# Private Learning and Exit Decisions in Collaboration

Preliminary, Comments welcome

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**Abstract**

We study a collaboration model in continuous time, with a positive arrival rate of a success in both the good and the bad state. If the project is bad, players may privately learn about it. At any time, players can choose whether to exit and secure the positive payoff of an outside option, or to stay with the project and exert costly effort. A player's effort not only increases the probability of success, but also serves as an investment in private learning.

We identify an equilibrium with three phases. In all phases, uninformed players exert positive effort. Players who become informed and learn that the project is bad never exert effort. Because players benefit from the effort of the others, informed players may not exit immediately. In the first, "no-exit" phase, informed players do not exit. In the subsequent, "gradual-exit" phase, they exit with a finite rate. In the final, "immediate-exit" phase, informed players exit immediately. We find that effort levels may increase in the no-exit phase, if the markup of effort in the bad state is positive. Surprisingly, increasing the payoff of the outside option encourages collaboration.

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# 1 Introduction

A wide range of team projects, from co-authorship to large-scale global corporate projects, exhibits the feature that team members have only very limited understanding of how hard it will be to complete the project. A priori the probability of a success is uncertain and collaborators may learn it over time. By working on the project, team members not only increase the probability of a success but also become more familiar with the project. They may privately learn about the challenge inherent in the project and hence find out that it has a low likelihood of success. How will an informed player use this information? Should he leave the partnership and opt for his outside option, or should he remain with the project and use the information to free-ride on his partner's effort? How does this affect the motivation of an uninformed team member to exert effort?

The situation described here is a problem that researchers commonly face in joint projects, entrepreneurs face when collecting funds to secure the survival of their joint venture, and companies face in product development or in the implementation of new software. For example, when implementing new software like SAP or Oracle, IT-consultants team up with experts in the company in order to adjust the software to the company's needs. At the beginning of such a large-scale project, there is uncertainty about its success rate. Throughout the course of the project, team members may learn about the challenges. For example, they may find out about compatibility issues or "white space risk"—some required activities that were not identified in advance. Privately observing such a bad signal reveals to them that the project has a low success rate. Still, even though keeping workers engaged in a project is costly, there are numerous examples of project managers holding on to projects even though various signs point to likely failure. Similarly, at the start of a research project, there is uncertainty about its success rate. By working on the project, co-authors increase the probability that a success will arrive, but may also discover, for example, tractability issues. This would reveal that the success rate of the project is low. An informed co-author now has the option to quit the project—thereby revealing his information—or to stay with the project and shirk. Some fixed costs are associated with staying with the project, such as e-mail correspondence and revising the paper.

In this paper, we are interested in understanding this kind of team problems. Specifically, we analyze the implications that arise from the new feature that players may privately learn about the state and can choose whether to disclose this information by exiting or not.

We consider a two-person team problem in which players can exert costly effort in order to increase the probability of a success. A success arrives according to a Poisson process and rewards both team members with a lump-sum payoff. The success rate depends on the total effort exerted by both players, and on the state of the project, which can be good or bad. The model has the following features: (i) If players exert effort, then in both states, there is a positive probability of success. The arrival rate of success is higher in the good state than in the bad state. Hence, a success is nonconclusive. (ii) If the state is bad, players who exert effort may observe a private, fully-revealing signal (a *bad-state-revealing signal*). Such a signal is conclusive but private. (iii) Players have a positive outside option and can exit. Exits are public and irreversible.

Notice that efforts serve a dual purpose. On the one hand, they are a contribution to the joint task and increase the probability of a success. On the other hand, they are an investment in private learning. Exerting effort increases the probability of observing a private signal, which opens up the option to free-ride. Specifically, a player who learns that the state is bad has two options: He can stop exerting effort but remain with the project, hoping that the other player' effort will result in a success. Alternatively, he can choose to exit and secure the positive payoff from the outside option.

Consider an informed player who has learned that the state is bad. Assume that for him it is not profitable anymore to actively engage in the project and to exert costly effort. Still, it is not obvious that an informed player's best option is to quit the project. If he quits, he secures the positive payoff of his outside option. On the other hand, an informed player may want to remain with the project, hoping that his collaborator's effort will eventually result in a success. An informed player has an incentive to free-ride in this way. Consider now an uninformed player, who is uncertain about whether his opponent is informed about the state. This uncertainty affects the uninformed player's incentive to put forth effort. In this paper, we analyze how an informed player's exit decision and an uninformed player's effort choice affect each other.

Our model is an inconclusive good-news model. Hence, if no success arrives, then players become more pessimistic about the state being good and hence about the arrival rate of a success being high. However, the bad-state-revealing signal creates a countervailing effect. If a player does not observe a bad-state-revealing signal, he becomes more optimistic about the state being good. In the analysis, we focus on the parameter region in which a single player's belief that the state is good is weakly decreasing if no success or signal arrives.[1]

We start by analyzing the special case in which a single player's belief of the good state stays constant if no success or bad-state-revealing signal arrives. We call this the *stationary case*. In this case, we identify a symmetric equilibrium which consists of two phases. The first is a *no-exit* phase, in which an informed player, who has observed a signal, does not exit. Instead, he remains with the project, exerts no effort, and free-rides on the effort exerted by

[1] This seems to be the natural assumption in the applications that we have in mind.

his opponent. Throughout this phase, both players get more and more pessimistic that their opponent is still uninformed and hence exerting effort. As time passes and no success arrives, the risk to an informed player of finding himself in an inactive project eventually becomes so high that an informed player exits with a positive probability. At this transition time, equilibrium play enters the second, *gradual-exit* phase. In this gradual-exit phase, the beliefs and the effort level of an uninformed player are constant, and an informed player exits at a constant, finite rate. If a player exits, this reveals to his opponent that the state is bad; the opponent then also exits immediately.

In the gradual-exit phase, the positive exit rate of an informed player helps to balance beliefs. In the absence of a success or a signal, an uninformed player gets more pessimistic about whether his opponent is still exerting effort. But now, the failure of the other player to exit is good news, indicating that the other player may still exert effort and the state may be good. This creates an encouragement effect: uninformed players are encouraged to keep exerting effort at a constant rate. For an informed player, knowing that his opponent exits with positive probability if he is informed makes it less risky to remain with the project and to free-ride. However, staying with the project is only attractive if the uninformed player is sufficiently optimistic, and hence exerts enough effort. In equilibrium, the exit rate and the effort level are such that an uninformed player is indifferent between exerting a bit more effort today or tomorrow, and an informed player is indifferent between staying with the project and exiting.

In the general, nonstationary case, the arrival rates are such that a single player gets more pessimistic about the state being good if no success or bad-state revealing signal arrives. In this case, we identify an equilibrium which consists of three phases, a *no-exit*, a *gradual-exit*, and an *immediate-exit* phase. The first two phases are parallel to the stationary case, with the difference that in the gradual-exit phase the effort level, exit rate, and beliefs are not constant. Instead, in the gradual-exit phase the belief that the state is good now decreases over time as more effort is put into the project. The equilibrium effort of uninformed players decreases. Hence, it becomes less attractive for informed players to stay with the project, and so the exit rate of informed players increases. As a consequence, if players observe no exit, their belief that the opponent is uninformed increases over time. At the transition time, the exit rate goes to infinity, and for players who are still with the project, the belief that their opponent is uninformed goes to one.

The equilibrium play then enters the immediate-exit phase, in which an informed player exits immediately. Hence, if a player observes that his opponent does not exit, he knows for sure that his opponent is uninformed. The situation is as if signals were public. Uninformed players exert positive effort, but the belief about the state being good and the effort level decrease over time. After some finite time, uninformed players are so pessimistic that they do not want to remain with the project, and both players exit.

There are two sources of inefficiencies in the present setting. Our setup is a team problem with moral hazard. Hence, it is known that players have incentives to reduce and postpone efforts.[2] The second inefficiency, *delayed information transmission*, arises from the new features in our model. A privately informed player has the incentive to delay exiting and to free-ride on the other player's effort.

The identified equilibrium exhibits various novel properties. In the no-exit phase, the effort level may be decreasing or increasing. This is in stark contrast to the findings in the previous literature, in which effort levels typically decrease as players become more pessimistic. Our model shares with the previous literature the feature that players have the incentive to procrastinate. If a player postpones exerting a bit more effort until tomorrow, then the effort exerted by his opponent today may yield a success or the opponent may exit. In both cases, this player saves the effort he had postponed. This creates an incentive to procrastinate. However, during the no-exit phase, an uninformed knows that his opponent, if informed, does not exit. Therefore, this uninformed player does not expect to learn from observing whether or not his opponent exits. At the same time, an uninformed player knows that it becomes more and more likely that his opponent is informed and exerts no effort. This further diminishes an uninformed player's incentive to procrastinate. Instead players may wish to compensate for the lack of effort of their informed opponents. Under certain parameters, the effort level during the no-exit phase increases.

At the transition point between the no-exit and the gradual-exit phases, the uninformed player's effort level drops discontinuously. Intuitively, if an informed player exits with a positive probability, an uninformed player has more incentive to postpone his effort in order to learn from the potential exit of his opponent. Hence, at the threshold time, effort levels must drop.

Finally, we find that increasing the payoff of the outside option, and hence making it more attractive for a player to leave the project, encourages collaboration. More specifically, increasing the payoff of the outside option diminishes both inefficiencies, procrastination and delayed information transmission. The ratio of the equilibrium payoff over the cooperative payoff is increasing in the outside option. This may be surprising at first, since making it more attractive for players to switch to the outside option will reduce players' incentives to remain with the project—an effect that is detrimental to a partnership. However, within the partnership, players have an incentive to procrastinate and also to delay revealing their private information that the state is bad. Increasing the payoff of the outside option dimin-

[2]This "procrastination" effect was identified and discussed in Bonatti and Hörner (2011).

ishes these two effects and leads to a better alignment of players' incentives. For sufficiently high payoffs of the outside option, the equilibrium payoff equals the cooperative payoff. Uninformed players exert full effort, and informed players exit immediately.

**Related Literature** Our paper contributes to the nascent literature on private learning in experimentation models. Some recent, related papers are Akcigit and Liu (2015a), Das (2014), and Bimpikis and Drakopoulos (2014). Akcigit and Liu (2015a) examine an innovation competition between two firms which decide whether to pursue a risky or a safe project. Only the first success of a project is rewarded. The risky project may be a success or a dead end, and firms may privately find out about dead ends. Since a firm benefits when its competitor works in a less rewarding direction, it never reveals dead-end findings—competition suppresses information sharing. By contrast, in our model information sharing may be delayed since an informed player has an incentive to free-ride on his opponent's effort. Das (2014) examines a situation in which two players can work on a risky project or a safe project, and only the first player who obtains a public success is rewarded. If the state is good, then in addition to a public success, the risky project may also generate private good news, which encourages an informed player to stay with the risky option forever. Depending on the prior, players experiment either too much or not enough. [3]

Bimpikis and Drakopoulos (2014) study a strategic experimentation model in which players' actions are private. Information generated through experimentation is private, but can be credibly disclosed. They show that efficiency is improved if all players commit to share no information up to a time and to fully disclose all available information at that time. Unlike our paper, their setting involves information externality only and no payoff externality. Heidhues et al. (2015) study a strategic experimentation game with observable actions and private payoffs. They show that private payoffs can diminish the free-rider problem, and identify cases in which the cooperative solution can be supported as a perfect Bayesian equilibrium.

Campbell et al. (2014) study a partnership in which players work on a joint project with a deadline and have private information about the success of their efforts. In equilibrium, players initially reveal their information but exert inefficiently low effort. As the deadline draws closer, players hide their information about successes to encourage their partners to work more. They show that private information about successes benefits welfare, compared to the case in which successes are public.

Our model also ties into the literature on dynamic games with exit options. McAdams

[3] Bergemann and Hege (2005) study agency problems regarding the timing of the termination of funding for R&D projects with uncertainty about the probability of success. They find that in equilibrium funding stops inefficiently early.

(2011) analyzes stochastic partnerships in which players can either stay with the current partner, or exit and get anonymously rematched. Players' actions are publicly observed; stage game payoffs vary stochastically and are common knowledge. McAdams (2011) shows that performance inside the partnership decreases with the attractiveness of players' outside options. By contrast, in our model we obtain the opposite effect: increasing the attractiveness of the outside option encourages collaboration within the partnership. Moscarini and Squintani (2010) study an R&D, winner-takes-all setting, in which players hold private information about the arrival rate of success. Staying in the race is costly, but players can choose to publicly exit. Players learn from exit decisions of their competitors, and the equilibrium exhibits a strong "herding" effect. Even if players differ strongly in their costs and benefits, they may exit at almost the same time. This is attributed to the survivor's curse: at any time in the game, a player is more optimistic about the state and his opponent's information than if he knew that his opponent would exit in the next instant. Murto and Välimäki (2011) examine information aggregation in an exit game in which players are uncertain about their payoff types, and their types are correlated.[4] Good types should stay in the game whereas bad types are better off exiting. By staying with the project, good-type players may privately learn about their type. They show that information aggregates in randomly occurring exit waves.

More broadly, this paper is related to the literature on experimentation. (See, for instance, Bolton and Harris (1999), Keller et al. (2005), and Bonatti and Hörner (2011)). Our model is based on the collaboration model of Bonatti and Hörner (2011). They analyze moral hazard in teams, and show that the incentive to free-ride on other players' efforts leads to reduction of effort and procrastination. Their model is incorporated as a special case in our setting, in which the payoff of the outside option and the arrival rates of a private signal or a success in the bad state are all zero. As in Keller and Rady (2010), and the related bad-news model Keller and Rady (2015), we assume that the arrival rate of a success is positive in both states.

# 2 The Model

There are two players, $i \in \{1, 2\}$, engaged in a common project. Time is continuous with infinite horizon, $t \in [0, \infty)$. At each instant $t$, a player first decides whether to remain engaged in the project, or to exit the project and take the outside option with (flow-)payoff $U > 0$. A player's exit decision is publicly observable.[5] Once a player exits the project, he cannot

[4]A related exit-game models with common values and private learning is studied in Rosenberg et al. (2007).

[5]The outside option can be interpreted as the expected payoff from starting a new project, or as the opportunity cost associated with staying with the project that can be avoided by quitting.

return to it. If a player decides to stay with the project, he chooses at which level to exert effort, $k_i(t) \in [0,1]$. Effort is costly, and the instantaneous cost to player $i$ of exerting effort $k_i(t)$ is $ck_i(t)$. The effort choice is, and remains, unobserved.

