The Value of Multistage Persuasion*

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Abstract

A sender persuades a receiver by disclosing information about a payoff-relevant state. The sender has full commitment, and decides how much information to provide on each day. The receiver chooses when to stop and what action to take if he stops. The receiver also has private information about his payoff from different actions. We characterize sufficient conditions under which the sender's optimal mechanism is static. We also present an example in which the sender benefits from multistage persuasion.

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1 Introduction

In this paper, we study the information design problem when a sender tries to influence the action of a receiver. The sender has full commitment power and designs a test on each day to reveal information about a payoff-relevant state. This state, also referred to as *the sender's type*, is drawn at the beginning. The receiver has private information about his payoff as

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well, referred to as *the receiver's type*. On each day, the receiver observes the sender's test and the realized signal. He decides when to stop, and what action to take when he stops. We study in which environment the sender's optimal disclosure mechanism is static.

The sender might benefit from dynamic mechanisms for the following intuitive reason. After the first test is done, some types of the receiver decide to stop and take some actions while other types may stay. If the receiver still stays, the sender can run a second test to further influence the receiver's action. However, the sender has to be careful when designing the second test. Given that the receiver is strategic instead of myopic, the sender has to make sure that those types who are supposed to stop after the first test have no incentives to wait for more information from the second one. In some environments, the incentive constraints for those types are so severe that the sender never finds it optimal to run a second test. In this paper, we identify sufficient conditions under which this is the case. We also present an example in which the sender strictly benefit from running more than one test.

This paper belongs to the literature on dynamic information design (e.g., Au (2015), Ely (2017), Renault, Solan and Vieille (2017), Orlov, Skrzypacz and Zryumov (2018)). Ely (2017) and Renault, Solan and Vieille (2017) study a long-run sender with full commitment persuading a sequence of short-run receivers (or a myopic receiver) who have no private information. Ely (2017) assumes that the sender's type changes at a random time, and Renault, Solan and Vieille (2017) assume that the sender's type follows a Markov chain. In our model, the sender's type is drawn at the beginning and is fully persistent. Our receiver has private information and is strategic instead of myopic. In Orlov, Skrzypacz and Zryumov (2018), the sender has incentive to engage in multiple tests since a payoff-relevant variable evolves exogenously and might trigger the stopping by the receiver. Their sender designs tests about another payoff-relevant variable in order to influence the receiver's stopping decision. In contrast to our full-commitment assumption, they assume that the sender cannot commit to future tests.

The most closely related paper is Au (2015). Au (2015) assumes that the sender's type and the receiver's type are independent. We examine environments where the two players' types are correlated. Our main motivation is that the receiver has access to external sources of information about the sender's type, and the receiver's type indicates how optimistic he is about the sender's type distribution.

Our paper is also related to the literature on dynamic pricing mechanism (e.g., Stokey (1979)). Stokey (1979) studies a fully-committed seller's optimal dynamic pricing mechanism when the buyer has private information about his willingness to pay. She shows that under regular conditions the optimal mechanism is a static one.

2 Model

Let $\mathcal{S}$ and $\mathcal{T}$ be finite sets of *Sender's types* and *Receiver's types*. Let $f \in \Delta(\mathcal{S} \times \mathcal{T})$ be the prior distribution. Let $\mathcal{A}$ be a finite set of *actions* and $u, v : \mathcal{S} \times \mathcal{T} \times \mathcal{A} \to \mathbf{R}$ be Sender's and Receiver's utility functions. We assume that the action set $\mathcal{A}$ includes an action 0 such that $u(s, t, 0) = v(s, t, 0) = 0$ for every s, t. Sender and Receiver have the same discount factor δ.

We consider an environment in which Sender has commitment power and he announces a sequence of signals whose joint distribution depends on his type. On any day Receiver may take an action, at which point the game ends and both players receive their discounted payoffs depending on players' types and the action. If Receiver never takes an action both players receive 0.

