Robust Monopoly Regulation

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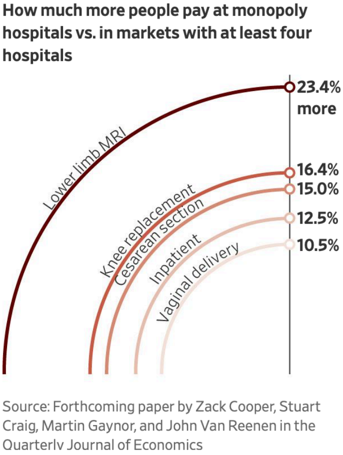
Regulating monopolies is challenging

- Cooper et al. (2018): prices at monopoly hospitals are 12% higher than those in markets with four or five rivals



Source: wsj

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- Protect consumer well-being *versus* not to distort production

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- If he wants a policy that works "fairly well" in all circumstances, what shall this policy look like?

What we do

- Regulator's payoff

$$\text{consumer surplus} \;+\; \phi \text{ firm's profit}, \;\phi \in [0, 1]$$

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- Optimal policy:

$$\underset{\text{policy}}{\textbf{minimize}} \underbrace{\max_{\text{demand,cost}} \text{regret}}_{\text{worst-case regret}}$$

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consumer surplus

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$\longleftarrow$ $\phi \in (0, 1)$ $\longrightarrow$

combination of price cap and capped subsidy

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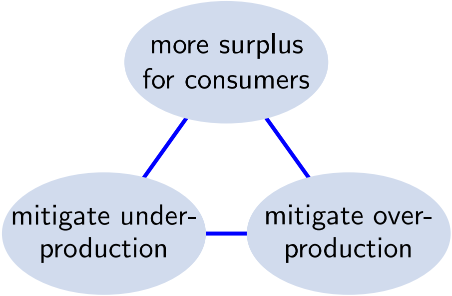

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Closest literature

- Monopoly regulation:
  Baron and Myerson (1982)
- Mechanism design with worst-case regret:
  Hurwicz and Shapiro (1978), Bergemann and Schlag (2008, 2011), Renou and Schlag (2011)
- Delegation:
  Holmström (1977, 1984)

Roadmap

- Environment
- Main result

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- A mass one of consumers

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  - (q, p) is feasible if $p \leqslant v(q)$
- $c : [0, 1] \to \mathbf{R}_+$ with $c(0) = 0$: an increasing l.s.c. cost function

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- Maximal total surplus is

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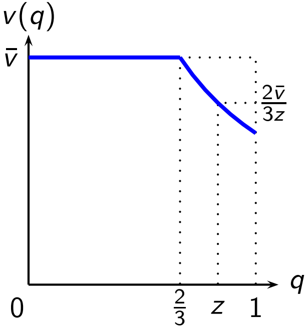
$$OPT = \max_{q \in [0,1]} \underbrace{\int_0^q v(z)\, dz}_{\text{total value to consumers}} - c(q)$$

- If the firm produces q, the distortion is

$$DSTR = OPT - \left(\int_0^q v(z)\, dz - c(q) \right)$$

Environment: two examples



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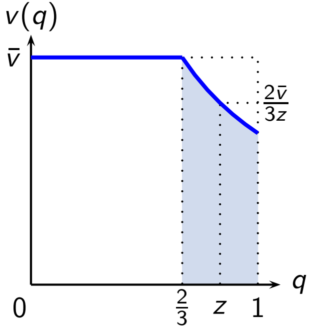

$c(q) = 0$

If $q = \frac{2}{3}$, $DSTR = \frac{2\bar{v}}{3} \int_{\frac{2}{3}}^{1} \frac{1}{z} \, \mathrm{d}z$

$$= -\frac{2\bar{v}}{3} \log \frac{2}{3}$$

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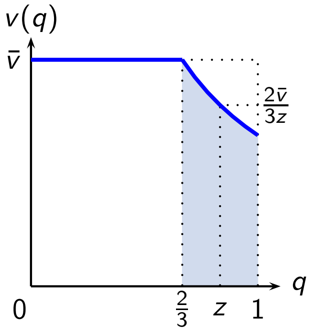