The probability of successfully completing the project depends on the players' efforts, and on an unknown binary state which is either good $g$ or bad $b$. Both players share a common prior belief $p_0 \in (0,1)$ of the state being good. At any time $t$ the instantaneous probability of success depends on players' efforts $\{k_1(t), k_2(t)\}$ and the state. If the state is good, the arrival rate of a success is $\lambda_h(k_1(t)+k_2(t))$; if the state is bad, the arrival rate of a success is $\lambda_\ell(k_1(t)+k_2(t))$, with $\lambda_h > \lambda_\ell > 0$. The arrival of a success is public, and a success is worth a net value of $h > 0$ to each of the players. As long as no success occurs, players reap no benefits from the project. The project generates at most one success. We assume that, for an individual player, exerting effort is ex-ante productive if and only if the state is good, $h\lambda_\ell < c < h\lambda_h$. Throughout the paper, we assume that the prior belief is high enough, such that a priori efforts at time 0 are productive, that is, $h\left(p_0\lambda_h + (1-p_0)\lambda_\ell\right) - c \geq 0$.

**Assumption 1** (Productive Efforts)**.** *At time* 0*, efforts are productive, that is, the prior belief satisfies*

$$p_0 \geq \frac{c - h\lambda_\ell}{h(\lambda_h - \lambda_\ell)}.$$

If the state is bad, and player $i$ exerts effort $k_i(t)$ at time $t$, then player $i$ may receive a private signal with instantaneous probability equal to $\beta k_i(t)$, with $\beta \geq 0$. In the good state such a signal is never realized, and hence the arrival of the signal reveals that the state is bad. We call the signal a *bad-state-revealing signal*.[6] Moreover, we say that a player who knows that the state is bad is *informed*, while a player who is uncertain about the state is *uninformed*.

Players discount future benefits and costs at a common discount rate $r$. If players exert effort $\{k_1(t), k_2(t)\}_{t\geq 0}$ and player $i$ exits at time $\tau \leq \infty$ before a success occurs, the normalized discounted payoff to player $i$ is

$$-r\int_0^\tau e^{-rs}ck_i(s)\mathrm{d}s + e^{-r\tau}U.$$

If a success occurs at time $t$ before player $i$ exits, player $i$'s payoff is

$$-r\int_0^t e^{-rs}ck_i(s)\mathrm{d}s + e^{-rt}(rh + U).$$

[6]For simplicity, we assume that a player obtains at most one private signal. Since the first signal fully reveals that the state is bad, this assumption does not affect our results.

We assume that every player takes the outside option immediately after a success occurs.[7] The player's objective is to maximize his expected payoff by choosing the optimal effort level and time to exit.

In our model, if a player exits, the information set of the other player changes. Similarly, if a player observes a private bad-state-revealing signal, his beliefs about the state and about the other player's information and past actions change. Hence, a player may want to react immediately to a bad-state-revealing signal or to another player's exit decision. It is well known that this may create modeling issues regarding the timing of events in continuous time models. To circumvent this problem, we adopt an approach similar to the one in Murto and Välimäki (2013) and Akcigit and Liu (2015a), and model the game as a stage game with a random number of stages. In our model, we have to keep track of public and private histories. At any time $t$, the public history $h_{p,t}$ captures whether and when a player has exited or a success has arrived. Player $i$'s private history $\hat{h}_{i,t}$ consists of his past efforts and whether and when he has observed a private signal. For player $i$, the history at time $t$ consists of both the public and his private history and is denoted $h_{i,t} = \left(h_{p,t}, \hat{h}_{i,t}\right)$.

We describe the stage game from the perspective of player $i$. The game begins with *Stage Null*, in which no success or private signal for player $i$ has yet arrived, and player $i$'s opponent has not yet exited. We let $\mathcal{H}^0_{i,t}$ be the set of player $i$'s possible histories at time $t$ in Stage Null. An element $h^0_{i,t}$ in $\mathcal{H}^0_{i,t}$ consists of player $i$'s effort level before time $t$, that is, $h^0_{i,t} = \{k_i(s)\}_{s \leq t}$. Player $i$ chooses an effort level and an exit rate as a function of time and history. More specifically, player $i$ chooses the measurable functions $k_i : \mathbb{R}_+ \times \mathcal{H}^0_{i,t} \to [0, 1]$, and $f_i : \mathbb{R}_+ \times \mathcal{H}^0_{i,t} \to [0, \infty]$, where $k_i(t, h^0_{i,t})$ is the instantaneous effort of player $i$ at time $t$, and $f_i(t, h^0_{i,t})$ is the instantaneous exit rate of player $i$ at time $t$, conditional on no success, signal, or exit having occurred. The game proceeds to the next stage if (i) player $i$ obtains a private signal, or (ii) player $j$ exits.[8] In each of these events, player $i$ updates his beliefs and immediately enters the next stage. Upon observing a private signal, player $i$ becomes informed and immediately enters *Stage Informed*. This transition does not affect the public history, so at any time $t$, $h^I_{p,t} = h^0_{p,t}$. In Stage Informed, the private history $\hat{h}^I_{i,t} = \left\{\{k_i(s)\}_{s \leq t}, \tau^I\right\}$ consists of past efforts and the transition time $\tau^I$ at which the private signal arrived. Upon observing an exit of player $j$, player $i$ immediately enters *Stage Exit*. In this case, the public history changes and is now given by the transition time, $h^E_{p,t} = \left\{\tau^E\right\}$, the time at which player $j$ has exited.

In every stage, a player chooses his effort level and exit rate as a function of history. A

[7]This is without loss of generality. Since the project can generate at most one success, it is dominant to take the outside option after the success occurs.

[8]It is possible that player $i$'s Stage Null ends because player $i$ himself has exited. In this case, there is no need to specify his strategy afterwards.

pure behavioral strategy for player $i$ in Stage $m$ is a tuple of measurable effort and exit rate functions:

$$k_i : \mathbb{R}_+ \times \mathcal{H}^m_{i,t} \to [0,1]$$
$$f_i : \mathbb{R}_+ \times \mathcal{H}^m_{i,t} \to [0,\infty]\,,$$

where $\mathcal{H}^m_{i,t}$ is the set of player $i$'s possible histories in Stage $m$ at time $t$. A strategy $(k_i, f_i)$ on the set of all histories $\mathcal{H}_{i,t} = \cup_m \mathcal{H}^m_{i,t}$ is defined by

$$k_i(t, h_{i,t}) = k_i(t, h^m_{i,t}) \text{ and } f_i(t, h_{i,t}) = f_i(t, h^m_{i,t}), \text{ whenever } h_{i,t} \in \mathcal{H}^m_{i,t}.$$

The further evolution of stages and public and private histories follows the same pattern. From Stage Exit, the game proceeds to the next stage, (Exit, Informed), if player $i$ obtains a private signal. From Stage Informed, the game proceeds to the next stage, (Informed, Exit), if player $j$ exits. The evolution of stages is illustrated in Figure 1. It should be noted that transitions induced by private signals lead to private stages. For example, the Stage Null and Stage Informed of player $i$ are indistinguishable for player $j$, and hence are private stages for player $i$.[9]

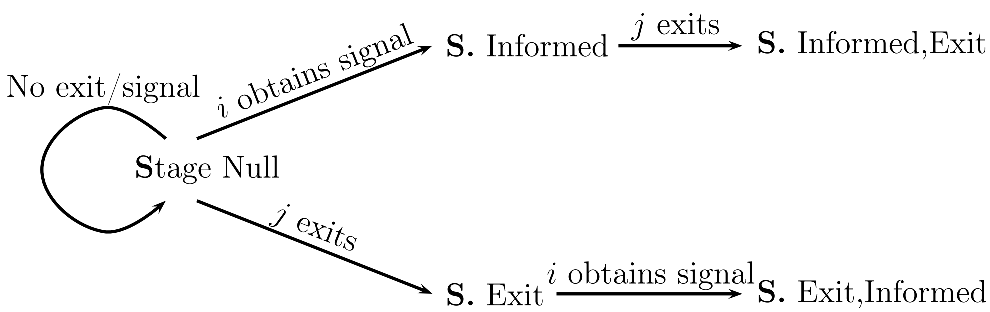


Figure 1: Stages of the game for player $i$.

The equilibrium concept is perfect Bayesian equilibrium. We focus on symmetric equilibria. Any strategy profile of effort and exit levels induces public and private beliefs of the players (Bayesian updating). A strategy profile $\{(k_i(t, h_{i,t}), f_i(t, h_{i,t})\}_{t\geq 0,\, i\in\{1,2\}}$ is a PBE of the game if (i) beliefs are consistent, and (ii) for all $i$ and all $(t, h_{i,t})$, $(k_i(t, h_{i,t}), f_i(t, h_{i,t}))$ is a best-response to player $j$'s strategy. In most of the paper, within each stage we focus on (pure) Markov strategies that depend only on the player's (public and private) beliefs.

Throughout most of the paper, it will be clear from the context in which stage players

[9]Similarly, Stages (Informed, Exit), Exit, and (Exit, Informed) are private stages of player $i$. The concept of private stages is also used in Akcigit and Liu (2015a).

are. Hence, by a slight abuse of notation, we will use $k_i(t)$ and $f_i(t)$ as the effort level of an uninformed player and the exit rate of an informed player, respectively.[10]

# 3 Single-Player Solution and Cooperative Solution

We first analyze two benchmark cases: the optimal policy for a single player, and the optimal cooperative strategy profile.

## 3.1 Single Player's Optimal Policy

By standard results, the single player's problem is Markovian with respect to his posterior belief of state $g$. If a player exerts effort $k_i(t)$ over the interval $[t, t+\mathrm{d}t)$, and observes neither a success nor a signal, then, by Bayes' rule, his posterior belief of state $g$ at $t+\mathrm{d}t$ is

$$p_{t+\mathrm{d}t} = \frac{p_t e^{-k_i(t)\lambda_h \mathrm{d}t}}{p_t e^{-k_i(t)\lambda_h \mathrm{d}t} + (1-p_t)e^{-k_i(t)(\lambda_\ell+\beta)\mathrm{d}t}}.$$

It is easy to see that the belief of state $g$ stays constant if $\beta = \lambda_h - \lambda_\ell$. In this case, the lack of a private signal offsets the lack of a success, and the belief of state $g$ stays constant as long as no success or signal arrives. We call this special case the *stationary case*. In the *general case*, we assume that if no success or signal arrives, the player gets more pessimistic, $\lambda_h - \lambda_\ell > \beta$.

For a single player, if a signal arrives and he learns that the state is bad, it is optimal to take the outside option, given our assumption that $\lambda_\ell h < c$. Given that a project generates at most one success, he also exits immediately after the arrival of a success. If no success or signal has arrived yet, a player's optimal action depends on his belief. When the belief $p_t$ that the state is good is high enough, the instantaneous payoff from exerting full effort $h\left(p_t\lambda_h + (1-p_t)\lambda_\ell\right) - c$ is higher than the payoff of the outside option $U$. Hence, it is optimal for the player to exert full effort. This is the case if and only if the belief $p_t$ is above the single-player belief threshold given by[11]

$$p^{s,*} := \frac{c - h\lambda_\ell + U}{h(\lambda_h - \lambda_\ell)}.$$

If the belief is below $p^{s,*}$, it is optimal for a single player to take the outside option.[12]

Note that the threshold belief $p^{s,*}$ does not depend on $\beta$, and it is the same for the stationary and nonstationary cases. However, in the stationary case, a single player's beliefs

[10] In equilibrium, informed players do not exert effort and uninformed players do not exit before a final time, at which time all players exit and the game ends.

[11] A formal discussion is provided in Appendix B.

[12] If the prior belief is below $p^{s,*}$, a single player takes the outside option at time 0.

do not change as long as no success or private signal arrives. Hence, if the prior belief is above $p^{s,*}$, the player stays with the project and exerts full effort until a success or a bad-state-revealing signal arrives.

## 3.2 Cooperative Solution

Suppose that the two players work cooperatively to maximize the average expected payoff by jointly choosing the strategy profile. In this case, players internalize the effect of their effort on the other player's payoff. Hence, given belief $p_t$ that the state is good, the flow payoff for an individual player $i$ from exerting effort $k_i$ is $[2h\,(p_t\lambda_h + (1-p_t)\lambda_\ell) - c]\,k_i$. A success generates a payoff of $h$ to both players, and player $i$ incurs cost $c$ per unit of effort. If $2\lambda_\ell h - c \geq U$, then the flow payoff (per player) from staying with the project and exerting full effort is higher than the outside option, even if the state is bad. In this case, the optimal cooperative strategy is for both players to exert full effort until they obtain a success.

For the rest of this subsection, we focus on the case with $2\lambda_\ell h - c < U$, in which it is optimal for the players to take the outside option if they learn that the state is bad. If at least one player is informed, the optimal continuation play is for the informed player to exit immediately, and for his opponent to follow. Hence, if no player has yet exited, this means that no bad-state-revealing signal has yet arrived. Players always share a common belief, and the motion of beliefs is given by

$$p_{t+\mathrm{d}t} = \frac{p_t e^{-2(k_i(t)+k_j(t))\lambda_h \mathrm{d}t}}{p_t e^{-2(k_i(t)+k_j(t))\lambda_h \mathrm{d}t} + (1-p_t)e^{-2(k_i(t)+k_j(t))(\lambda_\ell+\beta)\mathrm{d}t}}. \tag{1}$$

If the flow payoff for an individual player from exerting full effort is higher than the outside option, it is optimal for both players to exert full effort. This requires the belief of state $g$ to be sufficiently high. For lower beliefs, both players take the outside option.

**Proposition 1** (Cooperative Solution).
*In the cooperative problem, the belief of the good state evolves according to* (1). *There exists a cooperative threshold*

$$p^{c,*} := \frac{c - 2h\lambda_\ell + U}{2h(\lambda_h - \lambda_\ell)},$$

*such that whenever the belief is above this threshold, it is optimal for both players to exert full effort. If the belief is below* $p^{c,*}$*, it is optimal for both players to take the outside option.*

In the cooperative game, if the players decide to continue with the project, each player exerts full effort, and exits immediately if a signal is obtained. No player procrastinates in putting forth effort, and information transmission is not delayed. When $p_0 \leq p^{c,*}$, the flow payoff per player when both players exert full effort is less than $U$, and each player takes the outside option at time 0. Notice that the cooperative belief is lower than the single player's threshold, and does not depend on $\beta$.

