 4 applies to "small" shocks, numerical simulations suggest that it remains true even when shocks are moderately large. For instance, Figure 7 shows that it holds for a binary distribution over returns in which the high return is 35 percent higher than the low return. In this simulation, a household that stays in the community invests so little that it remains there forever *regardless* of the shocks. Further (unreported) simulations confirm that Proposition 4 holds widely for moderately large shocks.

In contrast, qualitatively different wealth dynamics can emerge when shocks are very large. In Figure 7, the household faces an extremely large binary shock: the high return is

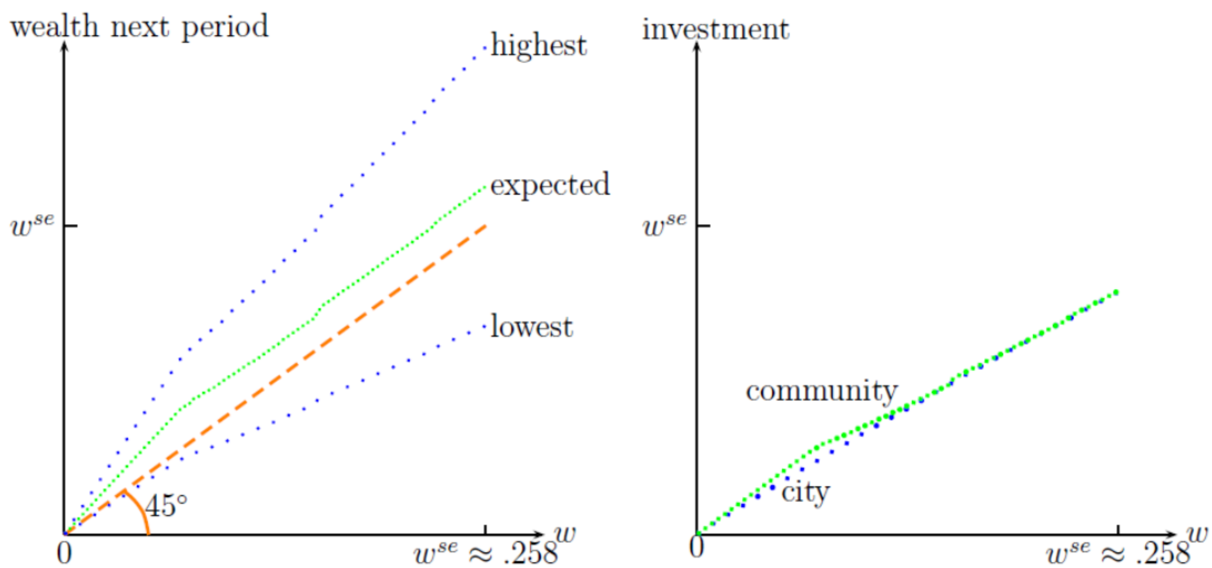

Figure 8: Wealth dynamics and investment in the model with very large shocks. The return distribution is again binary, with $R^H(I) = 1.4 * R(I)$ and $R^L(I) = 0.6 * R(I)$ each occurring with probability 1/2. Here, $R(w)$ is the return function from Figures 1-3.

133 percent higher than the low return. With such large shocks, a household that stays in the community invests *strictly more* than it would in the city. This stark transformation occurs because with very large shocks, even a wealthy household has substantial commitment power, since it is willing to repay favors today in order to keep accessing favor exchange following a negative shock. Similar to Section 5, this commitment power turns favor exchange into a force that accelerates wealth accumulation.

7 The Impossibility of Monetary Favor Exchange

A key feature of our model is that favors are in-kind rather than monetary, which distinguishes it from the literature on informal lending (e.g., Bulow and Rogoff 1989, Ligon et al. 2000). In this section, we show that this distinction is crucial: in sharp contrast to in-kind favors, *no* favor exchange would occur if favors were monetary.

To make this point, we consider a **model with monetary favors.** A long-lived household with initial wealth $w_0 \geq 0$ starts in the community. At the start of each period

$t \in \{0, 1, 2, ...\}$, it chooses to either stay in the community or permanently leave for the city. If it remains in the community in period t, then the household plays the following community game with a *long-lived* neighbor:

1. The household requests a payment $s_t \in [-w_t, \infty)$ from the neighbor, where $s_t < 0$ indicates that it pays the neighbor.

2. The neighbor accepts ($d_t = 1$) or rejects ($d_t = 0$) this request. If it accepts, then it pays s_t. If it rejects, then no transfer is made.

3. The household chooses an amount to consume, $c_t \geq 0$, where consumption cannot exceed its total wealth including the transfer: $c_t \leq w_t + s_t d_t$. Any remaining wealth is invested, so $I_t = w_t + s_t d_t - c_t$, resulting in period-$(t+1)$ wealth $w_{t+1} = R(I_t)$.

The household's stage-game payoff is $U(c_t)$, while the neighbor's stage-game payoff is $-s_t d_t$. The parties share a common discount factor $\delta \in [0, 1)$.

Once the household leaves the community, it thereafter plays the city game, which is identical to the community game except that $s_t = 0$ in every period and the household's payoff is $\hat{U}(c_t)$. We maintain the assumptions on $U(\cdot)$, $\hat{U}(\cdot)$, and $R(\cdot)$ from Section 2.

This model makes two changes to the model from Section 2. First, and most importantly, it replaces in-kind favors with monetary favors, s_t, which capture both borrowing and repayments by the household. Borrowed money can either be consumed or invested to generate returns according to $R(\cdot)$. The second change is that the neighbor is long-lived, which ensures that the household can delay repayments in order to invest borrowed money. Making the neighbor long-lived makes it easier to sustain favor exchange and so strengthens our impossibility result.

We show that favor exchange cannot occur in any equilibrium of this model.

Proposition 5 *In any equilibrium of the model with monetary favors, $s_t = 0$ in all $t \geq 0$.*

See online appendix B for the proof of this result, which is nearly identical to the proof of the impossibility result in Bulow and Rogoff [1989]. The intuition is that monetary favor exchange is inherently self-defeating, since the household becomes wealthier when it borrows money, which necessarily improves its outside option. In particular, if the household ever borrows money, then there must exist a period in which the discounted sum of future repayments strictly exceeds the discounted sum of future loans. In that period, the household prefers to renege on future repayments and leave for the city.

Why are Propositions 2 and 5 so different? The key is that in-kind favor exchange can increase *consumption utility* without increasing *wealth*. As the "too-big-for-their-boots" mechanism shows, the household can access favor exchange only if it keeps its wealth from increasing, which is possible with in-kind favors. Thus, while monetary favor exchange is self-defeating, in-kind favor exchange can be self-sustaining.

We view Proposition 5 as highlighting an important difference between in-kind and monetary favor exchange in settings where households have investment opportunities and the commitment problem is at play. This result is particularly stark because investment returns are deterministic. With stochastic returns, monetary favor exchange can insure the household in the same way that in-kind favor exchange does. If households cannot purchase insurance in the market, then the insurance role of monetary favor exchange can be enough to make it self-sustaining. See, e.g., Ligon et al. [2000].

8 Conclusion

Helping left-behind communities requires understanding the social constraints faced by those experiencing poverty. This paper argues that while favor exchange is an essential source of support in left-behind communities, it imposes hidden costs that can constrain wealth accumulation and deepen long-term inequality. More work remains to be done to understand how heterogeneity within communities—whether from heterogeneous access to favor-exchange networks, heterogeneous ability to repay favors, or another source—influences the constraints imposed by favor exchange and thereby affects economic outcomes.

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A Appendix: Omitted Proofs

A.1 Proof of Lemma 4

We break the proof of this lemma into four steps.

A.1.1 Step 1: Locally Bounding the Slope of $\Pi^*(\cdot)$ from Below

We claim that for any $w \in [0, w^{se})$, there exists $\epsilon_w > 0$ such that for any $\epsilon \in (0, \epsilon_w)$,

$$\Pi^*(w+\epsilon) - \Pi^*(w) > (1-\delta)\epsilon.$$

First, suppose $\hat{\Pi}(w) \geq \Pi_c(w)$, and let $\{w_t, c_t\}_{t=0}^{\infty}$ be the wealth and consumption sequences if the household enters the city. The proof of Lemma 3 implies that for any $w_0 < w^{se}$, $R(w_0 - \bar{c}) < w_0$. Proposition 1 says that $\{w_t\}_{t=0}^{\infty}$ is increasing, so $c_0 < \bar{c}$. Hence, there exists $\epsilon_w > 0$ such that $U'(c_0 + \epsilon_w) > 1$. Since $\hat{U}'(c) \geq U'(c)$ for all $c > 0$, $\hat{U}'(c_0 + \epsilon_w) > 1$.

For any $\epsilon < \epsilon_w$, if $w_0 = w + \epsilon$, then the household can enter the city and choose $\hat{c}_0 = c_0 + \epsilon$, with $\hat{c}_t = c_t$ in all $t > 0$. We can bound $\Pi^*(w+\epsilon)$ from below by the payoff from this strategy,

$$\Pi^*(w+\epsilon) \geq (1-\delta)\left(\hat{U}(c_0+\epsilon) - \hat{U}(c_0)\right) + \hat{\Pi}(w) > (1-\delta)\epsilon + \hat{\Pi}(w) = (1-\delta)\epsilon + \Pi^*(w).$$

We conclude that $\Pi^*(w+\epsilon) - \Pi^*(w) > (1-\delta)\epsilon$, as desired.

Now, suppose $\hat{\Pi}(w) < \Pi_c(w)$. Let $\{w_t, c_t, f_t\}_{t=0}^{\infty}$ be the wealth, consumption, and favor sequence in a household-optimal equilibrium. There exists $\tau \geq 0$ such that $f_\tau > 0$ for the first time in period τ; otherwise, the household could implement the same consumption sequence in the city. Choose $\epsilon_w > 0$ to satisfy $\epsilon_w < \delta^\tau f_\tau$.

For $\epsilon \in (0, \epsilon_w)$ and initial wealth $w_0 = w + \epsilon$, consider the perturbed strategy such that $\hat{p}_t = p_t$, $\hat{c}_t = c_t$, and $\hat{f}_t = f_t$ in every period *except* τ. In period τ, $\hat{f}_\tau = f_\tau - \frac{\epsilon}{\delta^\tau}$ and $\hat{p}_\tau = p_\tau + \chi$, where χ is chosen so that $\hat{w}_{t+1} = w_{t+1}$. Then, $\hat{c}_\tau = c_\tau + \chi - \frac{\epsilon}{\delta^\tau}$. Based on the proof of Proposition 2, $w_t < w^{se}$ for all $t \leq \tau$. This observation together with $w^{se} \leq \bar{w}$

and Assumption 1, implies that we can choose a sufficiently small ϵ_w such that the marginal return from capital in every $t < \tau$ is strictly higher than $\frac{1}{\delta}$ even if the initial wealth is $w + \epsilon$ rather than w. Hence, $\chi > \frac{\epsilon}{\delta^\tau}$.

Under this perturbed strategy, (2) is satisfied in all $t < \tau$ because $f_t = 0$ in these periods; in $t = \tau$ because $\hat{f}_\tau < f_\tau$ and $\hat{w}_{\tau+1} = w_{\tau+1}$; and in $t > \tau$ because play is unchanged after τ. Moreover, $f_\tau - \frac{\epsilon}{\delta^\tau} > 0$ because $\epsilon < \epsilon_w$, and $\hat{f}_\tau + \hat{p}_\tau = \hat{c}_\tau$, so this strategy is feasible. Thus, it is an equilibrium. Consequently, $\Pi^*(w+\epsilon)$ is bounded from below by the household's payoff from this strategy,

$$\Pi^*(w+\epsilon) > (1-\delta)\delta^\tau \frac{\epsilon}{\delta^\tau} + \Pi_c(w) = (1-\delta)\epsilon + \Pi^*(w),$$

as desired.

A.1.2 Step 2: Moving from Local to Global Bound on Slope

Next, we show that for any $0 \leq w < w' < w^{se}$, $\Pi^*(w') - \Pi^*(w) > (1-\delta)(w'-w)$.

Let

$$z(w) = \sup\{w'' | w < w'' \leq w^{se}, \text{ and } \forall w' \in (w, w''], \Pi^*(w') - \Pi^*(w) > (1-\delta)(w'-w)\}.$$

By Step 1, $z(w) \geq w$ exists. Moreover,

$$\Pi^*(z(w)) - \Pi^*(w) \geq \lim_{\tilde{w} \uparrow z(w)} \Pi^*(\tilde{w}) - \Pi^*(w) \geq (1-\delta)(z(w) - w),$$

where the first inequality follows because $\Pi^*(\cdot)$ is increasing, and the second inequality follows by definition of $z(w)$.

Suppose that $z(w) < w^{se}$. By Step 1, there exists $\epsilon_{z(w)}$ such that for any $\epsilon < \epsilon_{z(w)}$,

$$\Pi^*(z(w)+\epsilon) - \Pi^*(z(w)) > (1-\delta)\epsilon.$$

Hence,

$$\Pi^*(z(w)+\epsilon)-\Pi^*(w) = \Pi^*(z(w)+\epsilon)-\Pi^*(z(w))+\Pi^*(z(w))-\Pi^*(w) > (1-\delta)\epsilon+(1-\delta)(z(w)-w).$$

This contradicts the definition of $z(w)$, so $z(w) \geq w^{se}$.