Definition 1. A *(dynamic disclosure) mechanism* is given by sets $\mathcal{X}_0, \mathcal{X}_1, \ldots$ of *signals* and a Markov kernel κ from $\mathcal{S}$ to $\mathcal{X} = \mathcal{X}_0 \times \mathcal{X}_1 \ldots$: When his type is s, Sender randomizes the daily signals $x_0, x_1, \ldots$ according to $\kappa(\cdot|s)$. The mechanism is *static* if $\mathcal{X}_1 = \mathcal{X}_2 = \ldots$ are singletons, so that all information is delivered on day 0.

The space $\mathcal{X}$ of infinite signal sequences is equipped with a filtration $\mathcal{F}_0 \subseteq \mathcal{F}_1 \subseteq \ldots$, where $\mathcal{F}_n$ is generated by the signals $(x_0, \ldots, x_n)$ up to day n.

A *Receiver's strategy* is given by a pair (τ, α) where $\tau : \mathcal{X} \to \mathtt{N} \cup \{\infty\}$ is a *stopping time*, meaning that the event $\{\tau = n\}$ is $\mathcal{F}_n$-measurable, and $\alpha : \mathcal{X} \to \mathcal{A}$, is $\mathcal{F}_\tau$-measurable, meaning that $\{\tau = n\} \cap \{\alpha = a\}$ is $\mathcal{F}_n$-measurable for every n. When Receiver of type t uses this strategy, his expected payoff (up to a normalization factor that depends on t) is given by

$$U_t(\kappa, \tau, \alpha) = \sum_s f(s,t) \int \delta^{\tau(x)} u(s,t,\alpha(x)) \; \kappa(\mathrm{d}x|s). \tag{1}$$

We say that (τ, α) is *type t's best-response to κ* if $(\tau, \alpha) \in \arg\max U_t(\kappa, \tau, \alpha)$. If Receiver uses strategy (τ_t, α_t) when his type is t, then Sender's payoff is given by

$$V(\kappa, \{\tau_t, \alpha_t\}_{t \in \mathcal{T}}) = \sum_{s,t} f(s,t) \int \delta^{\tau_t(x)} v(s,t,\alpha_t(x)) \; \kappa(\mathrm{d}x|s) \tag{2}$$

Sender's *(dynamic disclosure) problem* is

$$\text{maximize } V(\kappa, \{\tau_t, \alpha_t\}_{t \in \mathcal{T}})$$

over all mechanisms κ and all strategies (τ_t, α_t) such that (τ_t, α_t) is Receiver of type t's best response to κ. A mechanism κ that achieves the maximum is called an *optimal mechanism.*

We study in which environment Sender's optimal mechanism is static.

Remark 1. A careful reader might notice that our definition of a static mechanism is not exactly equivalent to the standard definition of a static mechanism. The difference is that in our definition Receiver is still allowed to delay his action and only play it on a later day (and thus discount his and Sender's payoffs) or not play at all (in which case both players get payoff 0). Of course Receiver will only delay his action if it gives him a weakly negative expected payoff given his information.

In all games considered in this paper, Receiver has an action 0 such that $u(s,t,0) =$

$v(s,t,0) = 0$ for every s, t. For such games, it is easy to verify that for every best response of Receiver there is another best response that always acts on day 0, and gives Sender a weakly higher payoff. For this reason we can use the definition of a static mechanism given above.[1]

3 Promotion games

In a *promotion game*, Receiver has two actions, accept and reject. Both players receive payoff 0 from rejection. Sender's payoff from accepting is 1, and Receiver's payoff from accepting depends only on Sender's type.