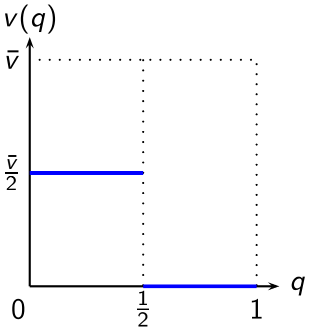

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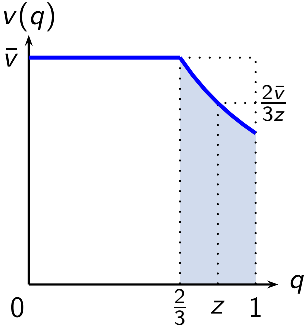

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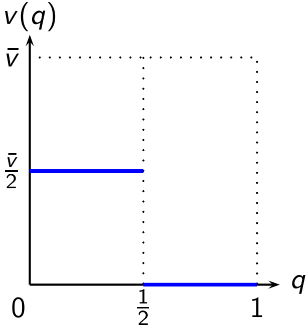

$c(q) = \frac{\bar{v}}{3}$

If $q = \frac{1}{2}$, $DSTR = \frac{\bar{v}}{3} - \frac{\bar{v}}{4}$

Regulation policy

- A policy is an u.s.c. function

$$\rho : [0,1] \times [0,\bar{v}] \to \mathbf{R}$$

  - if the firm sells q at price p, then it receives $\rho(q,p)$
  - e.g., if $\rho(q,p) > qp$, a subsidy of $\rho(q,p) - qp$
  - the firm is allowed to stay out of business with a profit of zero

- If $\rho(q,p) = qp, \forall q, p$, the firm is unregulated

Firm's best response and regulator's payoff

Fix ρ, v, c

- If the firm sells q at price p,
  the firm's profit and consumer surplus are:

$$FP = \rho(q, p) - c(q), \quad CS = \int_0^q v(z)\, \mathrm{d}z - \rho(q, p)$$

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- The regulator's payoff is

$$CS + \phi FP,\ \phi \in [0,1]$$

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Suppose that the regulator knows (v, c). Then

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  $\rho(q, p) = 0$ for $(q, p) \neq (q^*, v(q^*))$

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Solving for optimal policy

The regulator's problem is

$$\underset{\rho}{\textbf{minimize}} \max_{v,c} \ RGRT = \underbrace{DSTR}_{efficiency} + \underbrace{(1-\phi)FP}_{redistribution}$$

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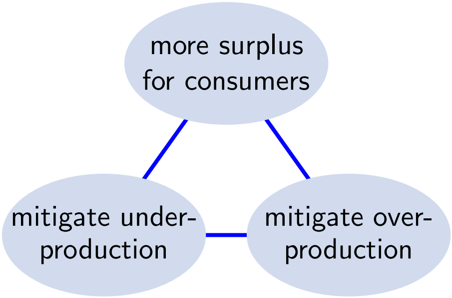

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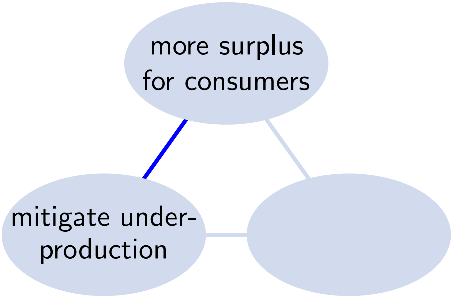

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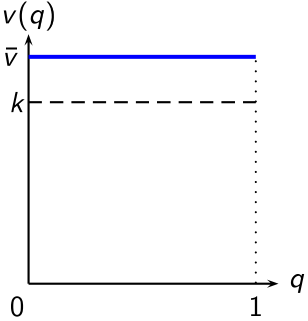