For any $w' < w^{se}$, $w' < z(w)$ and so $\Pi^*(w') - \Pi^*(w) > (1-\delta)(w'-w)$, as desired.

A.1.3 Step 3: Investment is Increasing in Wealth.

Consider two wealth levels, $0 \leq w_L < w_H < w^{se}$, and suppose that $\Pi_c(w_L) > \hat{\Pi}(w_L)$ and $\Pi_c(w_H) > \hat{\Pi}(w_H)$. Given any household-optimal equilibria, let p_H, p_L be the respective period-0 payments under w_H, w_L. We prove that $w_H - p_H \geq w_L - p_L$. Define $I_k = w_k - p_k$, $k \in \{L, H\}$. Towards a contradiction, suppose that $I_H < I_L$.

We first show that $c_H > c_L + (w_H - w_L)$. Suppose instead that $c_H \leq c_L + (w_H - w_L)$. Since $I_H < I_L$, we have $p_H > p_L + (w_H - w_L)$. But then $f_H < f_L$, since

$$f_H = c_H - p_H < c_H - (p_L + (w_H - w_L)) \leq c_L + (w_H - w_L) - (p_L + (w_H - w_L)) = f_L.$$

Consider the following perturbation: $\hat{p}_H = p_L + (w_H - w_L) \in (p_L, p_H)$, $\hat{f}_H = f_H + p_H - \hat{p}_H \geq f_H$, and $\hat{c}_H = c_H$. Under this perturbation, $\hat{I}_H = w_H - \hat{p}_H = I_L$. Thus, to show that the perturbation satisfies (2), we need only show that $\hat{f}_H \leq f_L$. Indeed:

$$\hat{f}_H = f_H + p_H - (p_L + (w_H - w_L)) = c_H - (c_L - f_L) - (w_H - w_L) = f_L + c_H - (c_L + w_H - w_L) \leq f_L,$$

where the final inequality holds because $c_H \leq c_L + (w_H - w_L)$ by assumption. Thus, this perturbation is also an equilibrium.

We claim that a household with initial wealth w_H strictly prefers this equilibrium to the original equilibrium, which is true so long as

$$
\begin{aligned}
& (1-\delta)(U(c_H)-\hat{f}_H)+\delta\Pi^*(R(I_L)) > (1-\delta)(U(c_H)-f_H)+\delta\Pi^*(R(I_H)) \\
\iff \quad & (1-\delta)(\hat{f}_H-f_H) < \delta(\Pi^*(R(I_L))-\Pi^*(R(I_H))) \\
\iff \quad & (1-\delta)(p_H-\hat{p}_H) < \delta(\Pi^*(R(I_L))-\Pi^*(R(I_H))).
\end{aligned}
$$

We know that $I_L = I_H + p_H - \hat{p}_H$. Since the household stays in the community, $I_H < I_L < w^{se}$, so $R'(I_H), R'(I_L) > \frac{1}{\delta}$. Thus,

$$
R(I_L)-R(I_H) > \frac{1}{\delta}(I_L-I_H) = \frac{1}{\delta}(p_H-\hat{p}_H).
$$

By Step 2, $\Pi^*(\cdot)$ increases at rate strictly greater than $(1-\delta)$, so we conclude

$$
\delta(\Pi^*(R(I_L))-\Pi^*(R(I_H))) > \delta(1-\delta)\frac{1}{\delta}(p_H-\hat{p}_H),
$$

as desired. Thus, if $I_H < I_L$, then $c_H > c_L + (w_H - w_L)$.

We are now ready to prove that $I_H < I_L$ contradicts household optimality. To do so, we consider two perturbations: one at w_L and one at w_H. At w_H, consider setting

$$
\begin{aligned}
\hat{c}_H &= c_L + (w_H - w_L) > c_L \geq 0, \\
\hat{p}_H &= p_L + w_H - w_L \in (p_L, w_H], \\
\hat{f}_H &= \hat{c}_H - \hat{p}_H = f_L.
\end{aligned}
$$

By construction, $w_H - \hat{p}_H = I_L$. Thus, $\hat{f}_H$ satisfies (2) because f_L does. Moreover, $\hat{p}_H + \hat{f}_H = \hat{c}_H$, so the neighbor is willing to accept. This perturbed strategy is therefore an equilibrium. For the original equilibrium to be household-optimal, we must therefore have

$$
(1-\delta)(U(c_H)-f_H)+\delta\Pi^*(R(I_H)) \geq (1-\delta)(U(\hat{c}_H)-\hat{f}_H)+\delta\Pi^*(R(\hat{I}_H)). \tag{3}
$$

At w_L, consider setting

$$\hat{c}_L = c_H - (w_H - w_L) > c_L \geq 0,$$
$$\hat{p}_L = p_H - (w_H - w_L) \in (p_L, w_L] \ ,$$
$$\hat{f}_L = \hat{c}_L - \hat{p}_L = f_H.$$

By construction, $w_L - \hat{p}_L = I_H$. Thus, $\hat{f}_L$ satisfies (2) because f_H does. This perturbed strategy is again an equilibrium, so the original equilibrium is household-optimal only if

$$(1-\delta)(U(c_L) - f_L) + \delta\Pi^*(R(I_L)) \geq (1-\delta)(U(\hat{c}_L) - \hat{f}_L) + \delta\Pi^*(R(\hat{I}_L)). \tag{4}$$

Combining (3) and (4) and plugging in definitions, we have

$$U(c_H) - U(c_H - (w_H - w_L)) \geq U(c_L + (w_H - w_L)) - U(c_L).$$

However, $c_H > c_L + w_H - w_L$ and $U(\cdot)$ is strictly concave, so this inequality cannot hold. Thus, if $I_H < I_L$, then at least one of the equilibria at w_H and w_L cannot be household-optimal.

A.1.4 Step 4: Establishing Monotonicity

We have shown that investment, $I(w)$, is increasing in w. Consider a household-optimal equilibrium with $w_1 \geq w_0$. Then, $I(w_1) \geq I(w_0)$, so $w_2 = R(I(w_1)) \geq R(I(w_0)) = w_1$. Thus, $w_2 \geq w_1$, and $w_{t+1} \geq w_t$ for all $t > 1$ by the same argument. Similarly, if $w_1 \leq w_0$, then $I(w_1) \leq I(w_0)$, $w_2 \leq w_1$, and $w_{t+1} \leq w_t$ in all $t \geq 0$. We conclude that $(w_t)_{t=0}^{\infty}$ is monotone in any household-optimal equilibrium. ■

A.2 Proof of Proposition 3

We show that there exists a household-optimal equilibrium such that if $p_t > 0$, then $f_{t'} = 0$ in all $t' > t$ so $\tau \leqslant t + 1$.

Define $\epsilon = \min\left\{p_t, \delta^{t'-t} f_{t'}\right\}$. Consider the following perturbation in periods t and t' (denoted by tildes): in period t, $\tilde{p}_t = p_t - \epsilon$, $\tilde{f}_t = f_t + \epsilon$, and $\tilde{c}_t = c_t$; in period t', $\tilde{p}_{t'} = p_{t'} + R(\tilde{w}_{t'-1}) - R(w_{t'-1})$, $\tilde{f}_{t'} = f_{t'} - \frac{\epsilon}{\delta^{t'-t}}$, and $\tilde{c}_{t'} = c_{t'} + (\tilde{p}_{t'} - p_{t'}) + (\tilde{f}_{t'} - f_{t'})$. This perturbation is feasible, and neighbors t and t' are willing to accept these requests. Thus, it is also an equilibrium.

Note that $c_t = \tilde{c}_t$ and $\delta^t \tilde{f}_t + \delta^{t'} \tilde{f}_{t'} = \delta^t f_t + \delta^{t'} f_{t'}$. Thus, this perturbation improves the household's payoff only if $\tilde{c}_{t'} \geq c_{t'}$, or $\tilde{p}_{t'} + \tilde{f}_{t'} \geq p_{t'} + f_{t'}$. But $\tilde{p}_{t'} - p_{t'} = R(\tilde{w}_{t'-1}) - R(w_{t'-1}) \geq \frac{\epsilon}{\delta^{t'-t}}$ because $R'(\cdot) \geq \frac{1}{\delta}$, while $\tilde{f}_{t'} = f_{t'} - \frac{\epsilon}{\delta^{t'-t}}$. Therefore, $\tilde{c}_{t'} \geq c_{t'}$, as desired. By similarly perturbing any periods $t < t'$ such that $p_t > 0$ and $f_{t'} > 0$, we can construct a household-optimal equilibrium with the property that if $p_t > 0$, then $f_{t'} = 0$ in all $t' > t$. If $\tau = \infty$ in this equilibrium, then $p_t = 0$ for every t which is clearly suboptimal. Hence, $\tau < \infty$ in this equilibrium and $p_t = 0$ for every $t < \tau - 1$. Since $I_t = w_t$ for every $t < \tau - 1$, the household invests strictly more than in the city with the same wealth level.

We next show that for $t = \tau - 1$, the household also invests strictly more. If $p_{\tau-1} = 0$, then this is clearly true. If $p_{\tau-1} > 0$, let $\{c_t\}_{t \geqslant \tau-1}$ be the consumption sequence starting from period $\tau - 1$. Optimality implies that

$$U'(c_{\tau-1}) = \delta R'(w_{\tau-1} - p_{\tau-1}) \hat{U}'(c_\tau).$$

Now consider a household in the city with wealth $w_{\tau-1}$. The household can consume $p_{\tau-1}$ today and continue with the consumption sequence $\{c_t\}_{t \geqslant \tau}$. Since $p_{\tau-1} < p_{\tau-1} + f_{\tau-1} = c_{\tau-1}$ and $\hat{U}' \geqslant U'$,

$$\hat{U}'(p_{\tau-1}) > \delta R'(w_{\tau-1} - p_{\tau-1}) \hat{U}'(c_\tau).$$

Hence, the household in the city with wealth $w_{\tau-1}$ would like to consume strictly more than $p_{\tau-1}$. Hence, the household would invest strictly less than $w_{\tau-1} - p_{\tau-1}$, which is the investment level in the community. ■

B Online Appendix: Additional Proofs

B.1 Proof of Proposition 1

Suppose that the household lives in the city. In any period t, since future vendors don't observe f_t, the household always chooses $f_t = 0$. Hence, vendor t accepts only if $p_t \geq c_t$. This means that $c_t \in [0, w_t]$ are the feasible consumption levels, so that the household's equilibrium continuation payoff is at most $\hat{\Pi}(w_t)$ given wealth w_t.

The following equilibrium gives the household an equilibrium continuation payoff of $\hat{\Pi}(w_t)$. In period t, (i) the household proposes $(c_t, p_t) = (\hat{C}(w_t), \hat{C}(w_t))$; (ii) vendor t accepts if and only if $p_t \geq c_t$. Vendor t has no profitable deviation. This strategy attains $\hat{\Pi}$, so the household has no profitable deviation either.

Let $\{c_t^*\}_{t=0}^{\infty}$ be the consumption sequence in the equilibrium above, given initial wealth w. If $w = 0$, then $c_t^* = 0$ in all $t \geq 0$, so $\hat{\Pi}(0) = \hat{U}(0) = 0$ is the unique equilibrium payoff. If $w > 0$, then it must be true that $c_t^* > 0$ in every $t \geq 0$. Suppose otherwise. Let $\tau \geq 0$ be the first period in which $\min\{c_\tau^*, c_{\tau+1}^*\} = 0$ and $\max\{c_\tau^*, c_{\tau+1}^*\} > 0$. If $c_\tau^* > 0$ and $c_{\tau+1}^* = 0$, consider the perturbation $c_\tau = c_\tau^* - \epsilon_1$, $c_{\tau+1} = c_{\tau+1}^* + \epsilon_2$ for some small $\epsilon_1, \epsilon_2 > 0$ such that the wealth $w_{\tau+2}$ stays the same. If $c_\tau^* = 0$ and $c_{\tau+1}^* > 0$, consider the perturbation $c_\tau = c_\tau^* + \epsilon_1$, $c_{\tau+1} = c_{\tau+1}^* - \epsilon_2$ for some small $\epsilon_1, \epsilon_2 > 0$ such that the wealth $w_{\tau+2}$ stays the same. In either case, the perturbation gives a strictly higher payoff, since $\lim_{c \downarrow 0} \hat{U}'(c) = \infty$.

Next, we show that $\hat{\Pi}(w)$ is the household's *unique* equilibrium payoff. At $w = 0$, $\hat{\Pi}(0) = 0$, so the household's unique equilibrium payoff is indeed $\hat{\Pi}(0)$. For $w > 0$, the household can choose $(c_t, p_t) = ((1-\epsilon)c_t^*, c_t^*)$ in every $t \geq 0$ for $\epsilon > 0$ small. Vendor t strictly prefers to accept. As $\epsilon \downarrow 0$, the consumption sequence $\{(1-\epsilon)c_t^*\}_{t=0}^{\infty}$ gives the household a payoff that converges to $\hat{\Pi}(w)$. So the household must earn at least $\hat{\Pi}(w)$ in any equilibrium.