Assumption 1. *[i.m.l.r.] The sets $\mathcal{S}$ and $\mathcal{T}$ are linearly ordered, Receiver's payoff from accepting is weakly increasing in s, and the prior f is* increasing in monotone likelihood-ratio order*. For every $t' < t$, the ratio $f(s,t)/f(s,t')$ is weakly increasing in s.*

Theorem 3.1. *In a promotion game that satisfies Assumption 1, Sender can achieve his optimal payoff with a static mechanism.*

4 Proof of Theorem 3.1

Let us say that a *direct static mechanism* is given by a function $\rho : \mathcal{S} \times \mathcal{T} \to [0,1]$. The interpretation is that when Sender's type is s and Receiver's type is t, then with probability $\rho(s,t)$ the mechanism recommends that Receiver accept. When Receiver is truthful and obedient, then Sender's payoff is

$$\bar{V}(\rho) = \sum_{s,t} f(s,t)\rho(s,t). \tag{3}$$

[1]Suppose that Receiver has a unique action which gives himself a payoff of -1 and Sender a payoff of 1. In the standard definition of a static mechanism, Receiver takes this action on day 0. In our definition, Receiver will delay his action forever. Suppose now that Receiver has another action 0 which gives both players a payoff of 0. In our definition, Receiver now acts on day 0.

A direct static mechanism is *downward incentive-compatible (or downward IC)* if the following conditions hold:

1.
$$\sum_s f(s,t)\rho(s,t') \leqslant \sum_s f(s,t)\rho(s,t)$$
for every types $t' \leqslant t$.

2.
$$\sum_s f(s,\underline{\mathrm{t}})\rho(s,\underline{\mathrm{t}}) \geqslant 0,$$
where $\underline{\mathrm{t}}$ is the minimal element in $\mathcal{T}$.

The interpretation is that no type wants to mimic a lower type and that the lowest type prefers to follow his recommendation rather than to always reject.

It is a standard revelation principle argument that if κ is a static mechanism according to Definition 1 and $(\tau_t, \alpha_t)_{t\in\mathcal{T}}$ is a best-response then the direct mechanism *induced* by $(\kappa, \{(\tau_t, \alpha_t)_{t\in\mathcal{T}}\})$, which is given by $\rho(s,t) = \kappa(\{x : \alpha_t(x) = \text{accept}\}|s)$ is downward IC. This mechanism gives Sender a weakly higher payoff than κ (as it may be higher if Receiver delays his acceptance decision). Note that the downward IC condition allows Sender to give different information to different types of Receiver, whereas in Definition 1 all information provided by Sender is public. Nevertheless, we proved in Guo and Shmaya (2018) that the optimal downward IC mechanism is induced by a static IC mechanism and some best response of Receiver.

We now return to the proof of Theorem 3.1. Consider a dynamic mechanism κ and strategies (τ_t, α_t) such that (τ_t, α_t) is Receiver t's best response to κ. By (Guo and Shmaya, 2018, Theorem 6.1), to prove Theorem 3.1, it is sufficient to construct a downward-IC direct static mechanism which gives Sender the same payoff under a truthful and obedient Receiver.

Consider the direct static mechanism given by

$$\rho(s,t) = \int \delta^{\tau_t(x)} 1_{\{x:\alpha_t(x)=\text{accept}\}}\ \kappa(\mathrm{d}x|s). \tag{4}$$

We first note that Sender's payoff under ρ is the same as under κ when Receiver of type t plays $\{(\tau_t, \alpha_t)\}_{t\in\mathcal{T}}$. Indeed

$$\bar{V}(\rho) = \sum_{s,t} f(s,t)\rho(s,t) = \sum_{s,t} f(s,t) \int \delta^{\tau_t(x)} v(s,t,\alpha_t(x))\ \kappa(\mathrm{d}x|s) = V(\kappa, \{\tau_t, \alpha_t\}_{t\in\mathcal{T}})$$

where the first equality follows from (3), the second from (4) and the third from (2).