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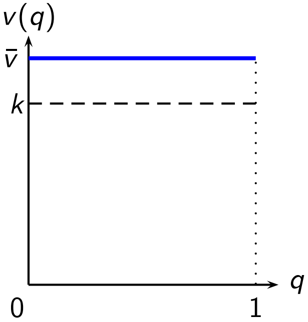

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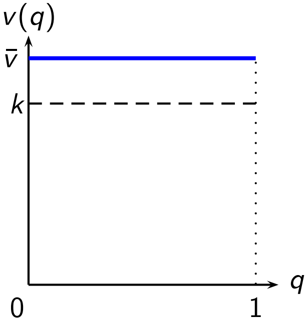

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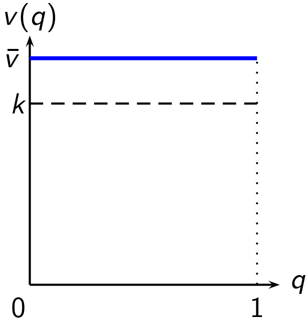

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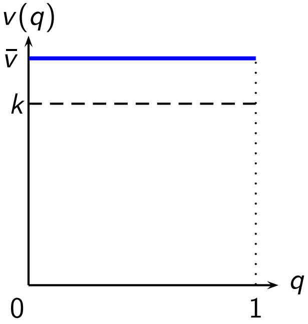

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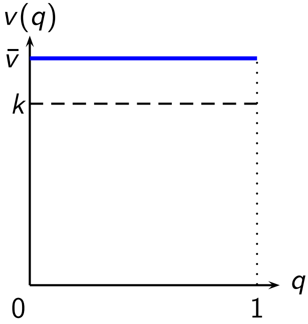

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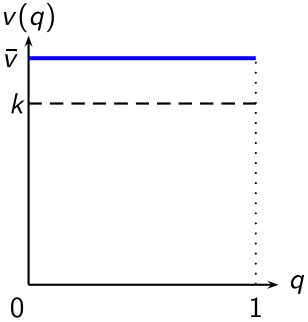

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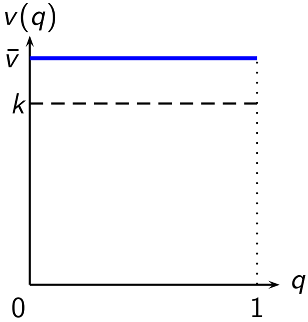

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$RGRT = \bar{v} - k$

Let $(1 - \phi)k_\phi = \bar{v} - k_\phi \implies k_\phi = \frac{\bar{v}}{2-\phi}$

Lower bound on worst-case regret

Theorem

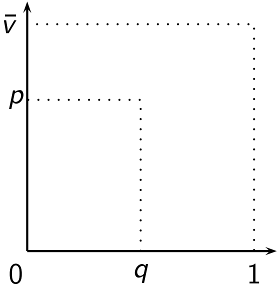
Let

$$L(q,p) = \min\left\{\, (1-\phi)qk_\phi - pq\log q,\ \ q(k_\phi - p)\,\right\}.$$

The worst-case regret under any policy is at least

$$\max_{q\in[0,1],\ p\in[0,k_\phi]} L(q,p).$$

Proof of lower bound



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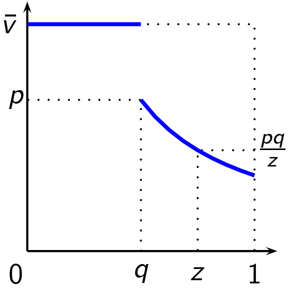

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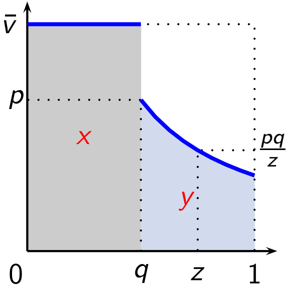

