The Interval Structure of Optimal Disclosure

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1 Online appendix

1.1 An atomic type space for Sender

In this section we describe the changes that are needed to extend theorem 3.1 to the case in which Sender's type space $\mathcal{S} \subset \mathbf{R}$ has a supporting measure μ that might include atoms. The main motivation is the atomic case in which $\mathcal{S}$ is a finite set with μ being the counting measure. Our definitions of a disclosure mechanism, incentive compatibility, and Sender's problem carry through to this framework. The main difference from the nonatomic case is that the optimal mechanism might not be deterministic. Therefore, we need to modify the definition of "accepting on intervals."

For a mechanism $(\mathcal{X}, \kappa, r)$, if type t follows the recommendation, then *type t's acceptance probability given s* is given by

$$\rho(s,t) = \int r(x,t)\ \kappa(s, \mathrm{d}x).$$

We say that the mechanism *recommends accepting on intervals* if, for every type t, there exist some $\underline{s} \leqslant \overline{s} \in \mathcal{S}$ such that $\rho(s,t) = 1$ whenever $s \in (\underline{s}, \overline{s})$, and $\rho(s,t) = 0$ whenever $s \notin [\underline{s}, \overline{s}]$. The acceptance probabilities at the endpoints $\underline{s}$ and $\overline{s}$ might be strictly between 0 and 1.

With these definitions we modify theorem 3.1 as follows: under assumption 1, the optimal IC mechanism is a cutoff mechanism that recommends accepting on intervals.

This holds because, by Skorokhod's representation theorem, every $\mathcal{S}$ can be transformed to an interval equipped with Lebesgue measure. For example, if $\mathcal{S} = \{s_-, s_+\}$ with a uniform prior, then one can think of Sender's type as a function of some $s \in [-1, 1]$ drawn from a

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uniform distribution. Sender's type is s_- if $s \in [-1, 0]$, and s_+ if $s \in (0, 1]$. Conversely, we can create $s \in [-1, 1]$ by randomizing from a uniform distribution on $[-1, 0]$ or $(0, 1]$ when Sender's type is s_- or s_+, respectively. This leads to a correspondence between mechanisms defined on $\mathcal{S} = \{s_-, s_+\}$ and those defined on $\mathcal{S} = [-1, 1]$. This correspondence preserves the IC properties and Sender's payoff (although it may transform a deterministic mechanism into a nondeterministic mechanism).

1.2 Counterexamples

The following example shows that without assumption 3, corollary 5.1 may not hold. This example satisfies assumptions 2 and 4, but not assumption 3.

Example 1. Let $\mathcal{S} = \{-4, 3, 4\}$ and $\mathcal{T} = \{H, M, L\}$. The density $f(s, t) = g(s, t)$ is given by table 1, which satisfies the i.m.l.r. assumption. When Sender's type is s, type H's payoff from accepting is s, type M's payoff from accepting is $s - 2$, and type L's payoff from accepting is $s - 4$. Sender's payoff from accepting is one.

$t \backslash s$	-4	3	4
H	$1/25$	$6/25$	$10/25$
M	$1/25$	$2/25$	$2/25$
L	$1/25$	$1/25$	$1/25$

Table 1: Density $f(s, t)$

$\rho(\cdot, t) \backslash s$	-4	3	4
$\rho(\cdot, H)$	1	1	1
$\rho(\cdot, M)$	2/7	1	13/14
$\rho(\cdot, L)$	0	0	1

Table 2: Optimal privately IC mechanism

Table 2 gives the optimal privately IC mechanism. The row for type t gives this type's acceptance probabilities for different Sender's types. Type H is indifferent between reporting H and M, type M is indifferent between reporting M and L, and type L is indifferent between accepting and not accepting when he receives an acceptance recommendation.

We now consider IC mechanisms. This example satisfies assumption 4, so cutoff mechanisms are sufficient. In table 3, the row for type t gives the probabilities that type t is the cutoff type for different Senders' types. In table 4, the row for type t gives this type's acceptance probabilities for different Sender's types. Comparing table 2 and 4, we conclude that the optimal privately IC mechanism gives Sender a strictly higher payoff. □

The following example shows that theorem 3.1 and corollary 5.1 may not hold without assumption 2. (The example satisfies assumption 3.) This is basically because, if assumption 2 fails, then the optimal IC mechanism needs not recommend acceptance on an interval even when $\mathcal{T}$ is a singleton.

t \ s	-4	3	4
H	$2/3$	0	$1/15$
M	$1/3$	1	0
L	0	0	$14/15$

Table 3: Optimal IC mechanism

$\rho(\cdot, t)$ \ s	-4	3	4
$\rho(\cdot, H)$	1	1	1
$\rho(\cdot, M)$	$1/3$	1	$14/15$
$\rho(\cdot, L)$	0	0	$14/15$

Table 4: $\rho(s, t)$ in optimal IC mechanism

Example 2. Assume that $\mathcal{S} = \{-2, -1, 1\}$ and $\mathcal{T} = \{L, H\}$. Receiver's payoff and Sender's payoff from accepting are $u(s, t) = s$ and $v(s, t) = 1$, respectively. Let Receiver's and Sender's beliefs be given by the following density functions:

Receiver's belief

	-2	-1	1
H	$1/10$	$2/10$	$2/10$
L	$4/12$	$1/12$	$1/12$

Sender's belief

	-2	-1	1
H	$8/20$	$4/20$	$4/20$
L	$2/20$	$1/20$	$1/20$

Receiver's belief satisfies the i.m.l.r. assumption, and Sender believes that Receiver's type and his own type are independent.

The unique optimal IC mechanism gives up on L and recommends that H accept if s is either -2 or 1, and reject if s is -1. Thus, the optimal IC mechanism does not recommend that H accept on an interval.

The unique optimal privately IC mechanism recommends that L accept if s is either -1 or 1, and reject if s is -2; it also recommends that H accept if s is either -2 or 1, and reject if s is -1.

□