# 4 Stationary Case: $\beta = \lambda_h - \lambda_\ell$

In this section, we identify a symmetric equilibrium for the stationary case in which $\beta = \lambda_h - \lambda_\ell$. For a single player, the nonarrival of a bad-state-revealing signal offsets the nonarrival of a success. A single player's belief of state $g$ does not change as long as no success or signal arrives.

When a player obtains a signal, he learns that the state is bad. In the bad state, the flow payoff from exerting effort is negative, i.e., $\lambda_\ell h - c < 0$. From an informed player's perspective, the effort input is not longer profitable, and hence it is optimal for him to stop exerting effort. However, it is unclear whether an informed player should take his outside option immediately. Instead, he may want to remain with the project, in the hope that his opponent is not informed yet, and hence is still exerting sufficiently high effort. More specifically, the flow payoff of an informed player from staying with the project is proportional to the product of (i) the probability that his opponent is exerting effort and (ii) his opponent's effort level. If this flow payoff is strictly higher than $U$, an informed player strictly prefers to stay with the project.

The highest flow payoff that an informed player can obtain from staying with the project is $\lambda_\ell h$. This is the payoff rate in the case that the informed player's opponent exerts full effort with probability one. If the outside option $U$ is higher than $\lambda_\ell h$, then it is a dominant strategy for an informed player to take the outside option immediately after he obtains a private signal. By exiting, the informed player then reveals that the state is bad. His opponent optimally follows suit and exits as well. This is as if the private signal had been publicly observed. If $U > \lambda_\ell h$, both informed players exit immediately. We discuss the details of this case in Subsection 4.2. First, we examine the case where $U < \lambda_\ell h$, in which an informed player may want to delay his exit.

## 4.1 Two-phase Equilibrium when $U < \lambda_\ell h$

In this section, we analyze the case where $U < \lambda_\ell h$. We present a symmetric equilibrium, which consists of two phases: the *no-exit* and the *gradual-exit* phase. The structure of this equilibrium is illustrated in Figure 2. In both phases, uninformed players do not exit and they exert positive effort. The uninformed player chooses his effort level such that his opponent has no incentive to either postpone or expedite his effort.

In the first, no-exit phase, an informed player knows that the state is bad. However, his belief that his opponent is still uninformed and hence is exerting effort is high enough that

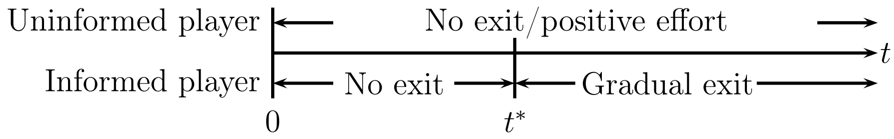


Figure 2: The two-phase equilibrium for the stationary case

the expected payoff from staying with the project is higher than the payoff from the outside option. Hence, an informed player wants to stay with the project and free-ride on the effort expected from his opponent instead of switching to the outside option.

Over time, players become more and more pessimistic about their opponent still being uninformed. For an informed player, it becomes more likely that the other player is also informed and hence the project has reached a deadlock. For an uninformed player, it becomes more likely that the state is bad and his opponent is free-riding. At some threshold time $t^* \in [0, \infty)$, equilibrium play enters the second, gradual-exit phase. In the gradual-exit phase, an informed player is indifferent between staying and exiting, and exits at a constant rate. Hence, observing that the opponent has not exited is good news and encourages uninformed players to keep exerting effort.

Throughout, we use superscripts $N, G$ to represent the no-exit and the gradual-exit phases, respectively. We now discuss the equilibrium behavior, based on heuristic arguments. The proofs are relegated to the appendix.

**No-exit phase:** In the no-exit phase, no player exits on the equilibrium path. The relevant probabilities that we have to keep track of are the uninformed player $i$'s posterior beliefs at any time $t \in [0, \infty)$ that (i) the state is good, (ii) the state is bad and player $j$ is informed, and (iii) the state is bad and player $j$ is uninformed. All of these beliefs are conditional on no success having arrived yet. We denote these beliefs by $p^g(t)$, $p^{bi}(t)$, $p^{bu}(t)$, respectively.

As discussed before, given that the flow payoff from exerting effort is negative in the bad state, an informed player never exerts effort. An informed player decides whether to exit or not, depending on (i) the effort level exerted by an uninformed player, and (ii) the probability that his opponent is uninformed conditional on the state being bad, which is

$$q^u(t) := \frac{p^{bu}(t)}{p^{bi}(t) + p^{bu}(t)}. \tag{2}$$

In the no-exit phase, informed players stay with the project and do not exit. Hence, for an informed player $i$, it must be the case that the flow payoff from staying with the project is (weakly) higher than the payoff from the outside option,

$$q^u(t) k_j(t) \lambda_\ell h \geq U.$$

An uninformed player assigns the same probability $q^u(t)$ to the event that his opponent is uninformed, conditional on the state being bad. Another conditional probability that is relevant for an uninformed player's decisions is the probability that the state is good, conditional on both player being uninformed:

$$q^g(t) := \frac{p^g(t)}{p^g(t) + p^{bu}(t)}. \tag{3}$$

If uninformed players $i, j$ exert effort $(k_i, k_j)$ over the interval $[t,\, t + \mathrm{d}t)$, then by Bayes' rule, conditional on no success, the uninformed player $i$'s posterior beliefs at time $t + \mathrm{d}t$ are

$$\begin{aligned} p^g(t + \mathrm{d}t) &= \frac{p^g(t)e^{-\lambda_h(k_i+k_j)\mathrm{d}t}}{p^g(t)e^{-\lambda_h(k_i+k_j)\mathrm{d}t} + p^{bi}(t)e^{-(\beta+\lambda_\ell)k_i\mathrm{d}t} + p^{bu}(t)e^{-(\beta k_i+\lambda_\ell(k_i+k_j))\mathrm{d}t}}, \\ p^{bi}(t + \mathrm{d}t) &= \frac{p^{bi}(t)e^{-(\beta+\lambda_\ell)k_i\mathrm{d}t} + p^{bu}(t)\left(1 - e^{-\beta k_j\mathrm{d}t}\right)e^{-(\beta k_i+\lambda_\ell(k_i+k_j))\mathrm{d}t}}{p^g(t)e^{-\lambda_h(k_i+k_j)\mathrm{d}t} + p^{bi}(t)e^{-(\beta+\lambda_\ell)k_i\mathrm{d}t} + p^{bu}_t e^{-(\beta k_i+\lambda_\ell(k_i+k_j))\mathrm{d}t}}, \\ p^{bu}(t + \mathrm{d}t) &= \frac{p^{bu}(t)e^{-(\beta+\lambda_\ell)(k_i+k_j)\mathrm{d}t}}{p^g(t)e^{-\lambda_h(k_i+k_j)\mathrm{d}t} + p^{bi}(t)e^{-(\beta+\lambda_\ell)k_i\mathrm{d}t} + p^{bu}(t)e^{-(\beta k_i+\lambda_\ell(k_i+k_j))\mathrm{d}t}}. \end{aligned} \tag{4}$$

Note that player $i$'s belief $q^g(t)$ that the state is good, conditional on neither player being informed, stays constant, due to the assumption that $\beta = \lambda_h - \lambda_\ell$. It is always equal to $p_0$. As a result, there is only one degree of freedom for the beliefs $p^g, p^{bi}, p^{bu}$.

An uninformed player decides at any instant how much effort to exert. For ease of exposition, we define the following arrival intensities as functions of the beliefs $p^g, p^{bi}, p^{bu}$:

$$\begin{aligned} \lambda^s(p^g) &:= p^g\lambda_h + (1 - p^g)\lambda_\ell \\ \lambda^{s,I}(p^g) &:= \lambda^s(p^g) + (1 - p^g)\beta \\ \lambda^U(p^g, p^{bu}) &:= p^g\lambda_h + p^{bu}\lambda_\ell. \end{aligned} \tag{5}$$

Here, $\lambda^s(p^g)$ is the intensity of an instantaneous success generated by player $i$'s own effort, and $\lambda^{s,I}(p^g)$ is the intensity of an instantaneous success or signal generated by player $i$'s own effort. Moreover, $\lambda^U(p^g, p^{bu})$ is the intensity of an instantaneous success generated by player $j$'s effort, given that player $j$ exerts effort only if he is uninformed.

In equilibrium, an uninformed player has no incentive to either postpone or advance efforts. Consider some time $t$ and suppose that an uninformed player $i$ exerts effort $k_i$ over the interval $[t,\, t + \mathrm{d}t)$ (today) and effort $k_i'$ over the interval $[t + \mathrm{d}t,\, t + 2\,\mathrm{d}t)$ (tomorrow). Now, consider the effect if player $i$ decreases his effort today by $\varepsilon$ and increases his effort tomorrow by the same amount. Note that, conditional on reaching $t + 2\mathrm{d}t$ without a success or a signal, the resulting beliefs are unchanged, and therefore so is the continuation payoff.

Exerting a bit more effort today increases the probability of the arrival of an instantaneous success or a bad-state-revealing signal, at rate $\lambda^{s,I}(p^g)\varepsilon$. In either event, player $i$ will save the costs of planned effort tomorrow, which is $ck_i$. If instead player $i$ waits and plans to increase tomorrow's effort by $\varepsilon$, then there is a chance that this extra effort will not have to be carried out. This is the case if a success or a bad-state-revealing signal arrives, the probability of which is $\lambda^{s,I}(p^g)k_i + \lambda^U(p^g, p^{bu})k_j$. The cost saved is $c\varepsilon$. Given that players are impatient, there is also another cost of postponing. The markup of effort $\left[\lambda^s(p^g)\left(h + \frac{U}{r}\right) - c\right] \cdot \varepsilon$ is delayed at a cost. Postponing effort to tomorrow is profitable if and only if[13]

$$\underbrace{\left(\lambda^{s,I}(p^g)k_i + \lambda^U(p^g, p^{bu})k_j\right)c}_{\substack{\text{saved costs upon arrival}\\ \text{of a success or signal}}} - \underbrace{r\left(\lambda^s(p^g)\left(h + \frac{U}{r}\right) - c\right)}_{\substack{\text{cost of delayed}\\ \text{markup of effort}}} \geq \underbrace{\lambda^{s,I}(p^g) \cdot ck_i.}_{\substack{\text{benefit of}\\ \text{advancing effort}}} \tag{6}$$

In equilibrium, the uninformed player $i$ has no incentive to either postpone or expedite effort. From (6), it follows that the equilibrium effort must satisfy

$$k_j^N = \frac{(hr + U)\lambda^s(p^g) - cr}{c\lambda^U(p^g, p^{bu})}. \tag{7}$$

Suppose that the above effort level is interior. For this case, by combining (7) with the evolution of the beliefs (4), we solve for the equilibrium effort level as a function of time:

$$k^{N,*}(t) = \frac{C_1}{C_2 e^{C_1 t} + \lambda_h}, \tag{8}$$

where

$$C_1 = \frac{(hr + U)\left(\lambda_\ell^2(1 - p_0) + \lambda_h^2 p_0\right)}{c\lambda^s(p_0)} - r, \quad C_2 = \frac{(1 - p_0)(\lambda_h - \lambda_\ell)(\lambda_\ell(hr + U) - cr)}{cr - (hr + U)\lambda^s(p_0)}. \tag{9}$$

In the no-exit phase, an uninformed player's effort level may increase or decrease as time passes. On the one hand, uninformed players get increasingly pessimistic about the state being good, and would like to decrease their effort.[14] On the other hand, an uninformed player knows that his opponent, if informed, exerts no effort. A potentially informed opponent gives player $i$ less incentive to postpone effort until tomorrow. As the probability that his opponent

[13] This is more formally derived in the appendix (Appendix A).

[14] Note that this differs from the single player's problem. A single player's belief of state $g$ stays constant in the absence of a success or a signal. Here, an uninformed player gets more pessimistic about state $g$ since he attaches a positive probability to the event that his opponent has observed a signal. By contrast, if signals were public and he knew for sure that his opponent had not obtained a signal, his belief of state $g$ would be constant.

is informed increases, the probability of an instantaneous success decreases, and hence also the probability of saving the cost of the planned extra effort. To compensate for this lack of effort from an informed opponent, the uninformed player may want to increase his effort. The combined effect depends on the parameters. In the stationary case, we have a clear-cut condition:

**Lemma 1.** *The (equilibrium) effort level* $k^{N,*}(t)$ *characterized in* (8) *increases over time if*

$$r < \frac{\lambda_\ell U}{c - h\lambda_\ell}, \tag{10}$$

*and decreases otherwise.*

Condition (10) can be easily interpreted as the case in which the markup of effort in the bad state is positive, that is, $r\left[\lambda_\ell\left(h + \frac{U}{r}\right) - c\right] > 0$. Consider an uninformed player. Conditional on his opponent being uninformed, his belief that the state is good remains constant, and so does his incentive to exert effort. Conditional on the event that his opponent is informed and not exerting effort, the uninformed player has to decide whether to compensate for this missing effort or not. If the markup of effort is positive in the bad state,[15] then it is optimal for an uninformed player to compensate for the lack of effort from his informed opponent, and his effort level increases over time. By contrast, if the markup of effort in the bad state is negative, an uninformed player does not wish to compensate for the lack of effort of his informed opponent, and so his own effort decreases.

The no-exit phase cannot last forever. From an informed player's perspective, it becomes increasingly likely that his opponent is also informed and provides no effort. The probability $q^u(t)$ that the opponent is uninformed, conditional on the state being bad, decreases in $t$. At some point, abandoning the project becomes a better option. Nonetheless, there cannot be a period of time during which (i) an informed player exits for sure, and (ii) an uninformed player never exits. If this were the case, then an uninformed player who does not observe his opponent exit at that time would believe that neither player has obtained a signal. Consequently, he would update his belief that the state is good to $p_0$, the prior belief at time 0. An uninformed player is then willing to exert sufficiently high effort, thereby diminishing an informed player's incentive to exit. This explains why, after the no-exit phase, equilibrium play enters a *gradual-exit* phase, in which informed players exit at a finite rate.