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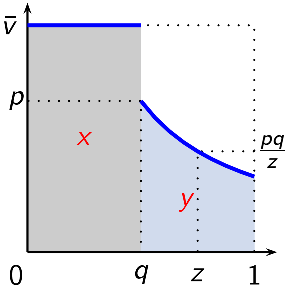

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1. If $\max\{x, y\} \leqslant qk_\phi$, a firm with fixed cost qk_ϕ won't produce:

$$RGRT = DSTR = q(\bar{v} - k_\phi) + \int_q^1 \frac{qp}{z} \, \mathrm{d}z$$
$$= q(1-\phi)k_\phi - pq\log(q) \geqslant L(q, p)$$

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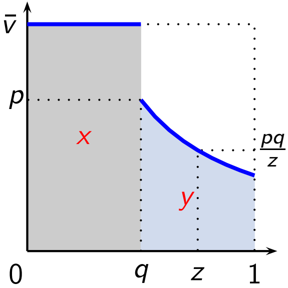

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2. If $\max\{x, y\} \geqslant qk_\phi$ and $x \geqslant y$, a firm with zero cost has $FP \geqslant qk_\phi$ and produces less than q:

$$RGRT \geqslant (1-\phi)qk_\phi + DSTR \geqslant (1-\phi)qk_\phi + \int_q^1 \frac{qp}{z}\,\mathrm{d}z$$
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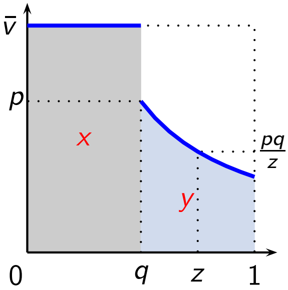

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3. If $\max\{x, y\} \geqslant qk_\phi$ and $y \geqslant x$, there exists q', p' in light-blue area such that $\rho(q', p') = y \geqslant qk_\phi$

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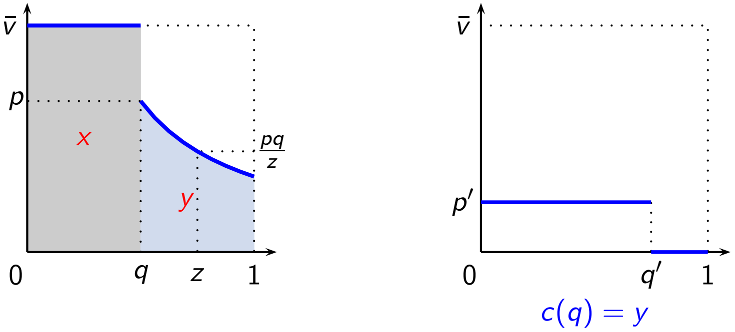

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Consider RHS firm:

$$RGRT = DSTR \geqslant qk_\phi - q'p' \geqslant q(k_\phi - p) \geqslant L(q, p)$$

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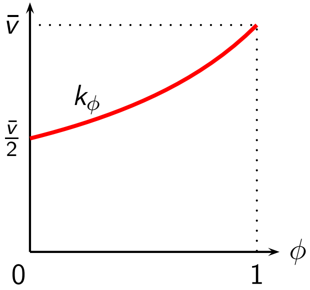

Roadmap

- Environment
- Main result
  - Lower bound on worst-case regret
  - Upper bound by our policy

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Theorem ($\phi = 0$)

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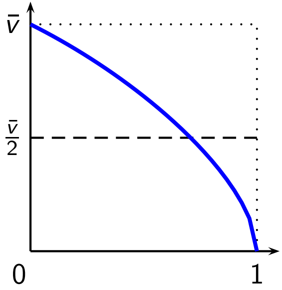

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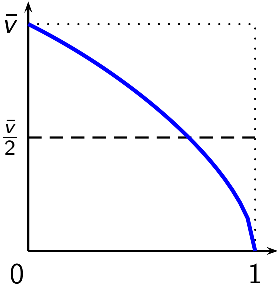
Proof idea:



if $q = 0$, for consumers with value $\leqslant \frac{\bar{v}}{2}$ each adds $\leqslant \frac{\bar{v}}{2}$ to total surplus;
for consumers with value $\geqslant \frac{\bar{v}}{2}$, average cost is $\geqslant \frac{\bar{v}}{2}$, so each adds $\leqslant \frac{\bar{v}}{2}$.