Turning to properties of $\hat{\Pi}(\cdot)$, we claim that $\hat{\Pi}(\cdot)$ is strictly increasing. Pick $0 \leq w < \tilde{w}$. Let $\{c_t^*\}_{t=0}^{\infty}$ be the sequence associated with w. If the initial wealth is $\tilde{w}$, it is feasible to choose $c_0 = c_0^* + \tilde{w} - w$ and $c_t = c_t^*$ for $t \geq 1$. Since $\hat{U}(\cdot)$ is strictly increasing, so too is $\hat{\Pi}(\cdot)$.

It remains to show that $\hat{\Pi}(\cdot)$ is continuous for all $w > 0$. If $w > 0$, then $\hat{C}(w) > 0$. For $\tilde{w}$ sufficiently close to w, setting $c_0 = \hat{C}(w) + (\tilde{w} - w)$ and $c_t = \hat{C}(w_t)$ for $t \geq 1$ is feasible. The household's payoffs converge to $\hat{\Pi}(w)$ as $\tilde{w} \to w$ under this perturbation, which means that $\lim_{\tilde{w}\uparrow w} \hat{\Pi}(\tilde{w}) \geq \hat{\Pi}(w)$ and $\lim_{\tilde{w}\downarrow w} \hat{\Pi}(\tilde{w}) \geq \hat{\Pi}(w)$. Since $\hat{\Pi}(\cdot)$ is increasing, we conclude that $\hat{\Pi}(\cdot)$ is continuous at every $w > 0$.

We now show that $\hat{\Pi}(\cdot)$ is continuous at $w = 0$. Consider $\lim_{w\downarrow 0} \hat{\Pi}(w)$. Since $R'(\bar{w}) = \frac{1}{\delta}$, the line tangent to $R(\cdot)$ at $\bar{w}$ is $\hat{R}(w) = R(\bar{w}) + \frac{w-\bar{w}}{\delta}$. Since $R(\cdot)$ is concave, $R(w) \leq \hat{R}(w)$ for all $w \geq 0$. Therefore, $\hat{\Pi}(w)$ is bounded from above by the household's maximum payoff if we replace $R(\cdot)$ with $\hat{R}(\cdot)$. For consumption path $\{c_t\}_{t=0}^{\infty}$ to be feasible under $\hat{R}(\cdot)$, it must satisfy

$$(1-\delta)\sum_{t=0}^{\infty} \delta^t c_t \leq (1-\delta)w_0 + \delta R(\bar{w}) - \bar{w}.$$

This means that the payoff of a household with initial wealth w_0 is at most

$$\hat{U}((1-\delta)w_0 + \delta R(\bar{w}) - \bar{w}).$$

Pick any small $\epsilon > 0$. There exists $T < \infty$ and sufficiently small $w_0 > 0$ such that

$$\delta^T \hat{U}\left((1-\delta)R^T(w_0) + \delta R(\bar{w}) - \bar{w}\right) < \frac{\epsilon}{2},$$

where $R^T(w_0)$ denotes the function that applies $R(\cdot)$ T-times to w_0.

Consider a hypothetical setting that is more favorable to the household: we allow the household to both consume *and* save its wealth until period T, after which it must play the original city game. The household's payoff from this hypothetical is strictly larger than $\hat{\Pi}(w_0)$ and is bounded from above by

$$(1-\delta)\sum_{t=0}^{T-1} \delta^t(\hat{U}(R^t(w_0)) + \delta^T \hat{U}((1-\delta)R^T(w_0) + \delta R(\bar{w}) - \bar{w}).$$

As $w_0 \downarrow 0$, $R^T(w_0) \downarrow 0$, so $R^t(w_0) \downarrow 0$ for any $t < T$. Thus,

$$\hat{\Pi}(w_0) \leq (1-\delta)\sum_{t=0}^{T-1} \delta^t \hat{U}(R^t(w_0)) + \delta^T \hat{U}((1-\delta)R^T(w_0) + \delta R(\bar{w}) - \bar{w}) < \epsilon.$$

This is true for any $\epsilon > 0$, so $\lim_{w \downarrow 0} \hat{\Pi}(w) = 0$.

Finally, consider any equilibrium in the city. If $w_0 = 0$, then $w_t = 0$ in any $t \geq 0$. If $w_0 > 0$, then we have shown that $c_t > 0$ in every $t \geq 0$, so $w_t > c_t > 0$. A standard argument (see below) implies the following Euler equation:

$$\hat{U}'(c_t) = \delta R'(w_t - c_t)\hat{U}'(c_{t+1}). \tag{5}$$

Together with $R'(\cdot) \geq \frac{1}{\delta}$ and $\hat{U}(\cdot)$ strictly concave, (5) implies $c_t \leq c_{t+1}$, and strictly so if $w_t - c_t < \bar{w}$.

Next, we argue that $\hat{C}(\cdot)$ is strictly increasing in w. Let $\{c_t\}_{t=0}^{\infty}$ and $\{\tilde{c}_t\}_{t=0}^{\infty}$ be the equilibrium consumption sequences for $w > 0$ and $\tilde{w} > w$, respectively. Suppose $c_0 \geq \tilde{c}_0$, and let $\tau \geq 1$ be the first period such that $c_t < \tilde{c}_t$, which must exist because $\hat{\Pi}(\cdot)$ is strictly increasing. Then, $c_{\tau-1} \geq \tilde{c}_{\tau-1}$, $w_{\tau-1} - c_{\tau-1} < \tilde{w}_{\tau-1} - \tilde{c}_{\tau-1}$, and $c_\tau < \tilde{c}_\tau$, so at least one of $(c_{\tau-1}, w_{\tau-1}, c_\tau)$ and $(\tilde{c}_{\tau-1}, \tilde{w}_{\tau-1}, \tilde{c}_\tau)$ violates (5). Hence, $\hat{C}(w)$ is strictly increasing in w. Therefore, $c_{t+1} \geq c_t$ implies $w_{t+1} \geq w_t$, with strict inequalities if $w_t \leq \bar{w}$.

Since $(w_t)_{t=0}^{\infty}$ is monotone, it converges to some $w^\infty \in \mathbb{R}_+ \cup \{\infty\}$. Suppose $w^\infty < R(\bar{w})$. Since $w_{t+1} = R(w_t - c_t)$, we must have c_t and $w_t - c_t$ converging, with $\lim_{t\to\infty}(w_t - c_t) < \bar{w}$. But then $R'(w_t - c_t)$ converges to a number strictly above $\frac{1}{\delta}$, which implies that (5) is violated as $t \to \infty$. We conclude that $\lim_{t\to\infty} w_t \geq R(\bar{w})$. ■

Deriving the Euler Equation

Consider a household in the city, and let its optimal consumption and wealth sequence be $\{c_t^*, w_t^*\}_{t=0}^{\infty}$. We prove that if $w_0 > 0$, then

$$\hat{U}'(c_t^*) = \delta R'(w_t^* - c_t^*)\hat{U}'(c_{t+1}^*)$$

in every $t \geq 0$.

The proof of Proposition 1 says that $c_t^* > 0$, $c_{t+1}^* > 0$, and $w_t^* - c_t^* > 0$. Suppose that $\hat{U}'(c_t^*) > \delta R'(w_t^* - c_t^*)\hat{U}'(c_{t+1}^*)$. Then, we can perturb (c_t^*, c_{t+1}^*) to $(c_t^* + \epsilon, c_{t+1}^* - \chi(\epsilon))$, where $\chi(\epsilon)$ is chosen such that w_{t+2}^* remains the same as before the perturbation. In particular,

$$R(w_t^* - (c_t^* + \epsilon)) - (c_{t+1}^* - \chi(\epsilon)) = R(w_t^* - c_t^*) - c_{t+1}^*.$$

Hence, $\chi'(\epsilon) = R'(w_t^* - (c_t^* + \epsilon))$.

As $\epsilon \downarrow 0$, this perturbation strictly increases the household's payoff:

$$\begin{aligned} & \lim_{\epsilon \downarrow 0} \left\{ \hat{U}'(c_t^* + \epsilon) - \delta \hat{U}'(c_{t+1}^* - \chi(\epsilon))\chi'(\epsilon) \right\} \\ = & \lim_{\epsilon \downarrow 0} \left\{ \hat{U}'(c_t^* + \epsilon) - \delta \hat{U}'(c_{t+1}^* - \chi(\epsilon))R'(w_t^* - c_t^* - \epsilon) \right\} \\ = & \hat{U}'(c_t^*) + \delta R'(w_t^* - c_t^*)\hat{U}'(c_{t+1}^*) > 0. \end{aligned}$$

This contradicts the fact that (c_t^*, c_{t+1}^*) is optimal. Using a similar argument, we can show that $\hat{U}'(c_t^*) < \delta R'(w_t^* - c_t^*)\hat{U}'(c_{t+1}^*)$ is not possible either. ■

B.2 Proof of Proposition 4

B.2.1 Uniform convergence of $\hat{\Pi}^{H,k}(\cdot)$ and $\hat{\Pi}^{L,k}(\cdot)$ to $\hat{\Pi}(\cdot)$.

Let $\hat{\Pi}(\cdot)$ be the equilibrium payoff in the city if the investment is given by $R(\cdot)$. Similarly, $\hat{\Pi}^{H,k}(\cdot)$ is the payoff under the return function $R^{H,k}(\cdot)$ and $\hat{\Pi}^{L,k}(\cdot)$ is the payoff under the return function $R^{L,k}(\cdot)$.

We want to show that $\forall w \geqslant 0$ and $\forall \epsilon > 0$, there exists K such that $\forall k > K$, $\hat{\Pi}^{H,k}(w) - \hat{\Pi}(w) < \epsilon$ and $\hat{\Pi}(w) - \hat{\Pi}^{H,k}(w) < \epsilon$. This shows pointwise convergence. By the monotonicity of $\hat{\Pi}^{H,k}(\cdot), \hat{\Pi}^{L,k}(\cdot)$ and Dini's theorem, we obtain uniform convergence.

We begin with the zero wealth $w = 0$. Since $R^{L,k}(0) \leqslant R(0) = 0$, $\hat{\Pi}^{L,k}(0) = 0$. We now consider the return function $R^{H,k}(\cdot)$ and construct the following more generous return function:

$$\hat{R}^{H,k}(w) = \frac{w - \bar{w}^{H,k}}{\delta} + R^{H,k}(\bar{w}^{H,k}).$$

Given the initial wealth w and the return function $\hat{R}^{H,k}(\cdot)$, the consumption sequence must satisfy the following condition:

$$(1-\delta)\sum_{t=0}^{\infty} \delta^t c_t = (1-\delta)w + \delta R^{H,k}(\bar{w}^{H,k}) - \bar{w}^{H,k}.$$

Hence, the equilibrium payoff giving the initial wealth w under $\hat{R}^{H,k}(\cdot)$ is:

$$\hat{U}\left((1-\delta)w + \delta R^{H,k}(\bar{w}^{H,k}) - \bar{w}^{H,k}\right).$$

We now allow the household to both save and consume its entire wealth from period 0 to $T-1$. From period T onward, we give the household the return function $\hat{R}^{H,k}(\cdot)$. The household's payoff given the initial wealth $w = 0$ is:

$$(1-\delta)\left(\sum_{t=0}^{T-1} \delta^t \hat{U}((R^{H,k})^t(0)) + \frac{\delta^T \hat{U}\left((1-\delta)(R^{H,k})^T(0) + \delta R^{H,k}(\bar{w}^{H,k}) - \bar{w}^{H,k}\right)}{1-\delta}\right). \tag{6}$$

Fix any T, as $k \to \infty$, $(R^{H,k})^T(0) \downarrow 0$, so $(R^{H,k})^t(0) \downarrow 0$ for any $t \leqslant T$. Moreover, there exists a T such that:

$$\delta^T \hat{U}\left(\delta R^{H,k}(\bar{w}^{H,k}) - \bar{w}^{H,k}\right) < \frac{\epsilon}{2}.$$

Therefore, for this value of T, there exists $K > 0$ such that for any $k > K$ the payoff (6) is at most ϵ, so the payoff $\hat{\Pi}^{H,k}(0)$ is at most ϵ. This concludes the proof of pointwise convergence at $w = 0$.

We now prove the pointwise convergence at $w > 0$. Let $\{c_t\}_{t=0}^{\infty}$ be the consumption sequence in the city under return function $R(\cdot)$. Recall that $c_0 > 0$ and c_t increases in t. For any ϵ, there exists a small enough $\gamma \in (0, c_0)$ such that:

$$(1-\delta)\sum_{t=0}^{\infty}\hat{U}(c_t - \gamma) > \hat{\Pi}(w) - \epsilon.$$

On the other hand, there exists $K > 0$ such that for any $k > K$, $R^{L,k}(w) > R(w) - \gamma$ for every $w \geqslant 0$. Under such $R^{L,k}(\cdot)$, the consumption sequence $\{c_t - \gamma\}_{t=0}^{\infty}$ is feasible, since the household can choose the same investment sequence as under $R(\cdot)$ and consume the rest in each period. Hence, $\hat{\Pi}^{L,k}(w) > \hat{\Pi}(w) - \epsilon$.