**Gradual-exit phase:** In the gradual-exit phase, informed players exit at a finite rate. Moreover, only informed players exit on the equilibrium path. Hence, an exit reveals to an uninformed player that the state is bad and so he also exits immediately. As in the no-exit

[15] That is, if $U$, $h$, $\lambda_\ell$ are sufficiently high, and $c$ and $r$ are sufficiently low.

phase, the relevant probabilities that we have to keep track of are player $i$'s posterior beliefs at any time $t \in [0, \infty)$ that (i) the state is good, (ii) the state is bad and player $j$ is informed, and (iii) the state is bad and player $j$ is uninformed. Again, all of these beliefs are conditional on no success having arrived yet, and we denote them by $p^g(t)$, $p^{bi}(t)$, $p^{bu}(t)$, respectively. Compared to the no-exit phase, the way beliefs are updated changes, since now we have to take into account the exit decision by informed players.

Suppose that over the interval $[t, t + \mathrm{d}t)$, uninformed players exert efforts $(k_i, k_j)$ and informed players exit at rates $(f_i, f_j)$. If the uninformed player $i$ observes no success or signal and his opponent does not exit, then player $i$'s updated beliefs at time $t + \mathrm{d}t$ are given as follows:

$$p^g(t+\mathrm{d}t) = \frac{p^g(t)e^{-\lambda_h(k_i+k_j)\mathrm{d}t}}{p^g(t)e^{-\lambda_h(k_i+k_j)\mathrm{d}t} + p^{bi}(t)e^{-(k_i(\beta+\lambda_\ell)+f_j)\mathrm{d}t} + p^{bu}(t)e^{-(\beta k_i+\lambda_\ell(k_i+k_j))\mathrm{d}t}}, \tag{11}$$

$$p^{bi}(t+\mathrm{d}t) = \frac{p^{bi}(t)e^{-((\beta+\lambda_\ell)k_i+f_j)\mathrm{d}t} + p^{bu}(t)\left(1-e^{-\beta k_j \mathrm{d}t}\right)e^{-(\beta k_i+\lambda_\ell(k_i+k_j))\mathrm{d}t}}{p^g(t)e^{-\lambda_h(k_i+k_j)\mathrm{d}t} + p^{bi}(t)e^{-(k_i(\beta+\lambda_\ell)+f_j)\mathrm{d}t} + p^{bu}(t)e^{-(\beta k_i+\lambda_\ell(k_i+k_j))\mathrm{d}t}},$$

$$p^{bu}(t+\mathrm{d}t) = \frac{p^{bu}(t)e^{-(\beta+\lambda_\ell)(k_i+k_j)\mathrm{d}t}}{p^g(t)e^{-\lambda_h(k_i+k_j)\mathrm{d}t} + p^{bi}(t)e^{-(k_i(\beta+\lambda_\ell)+f_j)\mathrm{d}t} + p^{bu}(t)e^{-(\beta k_i+\lambda_\ell(k_i+k_j))\mathrm{d}t}}.$$

In the gradual-exit phase informed players exit at a finite rate. Hence, they must be indifferent between exiting and staying. At any time $t$ during the gradual-exit phase, an informed player's flow payoff from staying with the project must be equal to the flow payoff of the outside option, that is:

$$q^u(t)k_j(t)h\lambda_\ell = U. \tag{12}$$

Moreover, the equilibrium effort level is such that an uninformed player $i$ has no incentive to either postpone or expedite effort. Again, we consider the effect if an uninformed player $i$ decreases his effort today by $\varepsilon$ and increases his effort tomorrow by the same amount. Conditional on reaching $t + 2\mathrm{d}t$ without a success, a signal, or an exit, the resulting beliefs are unchanged. In addition to the effects that appear in the no-exit section, now we have to take into account the effects resulting from the positive exit rates of informed players.

Exerting a bit more effort today increases the probability of the arrival of an instantaneous success or a bad-state-revealing signal, at rate $\lambda^{s,I}(p^g)$. In either event, player $i$ will save the cost of the planned effort tomorrow, which is $ck_i$. If instead a player $i$ waits and plans to increase tomorrow's effort by $\varepsilon$, the markup of effort is delayed at a cost, but player $i$ saves the costs of tomorrow's planned effort if a success or bad-state revealing signal occurs. These correspond to the third and the first term on the left-hand side of (13).

In the gradual-exit phase, if player $i$ chooses to wait, there is in addition a chance that his opponent exits today. The instantaneous probability of this event is $p^{bi}f_j$. If player $j$

exits, then player $i$ saves the cost of the planned effort tomorrow $c\varepsilon$, but he also forgoes the chance of an instantaneous success, which would yield an expected payoff $h\lambda_\ell\varepsilon$. It follows that postponing effort is profitable if and only if:

$$\underbrace{\left(\lambda^{s,I}(p^g)k_i + \lambda^U(p^g,p^{bu})k_j\right)c}_{\substack{\text{saved costs upon arrival}\\ \text{of a success or signal}}} + \underbrace{p^{bi}f_j\cdot(c-h\lambda_\ell)}_{\substack{\text{cost and benefit}\\ \text{of opponent's exit}}} - \underbrace{r\left[\lambda^s(p^g)\left(h+\frac{U}{r}\right)-c\right]}_{\substack{\text{costs of delayed}\\ \text{markup of effort}}} \geq \underbrace{\lambda^{s,I}(p^g)\cdot ck_i}_{\substack{\text{benefit of}\\ \text{advancing effort}}}. \tag{13}$$

In equilibrium, effort levels and exit rates are such that uninformed players have no incentive to postpone or expedite effort. Moreover, informed players are indifferent between exiting and not. Combining (12) and (13), we obtain that the equilibrium effort level and exit rate during the gradual-exit phase have to satisfy:

$$k_j^G = \frac{1-p^g}{p^{bu}}\cdot\frac{U}{h\lambda_\ell},\quad f_j^G = \frac{r\left[\lambda^s(p^g)\left(h+\frac{U}{r}\right)-c\right]-\lambda^U(p^g,p^{bu})c\frac{(1-p^g)U}{p^{bu}h\lambda_\ell}}{(1-p^g-p^{bu})(c-h\lambda_\ell)}. \tag{14}$$

We now need to determine the time at which the game proceeds from the no-exit phase to the gradual-exit phase. There exists a (unique) vector $(p^g, p^{bi}, p^{bu}; k_j, f_j)$ with $\frac{p^{bu}}{p^g+p^{bu}} = p_0$, such that this vector remains constant over time if beliefs evolve according to (11), and if effort and exit levels are given by (14). Let $k^{G,*}$ and $f^{G,*}$ denote the corresponding effort level and exit rate, respectively, given these beliefs. We let $q^{u,*}$ denote player $i$'s equilibrium belief that player $j$ is uninformed, conditional on state $b$. It is easily verified that $k^{G,*}$ is the unique positive root of the following equation:

$$\frac{hk^{G,*}\lambda_\ell(c-h\lambda_\ell)(k^{G,*}\lambda_h+r)}{U(\lambda_h(hr+U)-c(k^{G,*}\lambda_h+r))} = \frac{p_0}{1-p_0}, \tag{15}$$

and that $q^{u,*} = U/(hk^{G,*}\lambda_\ell)$. The constant exit rate is given by

$$f^{G,*} = \frac{k^{G,*}\lambda_\ell(hk^{G,*}\lambda_h-U)}{hk^{G,*}\lambda_\ell-U}.$$

The transition time $t^*$ from the no-exit to the gradual-exit phase is the time at which the belief $q^u(t)$ in the no-exit phase decreases to $q^{u,*}$. It is given by:

$$t^* = \frac{\log\left(\frac{\lambda_\ell(U-hk^{G,*}\lambda_h)}{C_2hk^{G,*}\lambda_\ell-U(C_2-\lambda_\ell+\lambda_h)}\right)}{C_1}, \tag{16}$$

with $C_1$ and $C_2$ as in (9).

To sum up, the game starts with the no-exit phase, in which uninformed players exert effort $k^{N,*}(t)$ and informed players do not exit. Over time, each player believes with a higher probability that his opponent is informed. The belief $q^u(t)$ decreases. At time $t^*$, the belief $q^u(t)$ has decreased to $q^{u,*}$, and the no-exit phase ends. Equilibrium play then immediately enters the gradual-exit phase, in which uninformed players choose the constant effort $k^{G,*}$, whereas informed players exit at the constant rate $f^{G,*}$.

Depending on the parameters and prior beliefs, it may be the case that the equilibrium does not exhibit two phases. In order for a two-phase equilibrium with a no-exit and a gradual-exit phase to exist, the prior belief must be high enough such that initially informed players want to remain with the project. This is the case if the prior belief satisfies:

$$p_0 \geq \frac{(c - h\lambda_\ell)}{h(\lambda_h - \lambda_\ell)} \cdot \frac{(h\lambda_\ell r + \lambda_h U)}{h\lambda_\ell r} =: \bar{p}^I. \tag{17}$$

For lower prior beliefs, there exists an equilibrium with just one, immediate-exit phase. This is discussed in more detail in Subsection 4.2.

We focus on the case in which the effort levels (during the no-exit and the gradual-exit phases) are always interior. The next lemma provides conditions that guarantee interior effort levels.

**Lemma 2.** *Suppose that* $0 < U < \lambda_\ell h$ *and* $p_0 > \bar{p}^I$*.*

*(i) If* $r \leq \min\{\frac{\lambda_\ell U}{c-h\lambda_\ell}, \frac{\lambda_h(c-U)}{\lambda_h h-c}\}$*, the equilibrium effort levels* (7) *and* (15) *are always interior.*

*(ii) If* $r \leq \frac{\lambda_\ell U}{c-h\lambda_\ell}$ *and* $r > \frac{\lambda_h(c-U)}{\lambda_h h-c}$*, the equilibrium effort levels* (7) *and* (15) *are always interior if and only if* $p_0 \leq \bar{p}$ *with* $\bar{p}$ *given by*

$$\frac{hk^{G,*}\lambda_\ell(1-\bar{p})(\lambda_\ell(hr+U)-cr)+\bar{p}U(\lambda_h(hr+U)-cr)}{cU\lambda^s(\bar{p})} = 1.$$

*(iii) If* $r > \frac{\lambda_\ell U}{c-h\lambda_\ell}$*, the equilibrium effort levels* (7) *and* (15) *are interior if and only if*

$$p_0 \leq \frac{\frac{cr}{c-hr-U} + \lambda_\ell}{\lambda_\ell - \lambda_h}.$$

In the first two cases, $(i)$ and $(ii)$, the markup of effort is positive in the bad state, and the effort level in the no-exit phase increases in $t$. Hence, the constraint $k_i(t) \leq 1$ does not bind if and only if it does not bind at $t^*$. This is the case whenever the markup of effort in the good state is lower than the potential costs saved by the possible arrival of a success if the state is good. If this is not the case, then for any prior belief below $\bar{p}$, equilibrium efforts are interior.

In case (*iii*), the markup of effort in the bad state is negative, and equilibrium effort levels decrease in $t$. Hence, if the effort level at time 0 according to (8) is less than 1, the constraint $k_i(t) \leq 1$ does not bind. This is the case if the prior belief is sufficiently low.

We are now ready to state the main result of this section.

**Proposition 2.** *Suppose that* $0 < U < \lambda_\ell h$ *and* $p_0 > \overline{p}^I$. *Suppose that one of the three conditions in Lemma 2 holds. There exists a symmetric perfect Bayesian equilibrium which consists of two phases: a no-exit phase,* $t \in [0, t^*)$, *and a gradual-exit phase,* $t \in [t^*, \infty)$. *The transition time* $t^* \in [0, \infty)$ *is given by* (16). *In equilibrium, we find the following:*

*(i) an uninformed player never exits, chooses the effort level* (8) *in the no-exit phase, and chooses the effort level* (15) *in the gradual-exit phase.*

*(ii) an informed player exerts no effort, does not exit in the no-exit phase, and exits at a constant rate* $f^{G,*}$ *in the gradual-exit phase.*

*(iii) if a player observes that his opponent exits, this player exits immediately.*

The equilibrium strategies of the equilibrium identified above in Proposition 2 are illustrated in Figure 3 and Figure 4. Figure 3 illustrates the equilibrium strategy when the

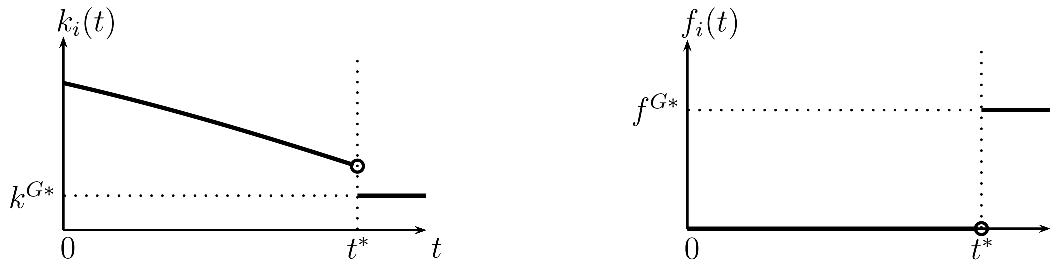


Figure 3: Equilibrium effort level and exit rate

markup of effort in the bad state is negative.[16] The left-hand side is the effort level of an uninformed player as a function of time, and the right-hand side is the exit rate of an informed player. Figure 4 illustrates equilibrium strategies when the markup of effort in the bad state is positive. In this case, the effort level increases initially.[17]

Notice that at the threshold time $t^*$, there is a discontinuity in the effort level. Intuitively, when the game transitions from the no-exit to the gradual-exit phase, an uninformed player has more incentive to procrastinate, since he can learn from observing whether or not his opponent exits. To counterbalance this effect, the effort level must drop at the transition time. The drop decreases the incentive of an uninformed player to procrastinate, since his opponent's lower effort level reduces the benefit from postponing his own effort.

[16] Parameters are $\lambda_h = 1, \lambda_\ell = 1/2, \beta = 1/2, h = 1, c = 2/3, U = 1/20, r = 1, p_0 = 1/2$.

[17] Parameters are $\lambda_h = 1, \lambda_\ell = 1/2, \beta = 1/2, h = 1, c = 2/3, U = 1/20, r = 1/10, p_0 = 4/5$.