$\phi = 0$: regulator cares about CS only

Theorem ($\phi = 0$)

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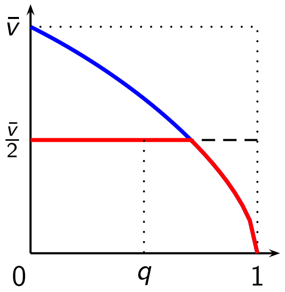

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for consumers with value $\geqslant \frac{\bar{v}}{2}$,
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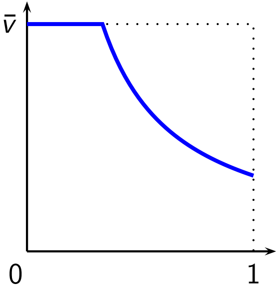
if $q > 0$, for consumers who are served,
regulator loses $p \leqslant \frac{\bar{v}}{2}$ each;
for consumers who are not served,
regulator loses $\leqslant \frac{\bar{v}}{2}$ each.

$\phi = 1$: regulator cares about total surplus



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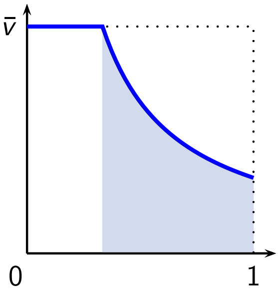
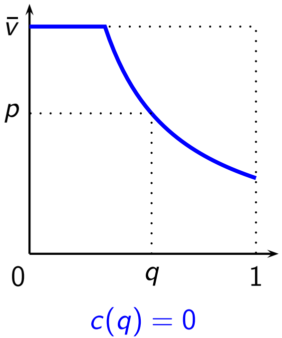


$c(q) = 0$

unregulated firm serve $\bar{v}$ consumers,

regulator loses surplus in light-blue area;

$\phi = 1$: regulator cares about total surplus



unregulated firm serve $\bar{v}$ consumers,

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If (q, p), subsidize $(\bar{v} - p)q$,

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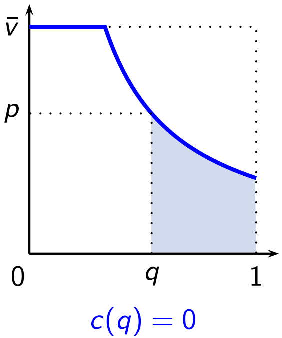

unregulated firm serve $\bar{v}$ consumers,
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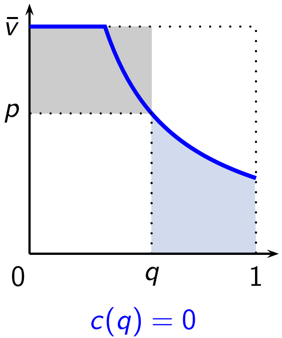

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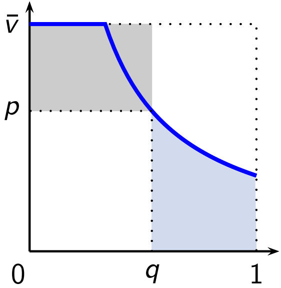

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Theorem ($\phi = 1$)

The worst-case regret is at most r_1 given the policy:

$$\rho(q, p) = \min(\ q\ \bar{v}\ ,\ qp + r_1\ ).$$

Upper bound on worst-case regret

Theorem ($0 \leqslant \phi \leqslant 1$)