Let $\{c_t^{H,k}\}_{t=0}^{\infty}$ be the consumption sequence under the return function $R^{H,k}(\cdot)$. By the next claim, claim 1, there exists a $\underline{c} > 0$ such that $c_0^{H,k} > \underline{c}$ for any k. By the Euler equation, $c_t^{H,k}$ increases in t. Fix any sufficiently small $\epsilon > 0$ and consider the number $\gamma_k > 0$ which solves the following equation:

$$\hat{U}(c_0^{H,k}) - \hat{U}(c_0^{H,k} - \gamma_k) = \epsilon.$$

Since $c_t^{H,k}$ increases in t, we have:

$$\hat{U}(c_t^{H,k}) - \hat{U}(c_t^{H,k} - \gamma_k) \leqslant \epsilon, \ \forall t \geqslant 0.$$

This implies that

$$\hat{\Pi}^{H,k}(w) - (1-\delta)\sum_{t=0}^{\infty}\hat{U}(c_0^{H,k} - \gamma_k) \leqslant \epsilon.$$

Moreover, since $c_0^k \geqslant \underline{c}$ for any sufficiently large k, there exists $\underline{\gamma} > 0$ such that $\gamma_k \geqslant \underline{\gamma}$ for any such k.

By the convergence of $R^{H,k}(\cdot)$ to $R(\cdot)$, there exists $K > 0$ such that for any $k > K$, we have

$$R(w) > R^{H,k}(w) - \underline{\gamma},\ \forall w \geqslant 0.$$

This implies that $\{c_t^{H,k} - \gamma_k\}_{t=0}^{\infty}$ is feasible under the return function $R(\cdot)$, so $\hat{\Pi}(w) > \hat{\Pi}^{H,k} - \epsilon$ for any $k > K$.

Claim 1: Fix the initial wealth $w_0 > 0$. There exists $\underline{c} > 0$ such that $c_0^{H,k} > \underline{c}$ for any $k \geqslant 0$.

Proof of claim 1: We proceed by bounding period-t consumption from above as a function of $c_0^{H,k}$. By (Euler), equilibrium consumption satisfies

$$\hat{U}'\left(c_t^{H,k}\right) = \delta(R^{H,k})'\left(w_t^{H,k} - c_t^{H,k}\right)\hat{U}'\left(c_{t+1}^{H,k}\right).$$

Since $w_t^{H,k} - c_t^{H,k}$ is increasing in t and $(R^{H,k})'(\cdot)$ is decreasing,

$$\hat{U}'\left(c_t^{H,k}\right) \leqslant \delta(R^{H,k})'\left(w_0 - c_0^{H,k}\right)\hat{U}'\left(c_{t+1}^{H,k}\right).$$

Moreover,

$$(R^{H,k})'(I) \leqslant \frac{R^{H,k}(I) - R^{H,k}(0)}{I} \leqslant \frac{R^{H,0}(I)}{I},$$

where the first inequality follows because $R^{H,k}$ is concave, and the second inequality follows because $R^{H,k}(0) \geqslant 0$ and $R^{H,k}(I)$ is decreasing in k. Thus, in equilibrium,

$$\hat{U}'\left(c_t^{H,k}\right) \leqslant \delta\frac{R^{H,0}(w_0 - c_0^{H,k})}{w_0 - c_0^{H,k}}\hat{U}'\left(c_{t+1}^{H,k}\right). \tag{7}$$

Let $\tilde{c}(c_t^{H,k}; w_0, c_0^{H,k})$ be the largest $c_{t+1}^{H,k}$ that satisfies (7), given $(c_t^{H,k}, w_0, c_0^{H,k})$. Since $\hat{U}'(\cdot)$ is decreasing, $\tilde{c}$ satisfies this inequality with equality. Notice that for a fixed $(w_0, c_0^{H,k})$, $\tilde{c}(\cdot; w_0, c_0^{H,k})$ is continuous, with $\lim_{c\downarrow 0}\tilde{c}(c; w_0, c_0^{H,k}) = 0$. Thus, for any $t \geqslant 1$, we have that $\lim_{c_0^{H,k}\downarrow 0}\tilde{c}^t(w_0, c_0^{H,k}) = 0$, where $\tilde{c}^t$ is defined inductively by $\tilde{c}^1 = \tilde{c}(c_0^{H,k}; w_0, c_0^{H,k})$ and

$\tilde{c}^t(w_0, c_0^{H,k}) = \tilde{c}\left(\tilde{c}^{t-1}(w_0, c_0^{H,k}); w_0, c_0^{H,k}\right)$ for $t > 1$.

For any $t \geqslant 1$, we claim that $\tilde{c}^t(w_0, c_0^{H,k}) \geqslant c_t^{H,k}$. We prove this inductively: for $t = 1$, $\tilde{c}^1(w_0, c_0^{H,k}) \geqslant c_1^{H,k}$ because $c_1^{H,k}$ satisfies (7) and $\tilde{c}^1$ is the largest consumption that satisfies this inequality. If $\tilde{c}^{t-1}(w_0, c_0^{H,k}) \geqslant c_{t-1}^{H,k}$, then (suppressing arguments):

$$\hat{U}'(c_t^{H,k}) \geqslant \frac{w_0 - c_0^{H,k}}{\delta R^{H,0}(w_0 - c_0^{H,k})}\hat{U}'(c_{t-1}^{H,k}) \geqslant \frac{w_0 - c_0^{H,k}}{\delta R^{H,0}(w_0 - c_0^{H,k})}\hat{U}'(\tilde{c}^{t-1}) = \hat{U}'(\tilde{c}^t),$$

where the first inequality is a rewriting of (7), the second inequality follows from $\tilde{c}^{t-1} \geqslant c_{t-1}^{H,k}$ and $\hat{U}'(\cdot)$ decreasing, and the final equality holds because $\tilde{c}^t = \tilde{c}\left(\tilde{c}^{t-1}; w_0, c_0^{H,k}\right)$. We conclude that $\tilde{c}^t \geqslant c_t^{H,k}$, proving the claim.

Define $\tilde{c}^0(w_0, c_0^{H,k}) = c_0^{H,k}$. We provide an upper bound on $\hat{\Pi}^{H,k}(w)$ using the following bounds on consumption: (i) for $t \in \{0, ..., T-1\}$, the household consumes $\tilde{c}^t$, which is greater than $c_t^{H,k}$; (ii) during these periods, the household saves its entire wealth using the return function $R^{H,0}(\cdot)$ which is greater than $R^{H,k}(\cdot)$, so that its wealth at the beginning of period T is $(R^{H,0})^T(w_0)$; and (iii) from period T onward, the return function is given by the more generous function $\hat{R}^{H,0}(w) = \frac{w - \bar{w}^{H,0}}{\delta} + R^{H,0}\left(\bar{w}^{H,0}\right)$. Thus, for any $T \geq 0$,

$$\hat{\Pi}^{H,k}(w_0) \leqslant (1-\delta)\sum_{t=0}^{T-1}\delta^t\hat{U}(\tilde{c}^t(w_0, c_0^{H,k})) + \delta^T\hat{U}\left((1-\delta)(R^{H,0})^T(w_0) + \delta R^{H,0}(\bar{w}^{H,0}) - \bar{w}^{H,0}\right). \tag{8}$$

Since $R^{H,0}(w) \leqslant \hat{R}^{H,0}(w)$,

$$(R^{H,0})^T(w) \leqslant (\hat{R}^{H,0})^T(w) = \frac{w}{\delta^T} + \frac{\delta\left(1-\delta^T\right)}{(1-\delta)\delta^T}\left(R^{H,0}\left(\bar{w}^{H,0}\right) - \frac{\bar{w}^{H,0}}{\delta}\right).$$

Consequently,

$$\begin{aligned} & \lim_{T\to\infty}\delta^T\hat{U}\left((1-\delta)(R^{H,0})^T(w_0) + \delta R^{H,0}(\bar{w}^{H,0}) - \bar{w}^{H,0}\right) \\ \leqslant\ & \lim_{T\to\infty}\delta^T\hat{U}\left((1-\delta)\tfrac{w}{\delta^T} + \tfrac{\delta\left(1-\delta^T\right)}{\delta^T}\left(R^{H,0}\left(\bar{w}^{H,0}\right) - \tfrac{\bar{w}^{H,0}}{\delta}\right) + \delta R^{H,0}(\bar{w}^{H,0}) - \bar{w}^{H,0}\right) \\ =\ & 0, \end{aligned}$$

where the equality follows from L'Hopital's rule and the assumption that $\hat{U}'(c) \to 0$ as $c \to \infty$. Thus, the final term in (8) goes to zero as $T \to \infty$.

We are now prepared to demonstrate that the lower bound $\underline{c} > 0$ exists. Since $R^{H,k}(\cdot) \geqslant R(\cdot)$, $\hat{\Pi}^{H,k}(w_0) \geqslant \hat{\Pi}(w_0)$. Choose $\epsilon \in (0, \hat{\Pi}(w_0))$. There exists T independent of k such that the final term in (8) is at most $\epsilon/2$. For this T, there exists $\xi > 0$ independent of k such that the first term in (8) is at most $\epsilon/2$ if $c_0^{H,k} < \xi$. This shows that for any k, $c_0^{H,k} > \xi$. Otherwise, there exists k such that $\hat{\Pi}^{H,k}(w_0) < \epsilon < \hat{\Pi}(w_0)$, leading to a contradiction. ■

B.2.2 Selection

For any k, consider the game with stochastic return F^k and let $\mathcal{W}^k$ be the collection of wealth levels such that the household stays in the community if its wealth is in $\mathcal{W}^k$ and leaves the community otherwise. Let $w^{se,k} = \sup \mathcal{W}^k$ denote the selection margin. We want to show that when k is large enough:

1. $\mathcal{W}^k$ includes $[0, \underline{w}]$ for some wealth level $\underline{w} > 0$ which is independent of k;

2. $w^{se,k} \leqslant \bar{w}^{H,k} + c^*$ which is smaller than $R^{H,k}(\bar{w}^{H,k})$.

We first show that $\mathcal{W}^k$ includes $[0, \underline{w}]$ for some fixed $\underline{w} > 0$. Recall that
$\Pi^*(0) = \max_c \left\{ U(c) - c | c \leqslant \frac{\delta}{1-\delta} \left(U(c) - c \right) \right\} > 0$ and that $\hat{\Pi}(w)$ is a continuous function with $\hat{\Pi}(0) = 0$. Fix a small $\gamma > 0$. Let $\underline{w} = \max\{w | \hat{\Pi}(w') \leqslant \Pi^*(0) - 2\gamma, \text{ for every } w' \in [0, w]\}$. We next show that there exists K such that for every $k \geqslant K$, $[0, \underline{w}] \subseteq \mathcal{W}^k$.

Denote by $\Pi^{*,k}(w)$ the household-optimal equilibrium payoff under the stochastic return $F^k(\cdot)$. Then

$$\Pi^{*,k}(w) \geqslant \max_c \left\{ U(c) - c | c \leqslant \frac{\delta}{1-\delta} \left(U(c) - c - \hat{\Pi}^{H,k}(R^{H,k}(0)) \right) \right\},$$

since the household can always consume all wealth in each period and rely entirely on favor exchange. Since $\hat{\Pi}^{H,k}$ is continuous, with $\hat{\Pi}^{H,k}$ and $R^{H,k}$ converging uniformly to $\hat{\Pi}$ and R, respectively, we have $\lim_{k \to \infty} \hat{\Pi}^{H,k}(R^{H,k}(0)) = \hat{\Pi}(R(0)) = 0$. Therefore, there exists K_1 such that if $k \geqslant K_1$, $\Pi^{*,k}(w) > \Pi^*(0) - \gamma$. Meanwhile, there exists K_2 such that if $k \geqslant K_2$, $\hat{\Pi}^{H,k}(w) - \hat{\Pi}(w) < \gamma$ for every $w \geqslant 0$. Therefore, if $k \geqslant \max\{K_1, K_2\}$, for any $w \in [0, \underline{w}]$:

$$\Pi^{*,k}(w) > \Pi^*(0) - \gamma \geqslant \hat{\Pi}(w) + \gamma > \hat{\Pi}^{H,k}(w),$$

where the second inequality follows from the definition of $\underline{w}$. This shows that $[0, \underline{w}] \subseteq \mathcal{W}^k$.

We now show that when k is large enough, $w^{se,k} \leqslant \bar{w}^{H,k} + c^*$. For any k, we define an auxiliary game which differs from the original game with the stochastic return F^k in two ways:

1. the investment return is given by the deterministic function $R^{H,k}(\cdot)$ instead of the stochastic return F^k in the original game;
2. in any period t, if the household stays in the community, he can commit to f_t. In other words, the household faces no dynamic enforcement constraint.