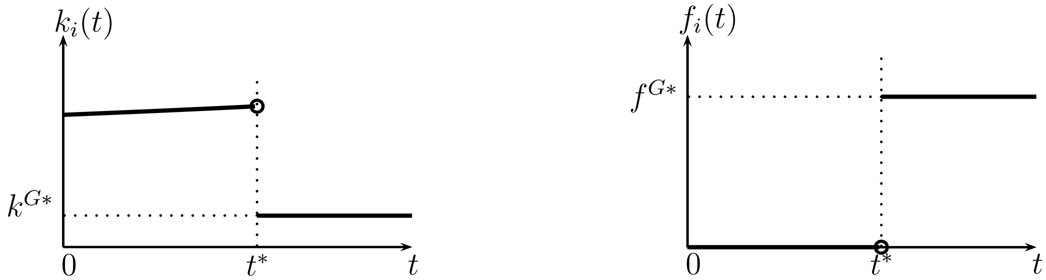


Figure 4: Equilibrium effort level and exit rate when $k_i(t)$ increases

## 4.2 Other Parameter Regions—Immediate-Exit Equilibrium

We now analyze the cases $p_0 \leq \overline{p}^I$ or $U \geq \lambda_\ell h$. In these cases, there exists an equilibrium with an immediate-exit phase. Throughout, we use superscript $I$ to represent the immediate-exit phase.

Suppose that upon observing a bad-state-revealing signal, an informed player exits immediately. His opponent optimally follows suit, since an exit reveals to him that the state is bad. This is as if the private signal were publicly observed. Consequently, at time $t$, a player assigns zero probability to the event that his opponent is informed. Given the assumption that $\beta = \lambda_h - \lambda_\ell$, the belief $p^g(t)$ that the state is good is always $p_0$, as long as no success, signal, or exit has yet occurred. The equilibrium is stationary. Hence, we only need to characterize an uninformed player's effort level and his exit decision. Depending on the parameters, an uninformed player may exert interior or full effort, or he exits at time 0.

We sketch the main steps of the equilibrium analysis; the formal proof is in the appendix. Let $V$ denote the value function. Suppose an informed player exits immediately. Then for an uninformed player $i$'s effort level $k_i$ to be a best-response against his opponent's strategy, the value function $V$ has to solve the following Bellman equation:[18]

$$V = \max_{\tilde{k}_i \in [0,1]} \left\{ r \left[ \lambda^s(p_0) h \left( \tilde{k}_i + k_j \right) - c\tilde{k}_i \right] + \lambda_h \left( \tilde{k}_i + k_j \right) (U - V) \right\}, \tag{18}$$

where, as before, $\lambda^s(p_0) = p_0 \lambda_h + (1 - p_0)\lambda_\ell$ is the expected arrival rate of a success, given belief $p_0$ that the state is good. Taking the derivative of the Bellman equation with respect to $\tilde{k}_i$ and setting it to zero allows us to solve for $V$:

$$V = \frac{r\left(h\lambda^s(p_0) - c\right) + \lambda_h U}{\lambda_h}.$$

[18]Notice, that in the stationary case $\lambda^{s,I}(p_0) = p_0 \lambda_h + (1 - p_0)(\lambda_\ell + \beta) = \lambda_h$.

Substituting $V$ into the Bellman equation, we solve for the symmetric effort level:

$$k_j^I = \frac{r(h\lambda^s(p_0) - c) + \lambda_h U}{c\lambda_h}. \tag{19}$$

Given the assumption that the effort is productive a priori, that is, $p_0 \geq \frac{c-h\lambda_\ell}{h(\lambda_h-\lambda_\ell)}$, this effort level is always greater than 0. This effort level is smaller than 1 if and only if

$$p_0 \leq \frac{c - h\lambda_\ell + (c-U)\lambda_h/r}{h(\lambda_h - \lambda_\ell)}. \tag{20}$$

When the derivative of the Bellman equation with respect to $\tilde{k}_i$ is strictly positive, it is optimal to exert full effort. Substituting the full effort level into the Bellman equation, we solve for the value function as follows:

$$V = \frac{r(2h\lambda^s(p_0) - c) + 2\lambda_h U}{2\lambda_h + r}.$$

This value function is greater than $U$ if and only if the prior belief is higher than the cooperative threshold, that is,

$$p_0 \geq \frac{c - 2h\lambda_\ell + U}{2h(\lambda_h - \lambda_\ell)}.$$

If this value function is less than $U$, both players exit at time 0.

We summarize the discussion above in the following proposition.

**Proposition 3.** *Suppose that $p_0 \leq \overline{p}^I$ or $U \geq \lambda_\ell h$. Suppose that the effort is productive a priori as in Assumption 1. There exists an immediate-exit equilibrium in which an informed player exits immediately and his opponent follows suit immediately.*

*(i) If $U \leq \lambda_\ell h$ and $p_0 < \overline{p}^I$, an uninformed player exerts effort $k_j^I$ as in* (19).

*(ii) If $\lambda_\ell h < U < c$, an uninformed player exerts effort $k_j^I$ as in* (19) *if*

$$p_0 \leq \frac{c - h\lambda_\ell + (c-U)\lambda_h/r}{h(\lambda_h - \lambda_\ell)},$$

*and exerts full effort otherwise.*

*(iii) If $U \geq c$ and $p_0 > p^{c,*}$, an uninformed player exerts full effort. If $U \geq c$ and $p_0 < p^{c,*}$, both players take the outside option at time* 0.

Figure 5 illustrates the different equilibria in Proposition 3, as the outside option (on the $x$-axis) and the prior belief (on the $y$-axis) vary. The solid line $AB$ corresponds to the condition that $p_0 = \overline{p}^I$ (or equivalently, (17) holds with equality). When $(U, p_0)$ lies below $AB$, an informed player exits immediately and an uninformed player chooses an interior effort level. The solid line $BC$ corresponds to the condition that (20) holds with equality. An uninformed player exerts full effort if $(U, p_0)$ lies above $BC$ and interior effort if below. The solid line $CD$ corresponds to the condition that $p_0$ is equal to $p^{c,*}$. An uninformed player exerts full effort if $(U, p_0)$ lies above $CD$, whereas both players exit at time 0 if $(U, p_0)$ lies above $CD$.[19]

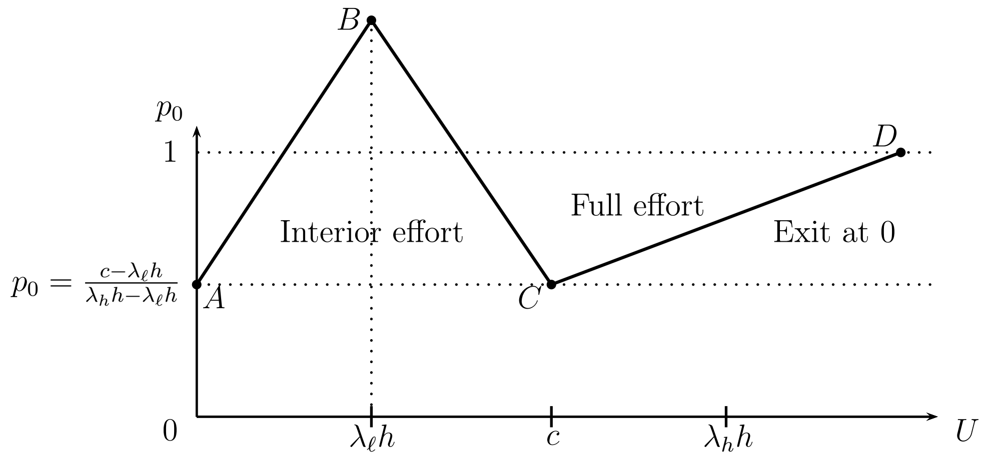


Figure 5: Immediate-exit equilibrium

### 4.3 Comparative statics

Figure 6 shows the ratio of the equilibrium payoff over the cooperative payoff as $U$ increases.[20] Here, $U_1$ is the value of the outside option at which (17) holds with equality, and $U_2$ is the value of the outside option at which (20) holds with equality. The value of $U_3$ is the value of the outside option at which $p_0 = p^{c,*}$. Both $U_1$ and $U_3$ are increasing functions of $p_0$, while $U_2$ decreases in $p_0$.

If $p_0 > (c - h\lambda_\ell)/(h(\lambda_h - \lambda_\ell))$, then $U_1 > 0$ and $U_2 < c < U_3$. On the other hand, $U_1 < U_2$ when $p_0$ satisfies the following condition

$$p_0 < \frac{(c - h\lambda_\ell)(\lambda_h + r)}{h(\lambda_h - \lambda_\ell)r}.$$

In this case, we have four types of equilibria as $U$ increases.

## 5 General Case: $\beta < \lambda_h - \lambda_\ell$

In this section, we extend the analysis to the general case, in which $\beta < \lambda_h - \lambda_\ell$. In the general case, if signals were public, then players would become more pessimistic about the

[19]The $y$-coordinates of point $A$ and $C$ are both equal to $\frac{c-\lambda_\ell h}{(\lambda_h - \lambda_\ell)h}$. The $x$-coordinate of $B$ is equal to $\lambda_\ell h$, and the $y$-coordinate of $B$ might be above or below 1.

[20]Parameters are $\lambda_h = 1, \lambda_\ell = 1/3, \beta = 2/3, h = 1, c = 2/3, r = 1/2, p_0 = 4/5$.

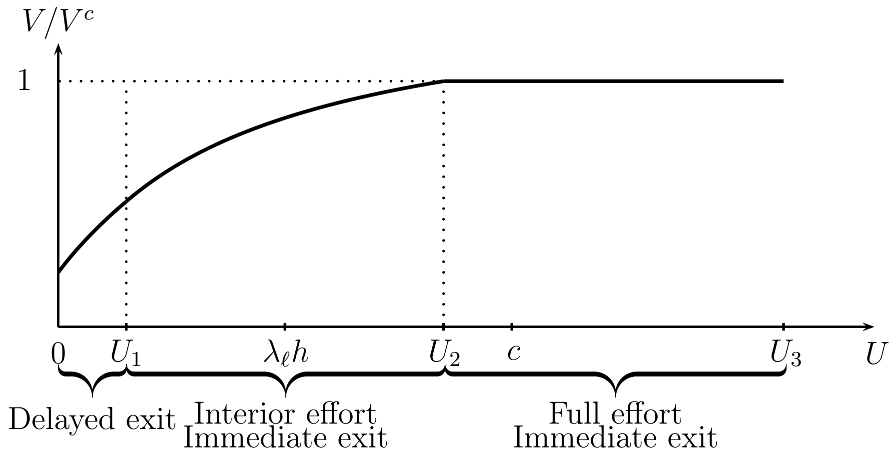


Figure 6: The ratio of equilibrium over cooperative payoff as $U$ varies

state being good, as long as no success or private, bad-state-revealing signal arrives. This is in contrast to the stationary case, in which the belief of state $g$ would remain constant. Still, if everything else is fixed, the lack of a signal makes a player more confident of the state being good when $\beta > 0$ than in the case in which $\beta = 0$. In the general case, however, the arrival rate of the signal is not high enough for the lack of a signal to fully offset the nonarrival of a success.

We present an equilibrium which consists of three phases: the *no-exit* phase, the *gradual-exit* phase, and the *immediate-exit* phase.[21] In contrast to the stationary case, an additional, immediate-exit phase is necessary, since in the general case the gradual-exit phase cannot last forever. When $\beta < \lambda_h - \lambda_\ell$, even if an uninformed player is certain that his opponent is also uninformed, the first player becomes more pessimistic about the state being good as more effort is put into the project. Therefore, there exists no pair of a constant effort level and exit rate under which players' beliefs stay constant in the absence of any success, signal, or exit. The incentives to exert effort during the no-exit and the gradual-exit phase are similar to the ones in the stationary case, described in Section 4. Unlike in the stationary case, the equilibrium exit rate increases to infinity during the gradual-exit phase. At the end of the gradual-exit phase, if a player's opponent has not exited, the player believes that his opponent is uninformed with probability one. At the same time, uninformed players become rather pessimistic about the state, and are not willing to exert high effort. The game proceeds to the immediate-exit phase: any player who becomes informed prefers to exit immediately, because the equilibrium effort is sufficiently low that the flow payoff from staying is strictly less than the level of the outside option, even if the opponent is uninformed and exerting effort with probability one. The immediate-exit phase lasts until the flow payoff

[21]This is the natural counterpart to, and generalization of the equilibrium discussed in Section 4.

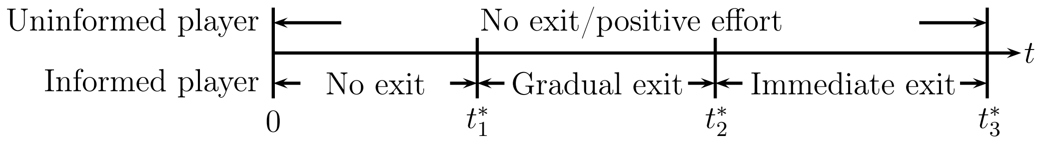


Figure 7: The three-phase equilibrium for the general case

to uninformed players from staying drops to the level of the outside option, at which point both players opt for the outside option.

In all three phases, uninformed players do not exit, and they exert positive effort until the flow payoff from the project drops to the same level as the outside option. At this point in time, the immediate-exit phase ends, and uninformed players take the outside option. We let $t_1^*$ and $t_2^*$ denote the threshold time at which the game proceeds from the no-exit to the gradual-exit phase, and from the gradual-exit to the immediate-exit phase, respectively. Let $t_3^*$ denote the time when the immediate-exit phase ends and uninformed players exit. The structure of this equilibrium is illustrated in Figure 7.

As for the stationary case, we first examine an equilibrium for the parameter region in which a three-phase equilibrium exists. We then discuss equilibria for other parameter regions in Subsection 5.2.

## 5.1 Three-phase Equilibrium

In this section, we examine a symmetric equilibrium with three phases for the case $U < \lambda_\ell h$, in which a player who observes a private signal may want to stay with the project instead of taking his outside option immediately. We first discuss each of the phases—the no-exit, the gradual-exit, and the immediate-exit phases—separately. (As in Section 4, we use superscripts $N, G, I$ to represent these three phases, respectively.) Then, we present the assumptions under which the equilibrium consisting of three phases exists.