The policy

$$\rho(q, p) = \min(\ q\ k_\phi\ ,\ pq + s\ )$$

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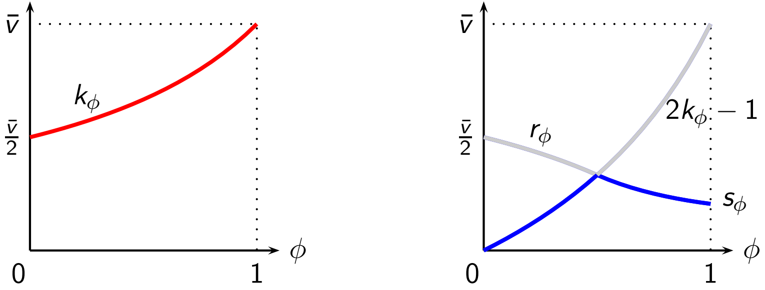

Upper bound on worst-case regret

Theorem ($0 \leqslant \phi \leqslant 1$)

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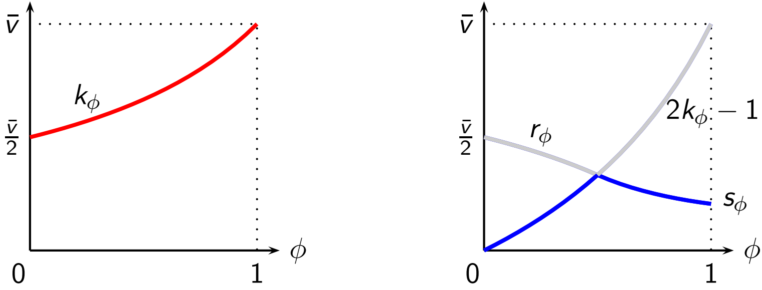
with $s_\phi \leqslant s \leqslant r_\phi$ achieves the worst-case regret r_ϕ.



Conclusion: our advocate for non-Bayesian approach

Armstrong and Sappington (2007):

1. Optimal policy under Bayesian approach is sensitive to how one models the regulator's knowledge

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1. Optimal policy under Bayesian approach is sensitive to how one models the regulator's knowledge
2. Multi-dimensional screening problems are typically difficult to solve

Conclusion: our advocate for worst-case regret

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$$\text{regret} = \underbrace{\text{distortion}}_{\textit{efficiency}} + \underbrace{(1-\phi)\ \text{firm's profit}}_{\textit{redistribution}}$$

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2. Worst-case regret is more relevant than worst-case payoff

Conclusion: our advocate for worst-case regret

3. Savage offers another interpretation, as observed by Linhart and Radner (1989):

   Suppose the [regulator] must justify his [policy] for a group of persons who have widely varying "subjective" probability distributions. In this case, the [regulator] might want to [regulate] in such a way as to minimize the maximum "outrage" felt in the group; here "outrage" is equated to regret.

Conclusion: three objectives

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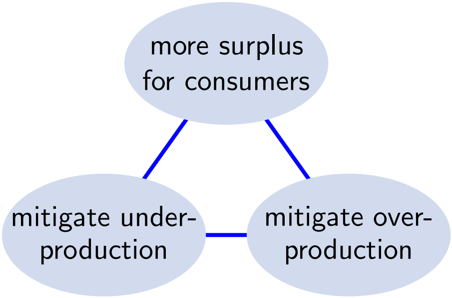

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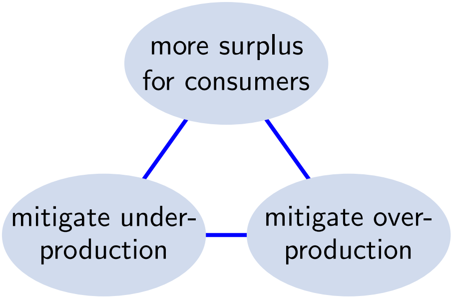

- Organizational economics

Thank you!