To complete the proof, we need show that the mechanism ρ is downward IC. Indeed,

$$\sum_{s,a} f(s,t)\rho(s,t') = U_t(\kappa, \tau_{t'}, \alpha_{t'}) \leqslant U_t(\kappa, \tau_t, \alpha_t) = \sum_{s,a} f(s,t)\rho(s,t),$$

The equalities follow from (4) and (1), and the inequality from the fact that (τ, α) is type t's best response to κ. The second condition in the definition of downward incentive-compatibility holds by a similar argument. ■

Remark 2. The proof of Theorem 3.1 relies only on the fact that the optimal downward-IC direct static mechanism is in fact IC. Therefore, the theorem extends to other environments where this property holds, such as Kolotilin et al. (2017) and Guo and Shmaya (2018). Note that both papers consider environments beyond promotion games.

5 A dynamic mechanism which is strictly better than static ones

The following example features a promotion game which does not satisfy Assumption 1. We show that Sender does strictly better in a dynamic mechanism than in the optimal static mechanism.

Example 1. Consider a promotion game with $\mathcal{S} = \{-2, -1, 1, 2\}$ and $\mathcal{T} = \{L, M, N\}$. Receiver's payoff from accepting is given by s. The discount factor is $\delta = 9/10$. The distribution f is:

	-2	-1	1	2
L	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
M	$\frac{3}{47}$	$\frac{6}{47}$	$\frac{5}{47}$	$\frac{5}{47}$
N	$\frac{2}{47}$	$\frac{1}{47}$	$\frac{1}{47}$	$\frac{1}{94}$

The optimal static mechanism use the signal set $\mathcal{X}_0 = \{\text{all accept}, L\&M \text{ accept}, \text{all reject}\}$. The Markov kernel κ from S to $\mathcal{X}_0$ is as follows:

	-2	-1	1	2
all accept	$\frac{1}{18}$	$\frac{7}{9}$	1	0
$L\&M$ accept	$\frac{8}{9}$	$\frac{2}{9}$	0	1
all reject	$\frac{1}{18}$	0	0	0

Type M and N are indifferent after the "all accept" signal. Type L is indifferent after the "$L\&M$ accept" signal. Sender's payoff is $\frac{2107}{2256}$.

Sender does better in the following dynamic mechanism. On day 0, Sender uses the same $\mathcal{X}_0$ as in the optimal static mechanism with the following mechanism:

	-2	-1	1	2
all accept	$\frac{1}{28}$	$\frac{1}{2}$	$\frac{9}{14}$	0
$L\&M$ accept	$\frac{13}{14}$	$\frac{1}{2}$	$\frac{5}{14}$	1
all reject	$\frac{1}{28}$	0	0	0

If the signal is "$L\&M$ accept" on day 0, then Sender uses the signal set $\mathcal{X}_1 = \{N \text{ accepts}, N \text{ rejects}\}$ with the following mechanism:

	-2	-1	1	2
N accepts	0	$\frac{5}{7}$	1	0
N rejects	1	$\frac{2}{7}$	0	1

At day 0, type M and N are indifferent after the "all accept" signal. On day 1, N is indifferent after the "N accepts" signal. After the "$L\&M$ accept" signal, type M is indifferent between accepting on day 0 and accepting on day 1 when N *rejects*. After the signal "$L\&M$ accept," type L's payoff on day 0 is zero. Also, his payoff on day 1 is zero after both the "N accepts" and "N rejects" signals. Sender's payoff is $\frac{9865}{10528}$, which is higher than his payoff in the optimal static mechanism. □

Remark 3. Our theorem 3.1 shows that the optimal dynamic mechanism consists of the optimal static mechanism on day 0, and never providing more information thereafter. Therefore, a sender who cannot commit to future tests does strictly worse than a static sender. We call a sender who cannot commit to future tests a *partially committed* sender.

Compared with a static sender, the advantage of a partially committed sender is that he can design multiple tests. The disadvantage is that he cannot commit to *not* disclosing more information in future. Our theorem 3.1 gives an environment where a static sender does strictly better than a partially committed sender. We suspect that there are environments where a partially committed sender does strictly better than a static sender.

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