**No-exit phase:** At time 0, the prior belief that the state is good is sufficiently high that players do not exit, but stay with the project and choose a positive effort level. Throughout the no-exit phase, conditional on the state being bad, the product of (i) the probability that a player is still uninformed and (ii) the effort level by an uninformed player is sufficiently high that an informed player strictly prefers to stay with the project. The equilibrium effort is given by the same expression (7) as in the stationary case. It guarantees that uninformed players have no incentive to either postpone or expedite effort. The motion of beliefs follows (4). However, in the general case, we cannot obtain a closed-form effort level as we did in (8). Nevertheless, the equilibrium effort level follows as a solution to an ODE. It is derived by combining (7) and the evolution of the beliefs (4).

**Gradual-exit phase:** Analogous to the stationary case, the equilibrium effort level and exit rate are given by (14). Beliefs evolve according to (11). The equilibrium effort level and exit rate are such that (i) an informed player is indifferent between staying with the project and exiting, and (ii) an uninformed player is indifferent between exerting a bit more effort today and doing so tomorrow. However, given the assumption that $\beta < \lambda_h - \lambda_\ell$, we cannot find a constant effort level and constant exit rate such that an uninformed player's beliefs about the state, and about his opponent's information about the state, stay constant over time. The reason the beliefs $(p^g, p^{bi}, p^{bu})$ cannot stay constant is that the probability $q^g(t)$ that the state is good, conditional on neither player being informed, decreases over time as more effort is put into the project. The equilibrium effort level decreases, and the equilibrium exit rate increases. Moreover, in the gradual-exit phase, an informed player's belief $q^u(t)$ that his opponent is uninformed increases over time. There exists a finite time $t_2^*$ at which the conditional belief $q^u(t)$ approaches one, and the equilibrium exit rate goes to infinity. As long as a player does not observe an exit, this player is certain that his opponent has obtained no signal and is still uninformed. At time $t_2^*$, the game proceeds to the immediate-exit phase.

**Immediate-exit phase:** In the immediate-exit phase, an informed player exits immediately. In this case, a player who has not yet observed an exit of his opponent believes that his opponent is uninformed with probability 1. Hence, from an uninformed player $i$'s perspective, it suffices to consider the belief $p^g$ that the state is good, and the belief $p^{bu}$ that the state is bad and his opponent is uninformed.[22] We thus have $p^g + p^{bu} = 1$. In the immediate-exit phase, on the equilibrium path, both players are uninformed.

Given player $i$'s belief $p^g(t)$, if the two players exert efforts $(k_i, k_j)$ over the interval $[t,\, t + \mathrm{d}t)$, conditional on no success, signal, or exit, player $i$'s updated belief at time $t + \mathrm{d}t$ is

$$p^g(t + \mathrm{d}t) = \frac{p^g(t)e^{-\lambda_h(k_i+k_j)\mathrm{d}t}}{p^g(t)e^{-\lambda_h(k_i+k_j)\mathrm{d}t} + (1 - p^g(t))\, e^{-(\beta+\lambda_\ell)(k_i+k_j)\mathrm{d}t}}. \tag{21}$$

To derive an uninformed player's effort level, we again analyze a player's incentive to advance or postpone efforts. Suppose an uninformed player decreases his effort today by $\varepsilon$ and increases his effort tomorrow by the same amount. Conditional on reaching time $t + 2\mathrm{d}t$ without a success, a signal, or an exit, that player's resulting beliefs at that time are unchanged.

Exerting a bit more effort today would increase the probability of the arrival of an instantaneous success or a signal at rate $\lambda^{s,I}(p^g)\varepsilon$. In this case, player $i$ will not have to pay the cost of the planned effort tomorrow, which is $k_i c$. If instead player $i$ waits and exerts a bit more effort tomorrow, then—as in the no-exit and gradual-exit phases—if a success or a signal arrives or player $j$ exits, the planned extra effort for tomorrow would not have to be carried out. The probability of this event is $\lambda^{s,I}(p^g)(k_i + k_j)$ and the cost saved is

[22] The probability $p^{bi}$ that the state is bad and the opponent is informed is zero.

$c\varepsilon$. Again, there is also a cost of postponing, given that players are impatient. This cost is proportional to the markup of effort, $r\left[\lambda^s(p^g)\left(h+\frac{U}{r}\right)+(1-p^g)\beta\frac{U}{r}-c\right]\varepsilon$. It follows that postponing effort is profitable if and only if

$$\underbrace{\lambda^{s,I}(p^g)(k_i+k_j)c}_{\substack{\text{saved costs upon arrival}\\ \text{of a success or signal}}} \underbrace{-r\left[\lambda^s(p^g)\left(h+\frac{U}{r}\right)+(1-p^g)\beta\frac{U}{r}-c\right]}_{\text{costs of delayed markup of effort}} \geq \underbrace{\lambda^{s,I}(p^g)k_ic.}_{\substack{\text{benefit of}\\ \text{advancing effort}}} \tag{22}$$

There are three differences between (22) and (6). First, the opponent is informed with probability zero; therefore, $p^{bu}$ equals $1-p^g$. Second, whenever the opponent is informed, he reveals the signal immediately by exiting. Hence, the postponed effort is saved in that event. Third, player $i$, if informed, also takes the outside option immediately, so the markup of effort is adjusted accordingly.

In equilibrium, the equilibrium effort level is chosen such that players have no incentive to postpone or expedite effort. From (22), we obtain that the equilibrium effort is given by:[23]

$$k_j^{I,*}=\frac{r(h\lambda^s(p^g)-c)}{c\lambda^{s,I}(p^g)}+\frac{U}{c}. \tag{23}$$

It is easily verified that the effort level (23) increases in $p^g$. Since the belief $p^g$ decreases over time, so does the equilibrium effort level.

We now need to determine the times $t_1^*, t_2^*$ at which the game proceeds from the no-exit to the gradual-exit phase, and from there to the immediate-exit phase; and the final exit time $t_3^*$ at which uninformed players exit. To determine the final exit time $t_3^*$, notice that for an informed player to be willing to exit immediately, it must be the case that $k_j^{I,*}\lambda_\ell h\leq U$.[24] When this is combined with (23), this imposes an upper bound on the belief that the state is good in the immediate-exit phase. It must be the case that:[25]

$$p^g(t)<\frac{(c-h\lambda_\ell)(U(\beta+\lambda_\ell)+h\lambda_\ell r)}{U(\beta+\lambda_\ell-\lambda_h)(c-h\lambda_\ell)+h^2\lambda_\ell r(\lambda_h-\lambda_\ell)}:=\overline{p}^I\quad\forall\, t\in[t_2^*,\, t_3^*)\,. \tag{24}$$

The immediate-exit phase lasts until the belief that the state is good drops to the level such that $h\lambda^s(p^g)-c=0$. At this point, the marginal benefit from effort, $h\lambda^s(p^g)$, is exactly equal to the marginal cost $c$. Uninformed players are indifferent between all effort levels and,

[23]Notice, that in the stationary case, with $\beta=\lambda_h-\lambda_\ell$ this reduces to (19).

[24]Recall that in the immediate-exit phase, on-path a player attaches probability 1 to his opponent being uninformed.

[25]Notice that this is equal to the lower bound imposed on the prior in (17). As was discussed in Subsection 4.2, for lower priors there exists an equilibrium with an immediate-exit phase. The upper bound in (24) is on the belief that the state is good in the immediate-exit phase of a three-phase equilibrium.

according to (23), choose the effort level at $k_j = U/c$. For an uninformed player $i$—who benefits from his opponent's effort—this effort level generates a flow payoff at the same level as the outside option, that is, $k_j \cdot h\lambda^s(p^g) = U/c \cdot c = U$. At this time, the belief that the state is good reaches a level such that the marginal benefit from effort equal the marginal cost. Players therefore take the outside option.

At the transition time, $t_2^*$, between the gradual-exit and the immediate-exit phases, the belief $q^u(t)$ that the opponent is uninformed, conditional on the bad state, must be one, which is equivalent to requiring that $p^{bi}(t)$ equals zero. If at time $t_2^*$, a player's opponent stays with the project, that player believes that his opponent is uninformed with probability one. Moreover, at the transition time $t_2^*$, (24) must be satisfied, that is, the belief $q^g(t)$ that the state is good, conditional on neither player being informed, must be below $\overline{p}^I$.[26] It turns out that there exists a unique transition time $t_1^*$ between the no-exit and the gradual-exit phases such that there exists a transition time $t_2^*$ at which beliefs satisfy these two required conditions: (i) $q^u(t) = 1$, and (ii) $p^g(t) \leq \overline{p}^I$. Moreover, the latter condition is binding.[27]

Depending on the parameter region, a three-phase equilibrium may not exist. In order for such an equilibrium to exist, the prior belief must be high enough for there to be an initial no-exit phase. This is the case if and only if the prior belief is above $\overline{p}^I$, as defined in (24).

For an immediate-exit phase to exist, the belief that the state is good must decrease to $p^g(t) \leq \overline{p}^I$. Hence, for a three-phase equilibrium to exist, it must be the case that $\overline{p}^I \in (0, 1)$. This imposes a lower bound on the discount rate $r$.

**Assumption 2.** *The discount rate satisfies:*

$$r > \frac{\lambda_h U(c - h\lambda_\ell)}{h\lambda_\ell(h\lambda_h - c)}.$$

Moreover, the prior belief $p_0$ has to be strictly above $\overline{p}^I$.

Again, we identify conditions that guarantee that the effort level in all three phases is interior, and hence that the boundary constraint $0 \leq k_i(t) \leq 1$ does not bind.

**Lemma 3.** *Suppose that Assumption 2 holds, $0 < U < \lambda_\ell h$, and $p_0 > \overline{p}^I$.*

*(i) If $r \leq \min\{\frac{\lambda_\ell U}{c - h\lambda_\ell}, \frac{\lambda_h(c-U)}{\lambda_h h - c}\}$, the equilibrium effort level in all three phases is interior.*

*(ii) If $r \leq \frac{\lambda_\ell U}{c - h\lambda_\ell}$ and $r > \frac{\lambda_h(c-U)}{\lambda_h h - c}$, there exists $\tilde{p}_0 \leq 1$ such that the equilibrium effort level is interior if $p_0 \leq \tilde{p}_0$.*

[26] Notice that in the immediate-exit phase $q^g(t) = \frac{p^g(t)}{p^g(t) + p^{bu}(t)} = p^g(t)$.

[27] The formal proof is in the appendix.

*(iii) If* $r > \frac{\lambda_\ell U}{c-h\lambda_\ell}$, *the equilibrium effort level is interior if and only if*

$$p_0 \leq \frac{\frac{cr}{c-hr-U} + \lambda_\ell}{\lambda_\ell - \lambda_h}.$$

We are now ready to state the main result for the general case.

**Proposition 4.** *Suppose that Assumption 2 holds,* $0 < U < \lambda_\ell h$, *and* $p_0 > \overline{p}^I$. *Suppose that one of the three conditions in Lemma 3 holds. There exists a symmetric perfect Bayesian equilibrium which consists of three phases: a no-exit phase,* $t \in [0,\, t_1^*)$*; a gradual-exit phase,* $t \in [t_1^*,\, t_2^*)$*; and an immediate-exit phase,* $t \in [t_2^*,\, t_3^*)$. *In equilibrium, we find the following:*

*(i) an uninformed player exerts the effort level* (7) *in the no-exit phase, the effort level* (14) *in the gradual-exit phase, and the effort level* (23) *in the immediate-exit phase. Both uninformed players exit at time* $t_3^*$.

*(ii) an informed player exerts no effort. He does not exit in the no-exit phase, exits at a finite rate given by* (14) *in the gradual-exit phase, and exits immediately in the immediate-exit phase.*

*(iii) if a player observes that his opponent exits, this player exits immediately.*

*(iv) the beliefs* $(p^g(t), p^{bi}(t), p^{bu}(t))$ *equal* $(p_0, 0, 1-p_0)$ *at time* 0. *The beliefs evolve according to* (4) *in* $[0, t_1^*)$, *according to* (11) *in* $[t_1^*, t_2^*)$, *and according to* (21) *in* $[t_2^*, t_3^*)$.

Figure 8 illustrates the equilibrium effort as a function of time.[28]

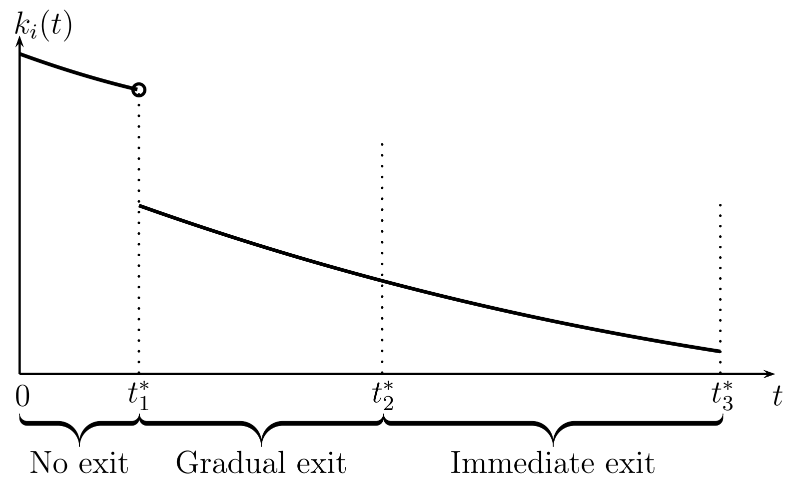


Figure 8: Equilibrium effort level for the general case

**Off-path beliefs and behavior.**

Here, we briefly discuss players' behavior off path. Suppose that an uninformed player deviated in such a way that, at time $t$, the aggregate effort of player $i$ over the interval $[0, t)$ is

[28]Parameters are $\lambda_h = 1, \lambda_\ell = 1/3, \beta = 1/3, h = 1, c = 2/5, U = 1/20, r = 1/10, p_0 = 1/4$.

lower than it would have been on path. This means that player $i$ is more optimistic than he would have been on path. His optimism leads him to exert maximal effort until the time at which his private belief reverts to the common belief. At this time he reverts to the common strategy. If a player deviates in such a way that his realized aggregate effort is greater than in equilibrium, he is more pessimistic and provides no effort until the private belief reverts to the common belief again. Regardless of his past deviation, an informed player assigns the same belief to the event that his opponent is informed. Therefore, off path, it is still optimal for him to follow the equilibrium exiting strategy. If the opponent has not exited by time $t > t_3^*$, an informed player believes that his opponent is exerting zero effort, and thus the informed player exits immediately. An uninformed player also believes that his opponent is exerting zero effort and decides whether to exit based on his private belief that the state is good. This private belief is calculated based on his own and his opponent's aggregate effort over the interval $[0, t_3^*)$. At time $t_3^*$ an uninformed player remains with the project and exerts effort if and only if his belief is above the single-player threshold.