Let $\tilde{\Pi}^{H,k}(\cdot)$ be the household's optimal equilibrium payoff in this auxiliary game if it initially lives in the community. Then, $\tilde{\Pi}^{H,k}(w) = \max\{\hat{\Pi}^{H,k}(w), \Pi_c^{H,k}(w)\}$, where $\Pi_c^{H,k}(w)$ is given by:

$$\Pi_c^{H,k}(w) = \max_{I \in [0,w], f \geqslant 0} \left\{ (1-\delta)\left(U(w+f-I) - I\right) + \delta \tilde{\Pi}^{H,k}(R^{H,k}(I)) \right\}. \qquad (9)$$

It follows that $\Pi_c^{H,k}(w)$ is an upper bound on the household's payoff if it optimally stays in the community in the original game. On the other hand, $\hat{\Pi}^{L,k}(w)$ is a lower bound on the household's payoff if it leaves for the city in the original game. If $\Pi_c^{H,k}(w) < \hat{\Pi}^{L,k}(w)$, then the household strictly prefers to leave for the city at the wealth level w in the original game. We now show that this is the case for any $w \geqslant \bar{w}^{H,k} + c^*$.

Claim 2: In the auxiliary game, given a wealth level $w_t \geqslant 0$, if the solution to (9) is such that $w_t + f_t - c_t > \bar{c}$, then $f_t = 0$ and $\tilde{\Pi}^{H,k}(R^{H,k}(I_t)) = \hat{\Pi}^{H,k}(R^{H,k}(I_t))$.

Proof of Claim 2: We will show that $w_t+f_t-I_t > \bar{c}$ implies that $f_{t'} = 0$ for all $t' \geqslant t$. This implies that the optimal continuation payoff at the beginning of $(t+1)$ is $\hat{\Pi}^{H,k}(R^{H,k}(I_t))$.

The first step is to show that $w_t+f_t-I_t > \bar{c}$ implies that $f_t = 0$. Suppose otherwise. Then we could decrease f_t by a small amount which will increase the household's period-t payoff at rate $-U'(w_t+f_t-I_t)+1 > 0$, where the inequality follows because $U'(w_t+f_t-I_t) < U'(\bar{c}) = 1$.

Now suppose that $f_t = 0$ and that there exists $t' > t$ such that $f_{t'} > 0$. Consider the following perturbation: (i) in period t, increase I_t by $\epsilon > 0$ and decrease c_t by the same amount; (ii) in period t' decrease $f_{t'}$ and $I_{t'}$ by $\frac{\epsilon}{\delta^{t'-t}}$. We claim that this perturbation is feasible for small ϵ. Indeed, $w_t-I_t > \bar{c}$ so we can increase I_t by ϵ. Moreover, this perturbation increases $w_{t'}$ by at least $\frac{\epsilon}{\delta^{t'-t}}$ because $\left(R^{H,k}\right)'(w) \geqslant \frac{1}{\delta}$, so decreasing $I_{t'}$ by $\frac{\epsilon}{\delta^{t'-t}}$ is feasible. As $\epsilon \to 0$, this perturbation increases the household's payoff at a rate which is at least:

$$-U'(w_t - I_t) + \delta^{t'-t}\frac{1}{\delta^{t'-t}} > 0,$$

where the inequality follows because $U'(w_t - I_t) < 1$.

We conclude that if the solution to (9) is such that $w_t + f_t - c_t > \bar{c}$, then $f_{t'}$ for all $t' \geqslant t$, which implies the claim. ■

Claim 3: Suppose that $w_0 \geqslant \bar{w}^{H,k} + c^*$. Then:

$$\Pi_c^{H,k}(w_0) < \hat{\Pi}^{H,k}(w_0) - (1-\delta)\left(\hat{U}(\bar{c}) - U(\bar{c})\right). \tag{10}$$

Proof of Claim 3: It suffices to show that the solution to (9) entails $w_0+f_0-I_0 > \bar{c}$. If so, the previous claim says that $f_0 = 0$ and the household's continuation payoff $\tilde{\Pi}^{H,k}(R^{H,k}(I_0))$ is equal to its payoff from the city $\hat{\Pi}^{H,k}(R^{H,k}(I_0))$. If the household instead leaves at the beginning of period 0, it could replicate the entire consumption sequence in the city. Its payoff from doing so is

$$(1-\delta)\hat{U}(c_0) + \delta\hat{\Pi}^{H,k}(R^{H,k}(I_0)),$$

which exceeds $\Pi_c^{H,k}(w_0)$ by $(1-\delta)(\hat{U}(c_0) - U(c_0)) > (1-\delta)(\hat{U}(\bar{c}) - U(\bar{c}))$, and is weakly smaller than $\hat{\Pi}^{H,k}(w_0)$. The inequality (10) then follows. For the rest of the proof, we will show that the solution to (9) entails $w_0 + f_0 - I_0 > \bar{c}$.

Suppose, towards contradiction, that the solution to (9) given the wealth level $w_0 \geqslant \bar{w}^{H,k} + c^*$ entails $w_0 + f_0 - I_0 \leqslant \bar{c}$. Then, $I_0 \geqslant w_0 - \bar{c} \geqslant \bar{w}^{H,k} + c^* - \bar{c} > \bar{w}^{H,k}$. So, $w_1 = R^{H,k}(I_0) > R^{H,k}(\bar{w}^{H,k}) > \bar{w}^{H,k} + c^*$. We now consider the auxiliary game starting from period 1. Either the household leaves the community in $t = 1$ or it stays. In either case, it must be unable to improve its payoff $\Pi_c^{H,k}(w_0)$ by decreasing I_0 slightly and increasing c_1 by an amount equal to the resulting decrease in w_1, which implies that:

$$U'(c_0) \leqslant \delta(R^{H,k})'(I_0)\hat{U}'(c_1) \tag{11}$$

if the household leaves the community in $t = 1$, or

$$U'(c_0) \leqslant \delta(R^{H,k})'(I_0)U'(c_1) \tag{12}$$

if the household stays in $t = 1$.

Suppose that the household leaves the community $t = 1$. We first argue that $c_1 > c^*$. If $c_1 \leqslant c^*$, then $I_1 = w_1 - c_1 > \bar{w}^{H,k}$, so $(R^{H,k})'(I_1) = \frac{1}{\delta}$. This implies that $c_2 = c_1$ by the Euler equation and that $I_2 = w_2 - c_2 > R^{H,k}(\bar{w}^{H,k}) - c^* > \bar{w}^{H,k}$. Iterating this argument, we can show that if $c_1 \leqslant c^*$ and $I_1 > \bar{w}^{H,k}$, then c_t is constant for all $t \geqslant 1$, which contradicts the fact that $w_1 > R^{H,k}(\bar{w}^{H,k}) > \bar{w}^{H,k} + c^*$. Therefore, $c_1 > c^*$. But then, (11) implies that $U'(c_0) \leqslant \delta(R^{H,k})'(I_0)\hat{U}'(c_1) < 1$, where the last inequality follows from $(R^{H,k})'(I_0) = 1$ and $\hat{U}'(c_1) < \hat{U}'(c^*) = 1$. This contradicts the presumption that $U'(c_0) \geqslant 1$. Therefore, the household cannot leave the community in $t = 1$ if $c_0 \leqslant \bar{c}$ and $w_1 > R^{H,k}(\bar{w}^{H,k})$.

Suppose instead that the household stays in the community in $t = 1$. Then, (12) implies that $U'(c_0) \leqslant U'(c_1)$ since $(R^{H,k})'(I_0) = \frac{1}{\delta}$. Therefore, $c_1 \leqslant c_0 \leqslant \bar{c}$ and $w_2 > R^{H,k}(\bar{w}^{H,k})$. Consequently, the argument in the previous paragraph implies that the household must stay in the community in $t = 2$ as well, with $c_2 \leqslant \bar{c}$ and $w_3 > R^{H,k}(\bar{w}^{H,k})$. We can iterate this argument to conclude that the household must stay in the community in every $t \geqslant 1$, with $c_t \leqslant \bar{c}$ in every $t \geqslant 1$. But this cannot be optimal, since the household can instead leave in $t = 1$ and receive a payoff strictly greater than $\hat{U}(c^*) > U(\bar{c})$. So the household cannot stay in the community in $t = 1$.

We have shown a contradiction to either the household leaving the community in $t = 1$ or staying. We conclude that $w_0 + f_0 - I_0 > \bar{c}$ in any household-optimal equilibrium, which proves Claim 3. ■

Completing the proof of Selection: Finally, we turn to the original game. Choose K such that for any $k \geqslant K$, $\hat{\Pi}^{H,k}(w) - \hat{\Pi}^{L,k}(w) \leqslant (1-\delta)(\hat{U}(\bar{c}) - U(\bar{c}))$ for any $w \geqslant 0$. For any such k, fix $w \geqslant \bar{w}^{H,k} + c^*$, and suppose that the household optimally remains in the community. As we noted previously, its payoff is then bounded from above by $\Pi_c^{H,k}(w)$, which by Claim 3, satisfies:

$$\Pi_c^{H,k}(w) < \hat{\Pi}^{H,k}(w) - (1-\delta)(\hat{U}(\bar{c}) - U(\bar{c}) < \hat{\Pi}^{L,k}(w).$$

But $\hat{\Pi}^{L,k}(w)$ bounds the household's payoff in the city from below. Therefore, it cannot be optimal for the household to remain in the community. We conclude that for any $k \geqslant K$ and $w \geqslant \bar{w}^{H,k} + c^*$, the household optimally leaves the community at wealth level w. ■

B.2.3 The "Too Big for their Boots" Mechanism

We want to prove the following claim. There exists $\gamma > 0$ and $K > 0$, such that for any $k > K$ in the game with stochastic return F^k, if $w_t \in [w^{se,k} - \gamma, w^{se,k}]$ and the household chooses to stay in the community in period t, then $R^{H,k}(I_t) < w^{se,k} - \gamma$.

We first explain how γ is chosen. We have shown in the selection proof that there exists

K_0 such that for any $k > K_0$, $w^{se,k} \geqslant \underline{w}$. Define the function $\Delta(\epsilon)$ as follows:

$$\Delta(\epsilon) = \max_{w \in [\underline{w}, \infty)} \left(\hat{\Pi}(w + \epsilon) - \hat{\Pi}(w - 2\epsilon) \right),$$

with $\lim_{\epsilon \to 0} \Delta(\epsilon) = 0$. Consider the following expression as a function of $w \in [w^{se,k} - \epsilon, w^{se,k}]$:

$$\hat{U}\left(w - R^{-1}(w^{se,k} + \epsilon)\right) - U\left(w - R^{-1}(w^{se,k} - 2\epsilon) + \frac{\delta}{1-\delta}\left(\Delta(\epsilon) + 2\epsilon\right)\right) \tag{13}$$

As $\epsilon \to 0$, the value of (13) converges to

$$\hat{U}\left(w^{se,k} - R^{-1}(w^{se,k})\right) - U\left(w^{se,k} - R^{-1}(w^{se,k})\right). \tag{14}$$

Since $(w - R^{-1}(w))$ is an increasing function of w and $w^{se,k} \geqslant \underline{w}$ for any $k > K_0$, the value of (14) is at least:

$$\hat{U}\left(\underline{w} - R^{-1}(\underline{w})\right) - U\left(\underline{w} - R^{-1}(\underline{w})\right) > 0.$$

Hence, there exists $\gamma \in (0, \underline{w})$ such that for any $k > K_0$, (13) is greater than $\frac{\delta}{1-\delta}\left(\Delta(\gamma) + 2\gamma\right)$ for every $w \in [w^{se,k} - \gamma, w^{se,k}]$.

Given this $\gamma > 0$, there exists $K_1 > 0$ such that for every $k > K_1$, $R^{H,k}(w) - R^{L,k}(w)$, $\hat{\Pi}^{H,k}(w) - \hat{\Pi}(w)$, and $\hat{\Pi}(w) - \hat{\Pi}^{L,k}(w)$ are all smaller than γ for every $w \geqslant 0$. For the rest of the proof, we will consider only $k > \max\{K_0, K_1\}$.

Suppose that $w_t \in [w^{se,k} - \gamma, w^{se,k}]$ and the household chooses to stay in the community in period t. Towards contradiction, suppose that $R^{H,k}(I_t) > w^{se,k} - \gamma$. This implies that $R^{L,k}(I_t) > w^{se,k} - 2\gamma$, so the wealth at the beginning of period $(t+1)$ is at least $\left(w^{se,k} - 2\gamma\right)$. On the other hand, the household will never choose I_t such that $R^{L,k}(I_t) > w^{se,k}$, since in that case $f_t = 0$ and the household could profitably deviate by leaving for the city at the beginning of period t. Hence, $R^{L,k}(I_t) \leqslant w^{se,k}$ and $R^{H,k}(I_t) \leqslant w^{se,k} + \gamma$, so the wealth at the beginning of period $(t+1)$ is at most $\left(w^{se,k} + \gamma\right)$. Then a necessary condition for the dynamic enforcement constraint is

$$f_t \leqslant \frac{\delta}{1-\delta}\left(\hat{\Pi}^{H,k}(w^{se,k}+\gamma) - \hat{\Pi}^{L,k}(w^{se,k}-2\gamma)\right). \tag{15}$$

A necessary condition for (15) is that:

$$f_t \leqslant \frac{\delta}{1-\delta}\left(\hat{\Pi}(w^{se,k}+\gamma) - \hat{\Pi}(w^{se,k}-2\gamma) + 2\gamma\right) \leqslant \frac{\delta}{1-\delta}\left(\Delta(\gamma)+2\gamma\right). \tag{16}$$

Here, the first inequality follows from the fact that $\hat{\Pi}^{H,k}(w^{se,k}+\gamma) < \hat{\Pi}(w^{se,k}+\gamma)+\gamma$ and that $\hat{\Pi}^{L,k}(w^{se,k}-2\gamma) > \hat{\Pi}(w^{se,k}-2\gamma)-\gamma$, and the second inequality follows from the definition of $\Delta(\gamma)$. Now we can bound the household's payoff at the beginning of period t:

$$\begin{aligned}
\Pi^{*,k}(w_t) &\leqslant (1-\delta)U(w_t - I_t + f_t) + \delta\Pi^{*,k}(w^{se,k}+\gamma) \\
&\leqslant (1-\delta)U\left(w_t - (R^{H,k})^{-1}(w^{se,k}-\gamma) + \tfrac{\delta}{1-\delta}\left(\Delta(\gamma)+2\gamma\right)\right) + \delta\hat{\Pi}^{H,k}(w^{se,k}+\gamma).
\end{aligned} \tag{17}$$

Here, the first inequality plugs in the definition of c_t and uses the fact that on $w_{t+1} \leqslant w^{se,k}+\gamma$; and the second inequality plugs in the lower bound on I_t implied by $R^{H,k}(I_t) \geqslant w^{se,k}-\gamma$, the upper bound on f_t given by (16), and $\Pi^{*,k}(w^{se,k}+\gamma) \leqslant \hat{\Pi}^{H,k}(w^{se,k}+\gamma)$.

Now, consider the following deviation: the household leaves the community in period t, invests I_t, and then continues to play according to the optimal equilibrium in the city. This deviation results in a payoff of at least:

$$\begin{aligned}
(1-\delta)\hat{U}(w_t - I_t) + \delta\hat{\Pi}^k(R^{L,k}(I_t)) &\geqslant (1-\delta)\hat{U}(w_t - I_t) + \delta\hat{\Pi}^{L,k}(w^{se,k}-2\gamma) \\
&\geqslant (1-\delta)\hat{U}(w - (R^{L,k})^{-1}(w^{se,k})) + \delta\hat{\Pi}^{L,k}(w^{se,k}-2\gamma),
\end{aligned} \tag{18}$$

where $\hat{\Pi}^k(\cdot)$ is a household's payoff in the city under the stochastic return F^k. The first inequality uses $\hat{\Pi}^k \geqslant \hat{\Pi}^{L,k}$ and $R^{L,k}(I_t) \geqslant R^{H,k}(I_t) - \gamma \geqslant w^{se,k} - 2\gamma$. The second inequality uses the fact that $I_t \leqslant (R^{L,k})^{-1}(w^{se,k})$.

Comparing the bounds in (17) and (18), this deviation is profitable so long as

$$(1-\delta)\hat{U}(w_t - (R^{L,k})^{-1}(w^{se,k})) + \delta\hat{\Pi}^{L,k}(w^{se,k} - 2\gamma) >$$
$$(1-\delta)U\left(w_t - (R^{H,k})^{-1}(w^{se,k} - \gamma) + \tfrac{\delta}{1-\delta}\left(\Delta(\gamma) + 2\gamma\right)\right) + \delta\hat{\Pi}^{H,k}(w^{se,k} + \gamma)$$

or, equivalently,

$$\hat{U}\left(w_t - (R^{L,k})^{-1}(w^{se,k})\right) - U\left(w_t - (R^{H,k})^{-1}(w^{se,k} - \gamma) + \tfrac{\delta}{1-\delta}\left(\Delta(\gamma) + 2\gamma\right)\right) >$$
$$\tfrac{\delta}{1-\delta}\left(\hat{\Pi}^{H,k}(w^{se,k} + \gamma) - \hat{\Pi}^{L,k}(w^{se,k} - 2\gamma)\right).$$

The definition $\Delta(\cdot)$ implies that $\hat{\Pi}^{H,k}(w^{se,k} + \gamma) - \hat{\Pi}^{L,k}(w^{se,k} - 2\gamma) \leqslant \Delta(\gamma) + 2\gamma$. Moreover, since $R^{H,k}(w) - R^{L,k}(w) < \gamma$, $(R^{L,k})^{-1}(w) < (R^{H,k})^{-1}(w + \gamma)$ for any $w \geqslant 0$. Thus,

$$\begin{aligned}
&\hat{U}\left(w_t - (R^{L,k})^{-1}(w^{se,k})\right) - U\left(w_t - (R^{H,k})^{-1}(w^{se,k} - \gamma) + \tfrac{\delta}{1-\delta}(\Delta(\gamma) + 2\gamma)\right) \geqslant \\
&\hat{U}\left(w_t - (R^{H,k})^{-1}(w^{se,k} + \gamma)\right) - U\left(w_t - (R^{L,k})^{-1}(w^{se,k} - 2\gamma) + \tfrac{\delta}{1-\delta}(\Delta(\gamma) + 2\gamma)\right) \geqslant \\
&\hat{U}\left(w_t - R^{-1}(w^{se,k} + \gamma)\right) - U\left(w_t - R^{-1}(w^{se,k} - 2\gamma) + \tfrac{\delta}{1-\delta}(\Delta(\gamma) + 2\gamma)\right).
\end{aligned} \tag{19}$$

Note that the last line of (19) is the same as (13) with $w = w_t$, which is great than $\frac{\delta}{1-\delta}(\Delta(\gamma) + 2\gamma)$. Hence, the deviation is indeed profitable.

B.3 Proof of Proposition 5

We first claim that if $w \geqslant R(\bar{w})$, then the household leaves the community. This shows that in the community, wealth is at most $R(\bar{w})$.

To prove this claim, we give the household a more generous return function $\hat{R}(w) = (w - \bar{w})/\delta + R(\bar{w})$. Suppose that the household starts in the community with some initial wealth $w \geqslant R(\bar{w})$. Then the discounted sum of consumptions is at most:

$$(1-\delta)\sum_{t=0}^{\infty}\delta^t c_t \leqslant (1-\delta)w + \delta R(\bar{w}) - \bar{w} + (1-\delta)\sum_{t=0}^{\infty}\delta^t s_t.$$

The neighbor's payoff at the beginning of period 0 is $-(1-\delta)\sum_{t=0}^{\infty}\delta^t s_t$, so it must be the case that $(1-\delta)\sum_{t=0}^{\infty}\delta^t s_t \leqslant 0$. Therefore, the discounted sum of consumptions is at most:

$$(1-\delta)\sum_{t=0}^{\infty}\delta^t c_t \leqslant (1-\delta)w + \delta R(\bar{w}) - \bar{w}.$$

The best consumption sequence given this constraint is to consume constantly $(1-\delta)w + \delta R(\bar{w}) - \bar{w}$ in every period. If the household with wealth w chooses to consume $(1-\delta)w + \delta R(\bar{w}) - \bar{w}$, then the investment is:

$$w - ((1-\delta)w + \delta R(\bar{w}) - \bar{w}) = \delta w + \bar{w} - \delta R(\bar{w}),$$

which is greater than $\bar{w}$ if $w \geqslant R(\bar{w})$. Hence, this constant consumption sequence can be sustained in the city under the return function $R(\cdot)$ for any $w \geqslant R(\bar{w})$. This shows that in the community, the wealth is at most $R(\bar{w})$.

Consider an equilibrium such that the household stays in the community. Define $L_t = (1-\delta)\sum_{\tau=t}\delta^{\tau-t}s_\tau$. We first show that $-L_t \leqslant R(\bar{w})$ for any $t \geqslant 0$. This is because:

$$\begin{aligned} L_t + R(\bar{w}) &= (1-\delta)\textstyle\sum_{\tau=t}\delta^{\tau-t}(s_\tau + R(\bar{w})) \geqslant (1-\delta)\sum_{\tau=t}\delta^{\tau-t}(s_\tau + w_t) \\ &\geqslant (1-\delta)\textstyle\sum_{\tau=t}\delta^{\tau-t}(s_\tau + w_t - I_t) = (1-\delta)\sum_{\tau=t}\delta^{\tau-t}c_t \geqslant 0. \end{aligned}$$

Here, the first inequality follows from the previous result that $w_t \leqslant R(\bar{w})$ in the community.

Next, we show that $-L_t \leqslant 0$ for any $t \geqslant 0$. Suppose not. Let $k \in (0,1]$ be the smallest number such that:

$$-L_t \leqslant kR(\bar{w}),\ \forall t \geqslant 0.$$

Therefore, there exists a period t' such that:

$$-L_{t'} > k\delta R(\bar{w}).$$

Consider the following deviation in period t': the household chooses $s_{t'} = 0$ and leaves for the city. In each $\tau \geqslant t'$, it chooses the investment $\tilde{I}_\tau$ given by:

$$\tilde{I}_\tau = I_\tau - s_\tau + \frac{k\delta R(\bar{w}) + L_\tau}{1-\delta},$$

with the remainder, $\tilde{c}_\tau$, being consumed. We show that this deviation (i) is feasible, and (ii) strictly improves the household's payoff.

Note that in each $\tau \geqslant t'$,

$$\tilde{I}_\tau - I_\tau = -s_\tau + \frac{k\delta R(\bar{w}) + L_\tau}{1-\delta} = \frac{k\delta R(\bar{w}) + L_\tau - (1-\delta)s_\tau}{1-\delta} = \delta\frac{kR(\bar{w}) + L_{\tau+1}}{1-\delta} \geqslant 0, \quad (20)$$

where the inequality holds by the definition of k. Thus, this deviation results in more investment, and so higher wealth, in each period $\tau \geqslant t'$.

Next, we show that this deviation is profitable. Using the definition of $\tilde{I}_\tau$ and the fact that $\tilde{w}_{t'} = w_{t'}$, period-t' consumption satisfies

$$\tilde{c}_{t'} = \tilde{w}_{t'} - \tilde{I}_{t'} = w_{t'} - I_{t'} + s_{t'} - \frac{k\delta R(\bar{w}) + L_{t'}}{1-\delta} > c_{t'}.$$

Here, the inequality follows from the fact that $k\delta R(\bar{w}) + L_{t'} < 0$ by choice of t'. So this deviation results in strictly higher consumption in period t'.

Now consider $\tau > t'$. From (20) and $R'(\cdot) \geqslant \frac{1}{\delta}$, we have that:

$$\tilde{w}_\tau - w_\tau = R(\tilde{I}_{\tau-1}) - R(I_{\tau-1}) \geqslant \frac{kR(\bar{w}) + L_\tau}{1-\delta}.$$

Therefore,

$$\begin{aligned}\tilde{c}_\tau &= \tilde{w}_\tau - \tilde{I}_\tau \geqslant w_\tau + \tfrac{kR(\bar{w})+L_\tau}{1-\delta} - \left(I_\tau - s_\tau + \tfrac{k\delta R(\bar{w})+L_\tau}{1-\delta}\right) \\ &= c_\tau + kR(\bar{w}) > c_\tau.\end{aligned}$$

Therefore, this deviation leads to strictly higher consumption in every $\tau \geqslant t'$. Since $\hat{U} \geq U$, we conclude that this deviation is profitable, which contradicts the presumption that the original strategy was an equilibrium. Thus, it must be that $-L_t \leqslant 0$ for any $t \geqslant 0$

Suppose that $s_t > 0$ in some period $t \geqslant 0$. For the neighbor to be willing to choose $s_t > 0$, it must be that her continuation payoff satisfies $-\sum_{\tau=t+1}^{\infty} \delta^{\tau-(t+1)} s_\tau > 0$, or $-L_{t+1} > 0$. This contradicts $-L_{t+1} \leqslant 0$. We conclude that $s_t \leqslant 0$ in all $t \geqslant 0$, which implies $s_t = 0$ for all $t \geq 0$, proving the proposition. ■

C Online Appendix: Extensions to the Model

C.1 Reversible Exit

This appendix shows that underinvestment occurs even if the household can return to the community after leaving for the city. Formally, we modify the game in Section 2 so that at the start of every period while the household is in the city, it can return to the community. If it does, then it plays the community game until it again chooses to leave for the city. Payoffs and information structures are the same as in Section 2, and so neighbors observe all of the household's interactions with neighbors, while vendors observe only their own interactions.

We impose a slightly stronger version of Assumption 1 and also assume that the household's marginal utility of consumption is strictly higher in the city.

Assumption 3 *Define $\bar{c}_m$ as the solution to $\hat{U}'(\bar{c}_m) = 1$, and let $\hat{w}_m$ satisfy $R(\hat{w}_m - \bar{c}_m) = \hat{w}_m$. Then, $R'(\hat{w}_m) > \frac{1}{\delta}$. Moreover, for every $c > 0$, $\hat{U}'(c) > U'(c)$.*

Under this assumption, we can prove that underinvestment occurs even if exit is reversible.