## 5.2 Other Parameter Regions—Immediate-Exit Equilibrium

As in the stationary case, it may be that for certain parameter regions, no three-phase equilibrium exists. In this section, we discuss equilibria for these parameter regions. As in Subsection 4.2 we focus on immediate-exit equilibria.

In any immediate-exit equilibrium, if a player becomes informed, he immediately exits and takes the outside option. Hence, the situation is as if signals were public, and the belief $p(t)$ that the state is good, conditional on no success, signal, or exit, is:

$$p(t) = \frac{p_0 e^{-\lambda_h \int_0^t 2k_i(s)\mathrm{d}s}}{p_0 e^{-\lambda_h \int_0^t 2k_i(s)\mathrm{d}s} + (1-p_0)e^{-(\beta+\lambda_\ell)\int_0^t 2k_i(s)\mathrm{d}s}}. \tag{25}$$

We only need to characterize an uninformed player's effort level and exit behavior. Players' incentives to exert effort are the same as in the immediate-exit phase. However, the effort level does not necessarily coincide with equation (23). When $U$ is sufficiently large, the effort level given by (23) exceeds 1. In this case, players initially exert the maximum effort level, until their belief that the state is good becomes sufficiently low that (23) drops below 1. Then, they exert the interior effort level (23). The equilibrium effort is $\min\{k_j^{I,*}(t),\ 1\}$, with $k_j^{I,*}(t)$ given by (23).

Over time, if no success, signal, or exit arrives, uninformed players get more pessimistic about the state being good. When the belief that the state is good decreases to $\frac{c-h\lambda_\ell}{h(\lambda_h-\lambda_\ell)}$, uninformed players are then indifferent among all effort levels, and their flow payoffs equal $U$. At this time, both players take the outside option.

**Proposition 5.** *Suppose that $p_0 \leq \overline{p}^I$ or $U \geq \lambda_\ell h$. Suppose that the effort is productive a priori as in Assumption 1. There exists an immediate-exit equilibrium in which an informed player exits immediately and his opponent follows suit immediately.*

*(i) If $U \leq \lambda_\ell h$ and $p_0 < \overline{p}^I$, an uninformed player exerts effort $k_j^{I,*}$ as in* (23)*, and exits when the belief of state $g$ decreases to $\frac{c-h\lambda_\ell}{h(\lambda_h-\lambda_\ell)}$.*

*(ii) If $\lambda_\ell h < U < c$, an uninformed player exerts effort $\min\{k_j^{I,*}, 1\}$, and exits when the belief of state $g$ decreases to $\frac{c-h\lambda_\ell}{h(\lambda_h-\lambda_\ell)}$.*

*(iii) If $U \geq c$ and $p_0 > p^{c,*}$, an uninformed player exerts full effort, and exits when the belief of state $g$ decreases to $p^{c,*}$.*

*(iv) If $U \geq c$ and $p_0 \leq p^{c,*}$, an uninformed player exits at time* 0*.*

# 6 Conclusion

In this paper, we studied a team problem in which the success rate of the joint project is unknown and collaborators may privately receive discouraging news. Players can choose whether and when to share this information with their collaborators by choosing when to exit the project. We analyzed how the possibility of receiving private discouraging news affects the incentive of collaborators to exert effort and the timing thereof, as well as players' optimal strategy for exiting and revealing discouraging news.

We characterized equilibria with no-exit, gradual-exit and immediate-exit phases and identified two types of inefficiencies. On the one hand, players have an incentive to procrastinate effort and to free-ride on the effort of their collaborators. On the other hand, equilibrium behavior displays delayed and diffused information transmission. This may lead to a deadlock of the project, in which both players do not exert effort anymore and the project is inactive. Remarkably, effort levels in the no-exit phase may increase, since players may want to compensate for the lack of effort of informed competitors. Moreover, increasing the payoff of the outside option diminishes both inefficiencies and encourages collaboration.

Our results raise a number of intriguing questions to explore in future research. We have already begun to investigate how to generalize the results to the $n$-player case, as well as to a larger set of arrival rates. Moreover, we plan to investigate the effect of deadlines, and the optimal transparency policy that a social planner would choose. Specifically, we are interested in understanding whether it is optimal to force team member to immediately make their information public, or if it may be beneficial to delay the sharing of discouraging news among team members.

# Appendix

## A Proofs

*Proof of Proposition 1.* In the cooperative game, it is without loss to focus on symmetric strategies. The belief of state $g$ evolves according to (1). Given the belief $p$ of state $g$, the flow payoff per player if both players choose the effort level $\tilde{k}$ is

$$(2h\lambda^s(p) - c)\,\tilde{k}.$$

By the Principle of Optimality, the value function of the cooperative game is given by

$$V(p) = \max_{\tilde{k}\in[0,1]} \left\{ r\,(2\lambda^s(p)h - c)\,\tilde{k}\mathrm{d}t + e^{-r\mathrm{d}t} 2\lambda^{s,I}(p)\tilde{k}\mathrm{d}t(U - V(p+\mathrm{d}p)) + e^{-r\mathrm{d}t}V(p+\mathrm{d}p) \right\}.$$

Substituting $V(p+\mathrm{d}p) = V(p) - V'(p)2\tilde{k}(1-p)p(\lambda_h - \lambda_\ell - \beta)\mathrm{d}t$ and rearranging, we obtain the Bellman equation:

$$V(p) = \max_{\tilde{k}\in[0,1]} \left\{ (2h\lambda^s(p) - c)\tilde{k} - \frac{1}{r}2\tilde{k}(1-p)p(\lambda_h - \lambda_\ell - \beta)V'(p) + \frac{1}{r}\,(U - V(p))\,2\tilde{k}\lambda^{s,I}(p) \right\}.$$

The linearity in $\tilde{k}$ of the maximand in the Bellman equation immediately implies that it is always optimal to choose either $\tilde{k} = 0$ or $\tilde{k} = 1$. In the latter case, $V$ satisfies the first-order ODE:

$$V(p) = 2h\lambda^s(p) - c + \frac{1}{r}\left[2\lambda^{s,I}(p)\,(U - V(p)) - 2(1-p)p(\lambda_h - \beta - \lambda_\ell)V'(p)\right].$$

Let $p^{c,*}$ denote the cutoff belief at which players are indifferent between staying with the project and exerting full effort and taking the outside option. The value matching $V(p^{c,*}) = U$ and smooth pasting $V'(p^{c,*}) = 0$ conditions allow us to solve for the cutoff belief $p^{c,*}$ and the constant of the integration in the solution to the above ODE. The cooperative threshold $p^{c,*}$ is chosen such that

$$2h\,(\lambda_h p^{c,*} + (1 - p^{c,*})\lambda_\ell) - c = U.$$

If the belief is above the cooperative threshold, players stay with the project and exert full effort. Otherwise they take the outside option. □

*Proof of Lemma 1.* The derivative of $k^{N,*}(t)$ as defined in (8) with respect to $t$ is

$$\frac{\mathrm{d}k^{N,*}(t)}{\mathrm{d}t} = -\frac{C_1^2 C_2 e^{C_1 t}}{(C_2 e^{C_1 t} + \lambda_h)^2},$$

which is positive if and only if $C_2 < 0$. It is easily verified that $C_2 < 0$ if and only if $r > \frac{\lambda_\ell U}{c - \lambda_\ell h}$. □

*Proof of Lemma 2.* We want to derive conditions under which the effort level in (8) is interior for all $t \in [0, t^*)$ with $t^*$ as in (16).
(*i*) If $r \geq \lambda_\ell U/(c - h\lambda_\ell)$, then by Lemma 1 effort level in (8) are decreasing. Hence, efforts are interior if and only if the boundary constraint $k_i(t) \leq 1$ does not bind at time 0. It is easy to check that the equilibrium effort in the no-exit phase given by (8) satisfies $k_i(0) \leq 1$, if and only if,

$$p_0 \leq \frac{\frac{cr}{c-hr-U} + \lambda_\ell}{\lambda_\ell - \lambda_h}.$$

(*ii*) and (*iii*): If $r < \lambda_\ell U/(c - h\lambda_\ell)$, then by Lemma 1 the effort level in (8) increases in $t$. Hence, the constraint $k_i(t) \leq 1$ does not bind if and only if it does not bind at $t^*$. Based on the formula in (8), the left-hand limit of $k_i(t)$ at $t^*$ is

$$\lim_{t \uparrow t^*} k^{N,*}(t) = \frac{hk^{G,*}\lambda_\ell(1-p_0)(\lambda_\ell(hr+U)-cr) + p_0 U(\lambda_h(hr+U)-cr)}{cU\lambda(p_0)}. \tag{26}$$

Given the assumption that $r < \lambda_\ell U/(c - h\lambda_\ell)$, this limit increases in $p_0$ and $k^{G,*}$. On the other hand, $k^{G,*}$ given by (15) increases in $p_0$. Therefore, the left-hand limit $\lim_{t\uparrow t^*} k^{N,*}(t)$ is an increasing function of $p_0$. It is easy to check that for $p_0 = 1$ the right-hand side of (26) is less or equal to 1 if and only if $r\left[\lambda_h\left(h + \frac{U}{r}\right) - c\right] \leq c\lambda_h$. Hence, under this condition, for any prior belief $p_0$, $k_i(t) \leq 1$ does not bind, given that the right-hand side of (26) is increasing in $p_0$.

Now suppose $r\left[\lambda_h\left(h + \frac{U}{r}\right) - c\right] > c\lambda_h$. In this case, given that (26) is increasing in $p_0$, there exists a unique $\bar{p}$ such that

$$\left.\frac{hk^{G,*}\lambda_\ell(1-p_0)(\lambda_\ell(hr+U)-cr) + p_0 U(\lambda_h(hr+U)-cr)}{cU\lambda(p_0)}\right|_{p_0=\bar{p}} = 1.$$

Whenever the prior belief is below $\bar{p}$, then equilibrium efforts are interior. □

*Proof of Proposition 2.*
We first discuss some details on how we obtain the equations that determine equilibrium

effort and exit rates. We then verify stage by stage that the strategy profile described in Proposition 2 is an equilibrium.

**Part 1: Effort levels and exit rates**

Consider the no-exit phase. For given effort levels $(k_i(t), k_j(t))$ the evolution of beliefs follows (4). Given the beliefs and the effort choice at time $t$, let $Q_1^N$ denote the probability that a success occurs, and $Q_2^N$ is the probability that no success occurs and player $i$ obtains a signal in interval $[t,\, t+\mathrm{d}t)$:

$$\begin{aligned} Q_1^N &= p^g\left(1-e^{-\lambda_h(k_i+k_j)\mathrm{d}t}\right)+p^{bi}\left(1-e^{-\lambda_\ell k_i \mathrm{d}t}\right)+p^{bu}\left(1-e^{-\lambda_\ell(k_i+k_j)\mathrm{d}t}\right), \\ Q_2^N &= \left(p^{bi}e^{-k_i\lambda_\ell \mathrm{d}t}+p^{bu}e^{-\lambda_\ell(k_i+k_j)\mathrm{d}t}\right)\left(1-e^{-\beta k_i \mathrm{d}t}\right). \end{aligned} \tag{27}$$

Player $i$'s continuations payoff at time $t$ can be written as

$$V_{i,t} = r(Q_1^N h - ck_i\mathrm{d}t) + e^{-r\mathrm{d}t}\left(Q_1^N U + Q_2^N W_{i,t+\mathrm{d}t} + (1-Q_1^N-Q_2^N)V_{i,t+\mathrm{d}t}\right),$$

where $W_{i,t+\mathrm{d}t}$ denotes player $i$'s continuations payoff at $t+\mathrm{d}t$ if he is informed.

We apply the same expansion to $V_{i,t+\mathrm{d}t}$ to obtain

$$\begin{aligned} V_{i,t} =& r(Q_1^N h - k_i c\mathrm{d}t) + e^{-r\mathrm{d}t}\left(Q_1 U + Q_2 W_{i,t+\mathrm{d}t}\right) + e^{-r\mathrm{d}t}(1-Q_1^N-Q_2^N) \\ &\cdot \left[r(Q_1^{N\prime}h - k_i' c\mathrm{d}t) + e^{-r\mathrm{d}t}(Q_1^{N\prime}U + Q_2^{N\prime}W_{i,t+2\mathrm{d}t} + (1-Q_1^{N\prime}-Q_2^{N\prime})V_{i,t+2\mathrm{d}t})\right], \end{aligned}$$

where $Q_1^{N\prime}$, $Q_2^{N\prime}$ denote the probability that a success occurs, respectively the probability that no success occurs and player $i$ obtains a signal in interval $[t+\mathrm{d}t,\, t+2\mathrm{d}t)$. Note that an informed player $i$'s continuation payoff $W_{i,t+\mathrm{d}t}, W_{i,t+2\mathrm{d}t}$ only depends on the probability that $j$ is uninformed, and on $j$'s effort level if uninformed. The effort choices $k_i, k_i'$ of an uninformed player do not affect $W_{i,t+\mathrm{d}t}, W_{i,t+2\mathrm{d}t}$. The evolution of $W_{i,t}$ is given by:

$$W_{i,t}' = rW_{i,t} - \frac{k_j\lambda_\ell p^{bu}(hr+U-W_{i,t})}{1-p^g} \tag{28}$$

The second term is proportional to the product of the probability that the other player's effort generates a success, conditional on the state being bad. A success yields payoff $hr+U$ but also has an opportunity cost equal to the continuation payoff $W_{i,t}$.