Proposition 6 *Impose Assumption 3. There exists a $w^{**} < \bar{w}$, a $w^{cc} < w^{**}$, and a positive-measure set $\mathcal{W} \subseteq [0, w^{**}]$ with $\sup \mathcal{W} = w^{**}$ such that (i) if $w_0 \notin \mathcal{W}$, the household permanently exits the community, with $\lim_{t\to\infty} w_t > \bar{w}$, and (ii) if $w_0 \in \mathcal{W}$, the household is in the community for an infinite number of periods.*

If $w_0 \in \mathcal{W}$, then for every $t \geq 0$, $w_t < w^{**}$. Moreover, $w_{t+1} < w_t$ whenever $w_t \in (w^{cc}, w^{**}) \cap \mathcal{W}$.

C.1.1 Proof of Proposition 6

Much like the proof of Proposition 2, we break this proof into a sequence of lemmas. We begin by showing that the household's worst equilibrium payoff equals $\hat{\Pi}(\cdot)$, its worst equilibrium payoff from the game in Section 2.

Lemma 5 *For any initial wealth $w \geq 0$, the household's worst equilibrium payoff is $\hat{\Pi}(\cdot)$.*

Proof of Lemma 5: This proof is similar to the proof of Proposition 1. It is an equilibrium for $f_t = 0$ in every $t \geq 0$, in which case it is optimal for the household to permanently leave the community. In the city, vendor t accepts only if $p_t \geq c_t$. Therefore, $\hat{\Pi}(\cdot)$ gives the maximum equilibrium payoff if the household permanently leaves the community. But as in the proof of Proposition 1, the household cannot earn less than $\hat{\Pi}(\cdot)$, because vendor t must accept whenever $p_t > c_t$. ■

Now, we turn to the household's maximum equilibrium payoff. Define $\Pi^{**}(w)$ as the maximum equilibrium payoff with initial wealth w. Define $\Pi_c^*(\cdot)$ identically to $\Pi_c(\cdot)$, except that $\Pi^*(\cdot)$ is replaced by $\Pi^{**}(\cdot)$. Define

$$\Pi_m^*(w) = \max_{0 \leq c \leq w} \left\{ (1-\delta)\hat{U}(c) + \delta \Pi^{**}(R(w-c)) \right\}$$

as the household's maximum equilibrium payoff if it chooses the city in the current period. The key difference between this model and the baseline model is that $\Pi_m^*(\cdot)$ might entail the household returning to the community to take advantage of relational contracts in the future. Therefore, $\Pi_m^*(\cdot) \geq \hat{\Pi}(\cdot)$, since the latter entails staying in the city forever.

Lemma 6 *Both $\Pi_c^*(\cdot)$ and $\Pi_m^*(\cdot)$ are strictly increasing. For all $w \geq 0$,*

$$\Pi^{**}(w) = \max\left\{\Pi_c^*(w), \Pi_m^*(w)\right\}.$$

Proof of Lemma 6: As in Lemma 1, conditional on choosing the community in period 0, the household's maximum equilibrium payoff equals $\Pi_c^*(w_0)$. If the household instead chooses the city in period 0, then $f_0 = 0$ in any equilibrium. Thus, the household optimally sets $p_t = c_t$, so its maximum equilibrium continuation payoff equals $\Pi^{**}(R(w-c))$. We conclude that $\Pi_m^*(w)$ is the household's maximum equilibrium payoff conditional on choosing the city. It then immediately follows that $\Pi^{**}(w) = \max\left\{\Pi_c^*(w), \Pi_m^*(w)\right\}$. Both $\Pi_c^*(\cdot)$ and $\Pi_m^*(\cdot)$ are strictly increasing by inspection. ■

Apart from some details of the proof, the next result is similar to Lemma 2.

Lemma 7 *If $\Pi^{**}(w_0) > \hat{\Pi}(w_0)$, then $\Pi^{**}(w_t) > \hat{\Pi}(w_t)$ in all $t \geq 0$ of any household-optimal equilibrium.*

Proof of Lemma 7: Suppose not, and let $\tau > 0$ be the first period such that $\Pi^{**}(w_\tau) = \hat{\Pi}(w_\tau)$. In period $\tau - 1$, (2) implies that $f_t = 0$ if the household stays in the community. Therefore, it is optimal for the household to leave the community in $\tau - 1$. But then it is optimal for the household to *permanently* leave the community in $\tau - 1$, since $\Pi^{**}(w_\tau) = \hat{\Pi}(w_\tau)$. So $\Pi^{**}(w_{\tau-1}) = \hat{\Pi}(w_{\tau-1})$, contradicting the definition of τ. ■

Lemma 8 *Suppose that $\Pi^{**}(w_0) > \hat{\Pi}(w_0)$. Then in every $t \geq 0$, (i) $\hat{U}'(c_t) \geq 1$, and (ii) there exists $\tau > t$ such that the household stays in the community in period τ.*

Proof of Lemma 8: Towards contradiction, suppose that there exists $t \geq 0$ such that $\hat{U}'(c_t) < 1$, so that *a fortiori*, $U'(c_t) < 1$. If $f_t > 0$, then we can decrease f_t and c_t by the same $\epsilon > 0$. This perturbation is also an equilibrium, and increases the household's period-t payoff at rate $1 - \hat{U}'(c_t) > 0$ as $\epsilon \to 0$. Thus, $f_t = 0$, which implies that $p_t = c_t > 0$.

By Lemma 7, $\Pi^{**}(w_{t+1}) > \hat{\Pi}(w_{t+1})$. Therefore, there exists a $\tau > t$ such that $f_\tau > 0$, since otherwise the household could do no better than exiting the city permanently. Let τ be the *first* period after t such that $f_\tau > 0$. Note that the household must be in the community in period τ.

Consider the following perturbation: decrease p_t and c_t by $\epsilon > 0$, and increase p_τ and decrease f_τ by $\chi(\epsilon)$, where $\chi(\epsilon)$ is chosen so that $w_{\tau+1}$ remains constant. Then, $\chi(\epsilon) \geq \frac{\epsilon}{\delta^{\tau-t}}$ because $R'(\cdot) \geq \frac{1}{\delta}$. As $\epsilon \to 0$, $\chi(\epsilon) \to 0$. Hence, this perturbation is feasible for small enough $\epsilon > 0$. It is an equilibrium, since (2) is trivially satisfied in all $t' \in [t, \tau - 1]$ because $f_{t'} = 0$ in those periods. This perturbation changes the household's period-t continuation payoff at rate no less than

$$-(1-\delta)\hat{U}'(c_t) + \delta^{\tau-t}(1-\delta)\frac{1}{\delta^{\tau-t}} > 0$$

as $\epsilon \to 0$. Thus, the original equilibrium could not have been household-optimal. ■

Next, we show that the household stays in the community for sufficiently low initial wealth levels.

Lemma 9 *The set*

$$\mathcal{W} = \left\{ w | \Pi^{**}(w) > \hat{\Pi}(w) \right\}$$

has positive measure, with

$$w^{**} = \sup \left\{ w | \Pi^{**}(w) > \hat{\Pi}(w) \right\} < \bar{w}.$$

Proof of Lemma 9: The proof that $\left\{ w | \Pi^{**}(w) > \hat{\Pi}(w) \right\}$ has positive measure is identical to the proof in Lemma 3, since the same constructions work at $w = 0$, $\Pi^{**}(\cdot)$ is increasing, and $\hat{\Pi}(\cdot)$ is continuous. If $w_0 \in \mathcal{W}$, then $c_t \leqslant \bar{c}_m$ in every t by Lemma 8, so $\Pi^{**}(w_0) \leqslant \hat{U}(\bar{c}_m)$. If w_0 is such that $R(w_0 - \bar{c}_m) > w_0$, then $\Pi^{**}(w_0) \geqslant \hat{\Pi}(w_0) > \hat{U}(\bar{c}_m)$. This implies that $R(w_0 - \bar{c}_m) \leqslant w_0$ for any $w_0 \in \mathcal{W}$. Hence, by Assumption 3, $w^{**} \leqslant \hat{w}_m < \bar{w}$.■

We are now in a position to prove Proposition 6. So far, the argument has hewn closely to the proof of Proposition 2. The rest of the proof marks a more substantial departure.

Lemma 9 shows that a positive-measure set $\mathcal{W}$ exists such that $\Pi^{**}(w_0) > \hat{\Pi}(w_0)$ for all $w_0 \in \mathcal{W}$ and $\sup \mathcal{W} < \bar{w}$. For any $w_0 \notin \mathcal{W}$, $\Pi^{**}(w_0) = \hat{\Pi}(w_0)$ and so the household permanently exits the community and has a long-term wealth strictly above $\bar{w}$. Lemma 7 implies that for any $w_0 \in \mathcal{W}$, we can construct an infinite sequence of periods such that the household remains in the community for each period in that sequence. This proves the first part of Proposition 6.

Suppose $w_0 \in \mathcal{W}$. Then, Lemma 7 and the definition of w^{**} immediately imply that $w_t < w^{**}$ in every $t \geq 0$. It remains to identify a $w^{cc} < w^{**}$ such that if $w_t \in \mathcal{W} \cap (w^{cc}, w^{**})$, then $w_{t+1} < w_t$. Since $\Pi^{**}(\cdot)$ is increasing, (2) holds only if

$$f_t \leq \Delta(w_t) = \frac{\delta}{1-\delta}(\Pi^{**}(w^{**}) - \hat{\Pi}(w_t)).$$

Proposition 1 implies that $\Delta(w_t)$ is strictly decreasing and continuous. Continuity implies that $\lim_{w \uparrow w^{**}} \hat{\Pi}(w) = \hat{\Pi}(w^{**})$. If $\Pi^{**}(w^{**}) > \hat{\Pi}(w^{**})$, then $\Pi^{**}(\cdot)$ increasing and $\hat{\Pi}(\cdot)$ continuous imply $\Pi^{**}(w) > \hat{\Pi}(w)$ just above w^{**}, contradicting the definition of w^{**}. Therefore, $\Pi^{**}(w^{**}) = \hat{\Pi}(w^{**})$ and $\Delta(w^{**}) = 0$.

For any $w \in [R^{-1}(w^{**}), w^{**}]$, define

$$G(w) = \hat{U}(w - R^{-1}(w^{**})) - (U(w^{**} - R^{-1}(w) + \Delta(w)) + \Delta(w)).$$

Then, $G(\cdot)$ is strictly increasing, continuous, and strictly crosses 0 from below. Define $w^{cc1} \in (R^{-1}(w^{**}), w^{**})$ as the unique wealth such that $G(w^{cc1}) = 0$.

Consider a household with $w_t > w^{cc1}$ such that $\Pi^{**}(w_t) = \Pi^*_c(w_t) > \hat{\Pi}(w_t)$, with corresponding household-optimal choices (c_t, p_t, f_t). Towards contradiction, suppose $w_{t+1} > w^{cc1}$. Then, this household can exit permanently, choose $\hat{c}_t = \hat{p}_t = p_t = c_t - f_t$ and $\hat{f}_t = 0$, where $p_t \geq 0$ because $w^{cc1} \geq R^{-1}(w^{**})$. This deviation leaves w_{t+1} unchanged and results in continuation payoff $\hat{\Pi}(w_{t+1})$.

We argue that this deviation is profitable:

$$\begin{aligned}
(1-\delta)\hat{U}(\hat{c}_t)+\delta\hat{\Pi}(w_{t+1}) &\geq (1-\delta)\hat{U}(c_t-f_t)+\delta\hat{\Pi}(w^{cc1}) \\
&> (1-\delta)(U(c_t)+\Delta(w^{cc1}))+\delta\hat{\Pi}(w^{cc1}) \\
&= (1-\delta)(U(c_t)+\Delta(w^{cc1}))+\delta\Pi^{**}(w^{**})-(1-\delta)\Delta(w^{cc1}) \\
&= (1-\delta)U(c_t)+\delta\Pi^{**}(w^{**}) \\
&\geq (1-\delta)U(c_t)+\delta\Pi^{**}(w_{t+1}) \\
&= \Pi^{**}(w_t).
\end{aligned}$$

Here, the first line follows from $\hat{c}_t = c_t - f_t$ and $w_{t+1} \geq w^{cc1}$; the second line is proven below; the third line follows from the definition of $\Delta(w^{cc1})$; and the fifth line because, by Lemma 7, $w_{t+1} \leq w^{**}$. The fourth and sixth lines are algebra.

The second line follows because for any $w_t > w^{cc1}$, $\hat{U}(c_t - f_t) > U(c_t) + \Delta(w^{cc1})$. To see this, note

$$w^{**} \geq w_{t+1} = R(w_t - p_t) > R(w^{cc1} - p_t).$$

Hence, $p_t > w^{cc1} - R^{-1}(w^{**})$. Similarly,

$$w^{cc1} < w_{t+1} = R(w_t - p_t) \leq R(w^{**} - p_t),$$

so $p_t < w^{**} - R^{-1}(w^{**})$. Therefore,

$$\begin{aligned}
\hat{U}(c_t-f_t)=\hat{U}(p_t) &> \hat{U}(w^{cc1}-R^{-1}(w^{**})) \\
&\geq U(w^{**}-R^{-1}(w^{cc1})+\Delta(w^{cc1}))+\Delta(w^{cc1}) \\
&> U(p_t+\Delta(w^{cc1}))+\Delta(w^{cc1}) \\
&\geq U(p_t+\Delta(w_t))+\Delta(w^{cc1}) \\
&\geq U(c_t)+\Delta(w^{cc1}).
\end{aligned}$$

Here, the first and third lines follow from $w^{**} - R^{-1}(w^{cc1}) > p_t > w^{cc1} - R^{-1}(w^{**})$; the second line from $G(w^{cc1}) = 0$; the fourth line from the fact that $w^{cc1} < w_t$ and $\Delta(\cdot)$ strictly decreasing; and the last line from $p_t + \Delta(w_t) \geq p_t + f_t = c_t$. The household therefore has a profitable deviation if $w_{t+1} > w^{cc1}$, so $w_{t+1} \leq w^{cc1}$ whenever $w_t \in (w^{cc1}, w^{**})$.

Define the function

$$F(w) = (1-\delta)\hat{U}(w^{**} - R^{-1}(w)) + \delta\hat{\Pi}(w^{**}) - \hat{\Pi}(w)$$

on $w \in [0, w^{**}]$. Then $F(\cdot)$ is strictly decreasing and continuous, with

$$F(0) = (1-\delta)\hat{U}(w^{**}) + \delta\hat{\Pi}(w^{**}) > 0.$$

At $w_0 = w^{**}$, it is feasible for the household to permanently leave the community and consume $c_t = w^{**} - R^{-1}(w^{**})$ in each $t \geq 0$. However, doing so violates (Euler), since $R'(w^{**}) > \frac{1}{\delta}$. Therefore, this consumption path must be dominated by some other feasible consumption path once the household permanently leaves the community, which implies

$$\hat{U}(w^{**} - R^{-1}(w^{**})) < \hat{\Pi}(w^{**}).$$

Consequently,

$$F(w^{**}) = (1-\delta)\hat{U}(w^{**} - R^{-1}(w^{**})) - (1-\delta)\hat{\Pi}(w^{**}) < 0.$$

We conclude that there exists a unique $w^{cc2} \in (0, w^{**})$ such that $F(w^{cc2}) = 0$.

Suppose the household with $w_t \in \mathcal{W}$ and $w_t > w^{cc2}$ chooses to live in the city in t. Towards contradiction, suppose that $w_{t+1} \geq w_t$. Therefore, the household's payoff satisfies

$$\begin{aligned} \Pi^{**}(w_t) &\leq (1-\delta)\hat{U}(w^{**} - R^{-1}(w_t)) + \delta\Pi^{**}(w^{**}) \\ &= (1-\delta)\hat{U}(w^{**} - R^{-1}(w_t)) + \delta\hat{\Pi}(w^{**}) \\ &< \hat{\Pi}(w_t), \end{aligned}$$

where the first inequality follows because $f_t = 0$, so that $p_t = c_t = w_t - R^{-1}(w_{t+1}) \leq w^{**} - R^{-1}(w_t)$; the equality follows because $\Pi^{**}(w^{**}) = \hat{\Pi}(w^{**})$, and the final, strict inequality follows because $w_t > w^{cc2}$ and so $F(w_t) < 0$. But $\Pi^{**}(\cdot) \geq \hat{\Pi}(\cdot)$, proving a contradiction. So $w_{t+1} < w_t$.

Set $w^{cc} = \max\{w^{cc1}, w^{cc2}\}$. We have shown that $w_{t+1} < w_t$ whenever $w_t > w^{cc}$. This completes the proof. ■

C.2 Non-Linear Favors

C.2.1 Model and Statement of Result

In this appendix, we show that a (slightly weaker version of) Proposition 2 holds if the household's cost in providing f_t is convex.

Consider the **game with non-linear favors,** which is identical to the game in Section 2 except that the household's stage-game payoff is

$$\pi_t = \begin{cases} U(c_t d_t) - k(f_t) & \text{in the community} \\ \hat{U}(c_t, d_t) - k(f_t) & \text{in the city.} \end{cases}$$

Assume that $k(\cdot)$ is strictly increasing, strictly convex, twice continuously differentiable, and satisfies $k'(0) = 0$, $k(0) = 0$, and $\lim_{f \to \infty} k'(f) = \infty$. We assume $\hat{U}'(\cdot) > U'(\cdot)$. A special case of this game has $k = U^{-1}$, which corresponds to a setting in which the neighbors value f_t according to the same utility function as the household's consumption.

Define $f^*(c)$ as the unique solution to $U'(c) = k'(f^*(c))$ and c^* as the solution to $U(c^*) = \hat{U}(c^* - f^*(c^*))$. Note that c^* exists and is unique because $\hat{U}'(c) > U'(c)$, $f^*(\cdot)$ is decreasing, and $\lim_{c \to \infty} f^*(c) = 0$. We impose the following assumption.

Assumption 4 *Define $\tilde{c}(w)$ as the unique solution to $R(w - \tilde{c}(w)) = w$. Assume that*

$$\hat{U}(\tilde{c}(\bar{w}) - f^*(\tilde{c}(\bar{w}))) - U(\tilde{c}(\bar{w})) > \frac{\delta}{1-\delta} U(c^*).$$

Suppose that the household has wealth $\bar{w}$, which is the wealth level at which strictly positive-return investments are exhausted. Assumption 4 ensures that this household's value from moving to the city and consuming $\tilde{c}(\bar{w}) - f^*(\tilde{c}(\bar{w}))$ forever after is substantially greater than its utility from staying in the community and consuming $\tilde{c}(\bar{w})$ forever. Note that $\tilde{c}(w)$ is increasing because $R'(\cdot) \geq \frac{1}{\delta} > 1$.

We now prove our main result for the game with non-linear favors.

Proposition 7 *Impose Assumption 4 in the game with non-linear favors. Then, there exists a $w^{se} < \bar{w}$, a $w^{tr} \in [0, w^{se})$, and a positive-measure set $\mathcal{W} \subseteq [0, w^{se})$, such that in any household-optimal equilibrium,*

1. ***Selection:*** *The household stays in the community forever if $w_0 \in \mathcal{W}$ and otherwise leaves immediately.*

2. ***"Too-big-for-their-boots:"*** *If $w_0 \in \mathcal{W}$ and $w_t \geq w^{tr}$ on the equilibrium path, then $w_{t+1} < w_t$.*

C.2.2 Proof of Proposition 7

We focus only on those parts of the proof that differ substantially from the proof of Proposition 2.

The proof of Proposition 1 goes through without change, since $f_t = 0$ in every period of any equilibrium in the city. Similarly, once we substitute the cost function $k(f)$ into the household's payoff, the proofs of Lemmas 1 and 2 go through without change.

The next step of the proof, which shows that wealthy households leave the community and poorer households stay, requires a new argument.

Lemma 10 *The set $\left\{w : \Pi^*(w) > \hat{\Pi}(w)\right\}$ has positive measure. Moreover, $w^{se} = \sup\left\{w : \Pi^*(w) > \hat{\Pi}(w)\right\}$ satisfies $0 < w^{se} < \bar{w}$.*

Proof of Lemma 10: The argument that $\Pi^*(0) > \hat{\Pi}(0) = 0$ goes through essentially without change. Thus, we need to show only that $w^{se} < \infty$.

Step 1: In any $t \geq 0$ of any household-optimal equilibrium, $f_t \leq f^*(c_t)$. If the household is in the city, $f_t = 0 \leq f^*(c_t)$. If it is in the community, then suppose that $f_t > f^*(c_t)$. Consider perturbing the equilibrium by decreasing c_t and f_t by $\epsilon > 0$. This perturbation is feasible as $\epsilon \to 0$, and moreover, increases the household's stage-game payoff at rate $k'(f_t) - U'(c_t) > k'(f^*(c_t)) - U'(c_t) = 0$. Thus, the original equilibrium was not household-optimal. This proves Step 1.

Step 2: For any $w \geq 0$, $\Pi^*(w) - \hat{\Pi}(w) \leq U(c^*)$. If $\Pi^*(w) = \hat{\Pi}(w)$, then the result is immediate. Suppose that $\Pi^*(w) > \hat{\Pi}(w)$. By Lemma 2, a household with $w_0 = w$ stays in the community in any $t \geq 0$ of any household-optimal equilibrium.

Fixing such an equilibrium, we can derive a lower bound on $\hat{\Pi}(w)$ using the following perturbation: the household leaves the community, chooses $\tilde{p}_t = p_t$ in each $t \geq 0$, and request consumption $\tilde{c}_t = c_t - f_t \geq 0$. This strategy is feasible, and the resulting payoff satisfies

$$\begin{array}{rcl} \sum_{t=0}^{\infty} \delta^t (1-\delta)\hat{U}(c_t - f_t) & = & \sum_{t|c_t \geq c^*} \delta^t(1-\delta)\hat{U}(c_t - f_t) + \sum_{t|c_t < c^*} \delta^t (1-\delta)\hat{U}(c_t - f_t) \\ & \geq & \sum_{t|c_t \geq c^*} \delta^t(1-\delta)\hat{U}(c_t - f^*(c_t)), \end{array}$$

where the inequality follows because $f_t \leq f^*(c_t)$ and $c_t - f_t \geq 0$. Therefore,

$$\begin{array}{rcl} \Pi^*(w) - \hat{\Pi}(w) & \leq & \sum_{t=0}^{\infty} \delta^t (1-\delta)\left(U(c_t) - \hat{U}(c_t - f_t)\right) \\ & \leq & \sum_{t|c_t \geq c^*} \delta^t(1-\delta)\left(U(c_t) - \hat{U}(c_t - f^*(c_t))\right) + \sum_{t|c_t < c^*} \delta^t(1-\delta)U(c_t) \\ & \leq & U(c^*), \end{array}$$

where the first two inequalities follow from our lower bound on $\hat{\Pi}(w)$, and the final inequality holds because $U(c_t) \leq \hat{U}(c_t - f^*(c_t))$ for all $c_t \geq c^*$, and $U(c_t) \leq U(c^*)$ for all $c_t < c^*$. This proves Step 2.

Step 3: There exists a w^{se} such that for all $w \geq w^{se}$, $\Pi^*(w) = \hat{\Pi}(w)$. Fix a wealth level such that $\Pi^*(w) > \hat{\Pi}(w)$ and $\hat{\Pi}(w) > U(c^*)$, and consider a household-optimal equilibrium with $w_0 = w$. By Lemma 2, the household must stay in the community forever. Therefore, there exists $t \geq 0$ such that $U(c_t) \geq \hat{\Pi}(w)$, so $c_t > c^*$.

In this period t, we must have $\Pi^*(w_{t+1}) > \hat{\Pi}(w_{t+1})$. Since the household can always leave the community, consume $c_t - f^*(c_t)$, and earn $\hat{\Pi}(w_{t+1})$, $\Pi^*(w_t) > \hat{\Pi}(w_t)$ holds only if

$$(1-\delta)U(c_t) + \delta\Pi^*(w_{t+1}) > (1-\delta)\hat{U}(c_t - f^*(c_t)) + \delta\hat{\Pi}(w_{t+1})$$

or

$$\frac{\delta}{1-\delta}\left(\Pi^*(w_{t+1}) - \hat{\Pi}(w_{t+1})\right) > \hat{U}(c_t - f^*(c_t)) - U(c_t).$$

By Step 2, this inequality holds only if

$$\frac{\delta}{1-\delta}U(c^*) > \hat{U}(c_t - f^*(c_t)) - U(c_t). \tag{21}$$

Now, choose w^{se} such that $\hat{U}(\tilde{c}(w^{se}) - f^*(\tilde{c}(w^{se}))) - U(\tilde{c}(w^{se})) = \frac{\delta}{1-\delta}U(c^*)$; note that w^{se} exists and satisfies $w^{se} < \bar{w}$ by Assumption 4. For any $w \geq w^{se}$, (21) cannot hold, which means that in any household-optimal equilibrium, there exists a $t \geq 0$ such that $\Pi^*(w_t) = \hat{\Pi}(w_t)$. Lemma 2 then implies that for all $w \geq w^{se}$, $\Pi^*(w_0) = \hat{\Pi}(w_0)$. We conclude that $w^{se} < \bar{w}$, as desired. ■

We can now complete the proof of Proposition 7. As in the proof of Proposition 2, **selection** follows from Lemmas 2 and 10. The argument for **"too-big-for-their-boots"** is similar to the argument in Proposition 6, with one change. We must substitute $k(f_t)$ into the household's payoff. With this change, we can construct a similar w^{cc1} as in Proposition 6 so $w_{t+1} < w_t$ whenever $w_t > w^{cc1}$. We can therefore take $w^{tr} = w^{cc1}$ to prove Proposition 7. ■