In order to analyze the effect of postponing effort, consider decreasing $k_i$ by $\varepsilon$ and increasing $k_i'$ by the same amount. Note that, conditional on reaching $t+2\mathrm{d}t$ without a breakthrough and without becoming informed, the resulting beliefs are unchanged, and therefore so is the continuation payoff $V_{i,t+2\mathrm{d}t}$. To ease interpretation of the effect of postponing effort, we use the Taylor expansion to the third order.[29] Assuming that $k_i, k_j, W_{i,t}$ are continuous, and letting $\mathrm{d}t \to 0$, we are left with

$$\frac{\mathrm{d}V_{i,t}/\mathrm{d}\varepsilon}{\mathrm{d}t^2} = \beta(1-p^g)W'_{i,t} + \left[\beta\lambda_\ell p^{bu}(hr+U-W_{i,t}) + cr(\lambda_\ell p^{bu} + \lambda_h p^g)\right] k_j \\ + r\left[cr - \beta(1-p^g)W_{i,t} - (hr+U)(p^g\lambda_h + (1-p^g)\lambda_\ell)\right]. \tag{29}$$

Postponing effort is profitable for player $i$ if and only if $\frac{\mathrm{d}V_{i,t}/\mathrm{d}\varepsilon}{\mathrm{d}t^2} \geq 0$. By substituting (28) into (29) and rearranging, we obtain (6). In equilibrium effort levels are such that uninformed players have no incentive to postpone or expedite effort. It follows that effort levels must satisfy (7). If one of the conditions in Lemma 2 holds, the effort level is interior. We obtain that the effort level is given by (8).

Consider the gradual-exit phase. Given effort levels $k_i, k_j$, exit rates $f_i, f_j$ and believes at time $t$, the probability $Q_1^G$ that a success occurs during $[t, t+\mathrm{d}t)$ is given by the same expression as in the no-exit phase. Let $Q_2^G$ denote the probability that (i) no success occurs and (ii) player $i$ becomes informed or player $j$ exits:

$$Q_2^G = p^{bi}e^{-k_i\lambda_\ell \mathrm{d}t}\left(1-e^{-(f_j+\beta k_i)\mathrm{d}t}\right) + p^{bu}\left(1-e^{-\beta k_i \mathrm{d}t}\right)e^{-\lambda_\ell(k_i+k_j)\mathrm{d}t}.$$

If no success occurs, player $i$ is not informed, and player $j$ does not exit, player $i$'s updated beliefs at time $t+\mathrm{d}t$ are given by (11).

Player $i$'s continuation payoff at time $t$ has to satisfy the following recursion:

$$V_{i,t} = r(Q_1^G h - k_i c\mathrm{d}t) + e^{-r\mathrm{d}t}(Q_1^G + Q_2^G)U \\ + e^{-r\mathrm{d}t}(1-Q_1^G-Q_2^G)\left(r(Q_1^{G\prime}h - k_i' c\mathrm{d}t) + e^{-r\mathrm{d}t}((Q_1^{G\prime}+Q_2^{G\prime})U + (1-Q_1^{G\prime}-Q_2^{G\prime})V_{i,t+2\mathrm{d}t})\right).$$

Note that if player $i$ becomes informed, his continuation payoff is $U$, since an informed player is indifferent between exiting and not.

Again, we consider the effect of decreasing $k_i$ by $\varepsilon$ and increasing $k_i'$ by the same amount. It is given by:

$$\frac{\mathrm{d}V_{i,t}/\mathrm{d}\varepsilon}{r\mathrm{d}t^2} = p^{bi}(c-h\lambda_\ell)f_j + (\beta p^{bu}h\lambda_\ell + c(\lambda_\ell p^{bu} + \lambda_h p^g))k_j - \beta(p^{bi}+p^{bu})U \\ - r\left[\lambda(p^g)\left(h+\frac{U}{r}\right) - c\right]. \tag{30}$$

[29] For instance,

$$e^{-r\mathrm{d}t} = 1 - r\mathrm{d}t + \frac{r^2\mathrm{d}t^2}{2} + O\left(\mathrm{d}t^3\right).$$

Substituting (12) into (30), we obtain that postponing effort is profitable if and only if (13) holds. The equilibrium effort level is such that an uninformed player $i$ has no incentive to postpone or expedite effort. Combining this with the condition (12) which guarantees that informed players are indifferent between staying with the project and exiting yields (14).

**Part 2: Verifying the equilibrium strategy profile**

We now verify stage by stage that the strategy profile described in Proposition 2 is an equilibrium.

**Step 1:** We begin with Stage Informed, the stage in which player $i$ has observed a private signal, but no success or exit yet. If player $i$ enters Stage Informed at time $\hat{t}$, he learns that the state is bad. Define $t^*$ to be the time, at which play enters the gradual-exit phase from the no-exit phase. That is, for all $t < t^*$, the exit rate is zero, $f^{N,*}(t) = 0$, and the flow payoff of an informed player is weakly higher than the outside option.

Suppose that $\hat{t} < t^*$, and that player $j$ follows the equilibrium strategy, that is, if he is uninformed his effort $k_j^N(t)$ is given by (7). Then conditional on no success and the fact that he himself is informed, player $i$ assigns the following probability to the event that player $j$ is uninformed:

$$q^u(t) = \frac{p^{bu}(t)}{p^{bi}(t) + p^{bu}(t)} = \frac{\lambda_h e^{-\lambda_h \int_0^{\hat{t}} k_j(s) \mathrm{d}s}}{\lambda_h - \lambda_\ell \left(1 - e^{-\lambda_h \int_0^{\hat{t}} k_j(s) \mathrm{d}s}\right)}, \tag{31}$$

Note that the belief in (31) does not depend on the amount of experimentation that player $i$ has conducted before time $t$. The flow payoff that player $i$ obtains from staying with the project is given by:

$$q^u(t) k_j^{N,*}(t) \lambda_\ell h = \frac{C_1 h \lambda_\ell}{e^{C_1 t}(C_2 - \lambda_\ell + \lambda_h) + \lambda_\ell},$$

which, as discussed, is weakly greater than $U$. Hence, an informed player $i$ finds it optimal not to exit. At time $t^*$, the conditional probability that player $j$ is uninformed, as specified in (31), decreases to $U/(hk^{G,*}\lambda_\ell)$. From $t^*$ on, play enters the gradual-exit phase, in which an informed player $j$ exits at a constant rate $f^{G,*} > 0$. Player $i$ becomes more confident that player $j$ is uninformed if player $j$ has not exited. On the other hand, player $i$ becomes less confident that player $j$ is uninformed given that there is no success. The exit rate $f^{G,*}$ and the effort level $k^{G,*}$ in (15) are chosen so that the probability that player $i$ assigns to the event that player $j$ is uninformed, conditional on no exit and no success, stays constant at $U/(hk^{G,*}\lambda_\ell)$. Also, given that an uninformed player $j$'s effort level is $k^{G,*}$, the flow payoff that player $i$ obtains from staying with the project is exactly $U$. Hence, player $i$ is indifferent between exiting and staying.

If player $i$ enters Stage Informed at $t > t^*$, it is easy to check that he believes that player $j$ is uninformed with probability $U/(hk^{G,*}\lambda_\ell)$, so he is indifferent between exiting and staying.

If player $i$ observes the exit by an opponent (that is, entering Stage Exit or (Informed, Exit)), then he assigns probability 1 to the bad state, and it is optimal for him to take the outside option.

**Step 2:** Next we show that for an uninformed player it is optimal to choose the effort level as specified in Stage Null. Recall that Stage Null is the stage in which no success or signals has occurred yet. Hence, in Stage Null, the initial values are $p^g(0) = p_0 = 1 - p^{bu}(0), p^{bi}(0) = 0$, and the sum of $p^g(t), p^{bi}(t), p^{bu}(t)$ always equals 1. Notice that the evolution of these probabilities given player $j$'s strategy does not depend on the effort that player $i$ actually exerts. Indeed, in the no-exit phase, that is when $t \leq t^*$, if player $i$ chooses the effort level $\tilde{k}_i(t)$ over the interval $[t, t + \mathrm{d}t)$, his updated beliefs given that he obtains no public success or private signal are given by (4) (with $k_i = \tilde{k}_i$). Substituting $\beta = \lambda_h - \lambda_\ell$ and $p^{bi}(t) = 1 - p^g(t) - p^{bu}(t)$, the corresponding derivatives are[30]

$$\begin{aligned}
p^{g\prime}(t) &= -k_j(t)p^g(t)(\lambda_h(1 - p^g(t)) - \lambda_\ell p^{bu}(t)), \\
p^{bu\prime}(t) &= -k_j(t)p^{bu}(t)(\lambda_h(1 - p^g(t)) - \lambda_\ell p^{bu}(t)),
\end{aligned}$$

which do not depend on $\tilde{k}_i(t)$. Substituting the equilibrium effort level $k_j^N(t)$ from (7) and the initial values, we derive explicitly player $i$'s beliefs in the no-exit phase (for $t \leq t^*$):

$$\begin{aligned}
p^g(t) &= \frac{p_0 \left(C_2 e^{C_1 t} + \lambda_h\right)}{e^{C_1 t}(C_2 + (1 - p_0)(\lambda_h - \lambda_\ell)) + \lambda_\ell(1 - p_0) + \lambda_h p_0}, \\
p^{bu}(t) &= \frac{(1 - p_0) \left(C_2 e^{C_1 t} + \lambda_h\right)}{e^{C_1 t}(C_2 + (1 - p_0)(\lambda_h - \lambda_\ell)) + \lambda_\ell(1 - p_0) + \lambda_h p_0}.
\end{aligned}$$

At time $t^*$, these beliefs are

$$p^g(t^*) = \frac{p_0 U}{p_0 U + h k^* \lambda_\ell (1 - p_0)}, \quad p^{bu}(t^*) = \frac{U(1 - p_0)}{p_0 U + h k^* \lambda_\ell (1 - p_0)}. \tag{32}$$

In the gradual-exit phase, when $t > t^*$, suppose that player $j$ follows the equilibrium strategy, that is, chooses the effort level $k^{G,*}$ if uninformed, and exits at the rate $f^{G,*}$ if informed, given by (15). If an uninformed player $i$ chooses the effort level $\tilde{k}_i(t)$, the derivatives of his beliefs given that he obtains no success or signal and that player $j$ has not exited are:

$$\begin{aligned}
p^{g\prime}(t) &= p^g(t) \left[f^{G,*} \left(1 - p^g(t) - p^{bu}(t)\right) - k^{G,*} \left(\lambda_h(1 - p^g(t)) - \lambda_\ell p^{bu}(t)\right)\right], \\
p^{bu\prime}(t) &= p^{bu}(t) \left[f^{G,*} \left(1 - p^g(t) - p^{bu}(t)\right) - k^{G,*} \left(\lambda_h(1 - p^g(t)) - \lambda_\ell p^{bu}(t)\right)\right].
\end{aligned}$$

[30] We omit the sequence $p^{bi}(t)$ since for every $t \leq t^*$, $p^{bi}(t) = 1 - p^g(t) - p^{bu}(t)$.

Substituting $k^{G,*}$, $f^{G,*}$ and the initial values $p^g(t^*), p^{bu}(t^*)$, we obtain that beliefs stay constant: $p^g(t) = p^g(t^*)$ and $p^{bu}(t) = p^{bu}(t^*)$ for $t \geq t^*$. This shows that, on and off path, $p^g(t), p^{bi}(t), p^{bu}(t)$ are constant. For ease of exposition, we denote these probabilities by $p^{g,*}$, $p^{bi,*}$, $p^{bu,*}$.

We first analyze player $i$'s incentive to exert effort when $t \geq t^*$. Let $V(t)$ denote the (normalized) continuation payoff of player $i$ at time $t > t^*$ if he is uninformed and his opponent has not exited yet. The equilibrium is stationary, so we can ignore the subscript $t$. The payoff must satisfy the Bellman equation:

$$V = \max_{\tilde{k}_i \in [0,1]} \Big\{ r \left[ (p^{g,*}\lambda_h + p^{bu,*}\lambda_\ell)(\tilde{k}_i + k^{G,*})h + p^{bi,*}\lambda_\ell \tilde{k}_i h - c\tilde{k}_i \right] \mathrm{d}t + e^{-r\mathrm{d}t} V$$
$$+ e^{-r\mathrm{d}t} \left[ (p^{g,*}\lambda_h + p^{bu,*}\lambda_\ell)(\tilde{k}_i + k^{G,*}) + p^{bi,*}\lambda_\ell \tilde{k}_i + p^{bi,*} f^{G,*} + (p^{bi,*} + p^{bu,*})\tilde{k}_i \beta \right] (U - V)\mathrm{d}t \Big\}.$$

Note that if a success occurs, or player $i$'s opponent exits, or if player $i$ becomes informed, the continuation payoff of player $i$ is equal to the outside option $U$. Otherwise, the continuation payoff is $V$. Substituting $e^{-r\mathrm{d}t} = 1 - r\mathrm{d}t$ and rearranging, we obtain the Bellman equation:

$$V = \max_{\tilde{k}_i \in [0,1]} \Big\{ \left[ (p^{g,*}\lambda_h + p^{bu,*}\lambda_\ell)(\tilde{k}_i + k^{G,*})h + p^{bi,*}\lambda_\ell \tilde{k}_i h - c\tilde{k}_i \right]$$
$$+ \frac{1}{r} \left[ (p^{g,*}\lambda_h + p^{bu,*}\lambda_\ell)(\tilde{k}_i + k^{G,*}) + p^{bi,*}\lambda_\ell \tilde{k}_i + p^{bi,*} f^{G,*} + (p^{bi,*} + p^{bu,*})\tilde{k}_i \beta \right] (U - V) \Big\}.$$

Substituting the probabilities in (32) and the equilibrium effort level $\tilde{k}_i = k^*$, we solve for $V$ and obtain:

$$V = \frac{k^{G,*}h\lambda_h(\lambda_\ell(hr + U) - cr) - U(\lambda_h(hr + U) - cr)}{k^{G,*}h\lambda_\ell\lambda_h + hr(\lambda_\ell - \lambda_h) - \lambda_h U}. \tag{33}$$

Substituting $V$ into the Bellman equation, we verify that the FOC with respect to $\tilde{k}_i$ indeed equals zero:

$$\left[ h(\lambda_\ell(p^{bi,*} + p^{bu,*}) + \lambda_h p^{g,*}) - c \right] - \frac{1}{r}((\beta + \lambda_\ell)(p^{bi,*} + p^{bu,*}) + \lambda_h p^{g,*})(V - U) = 0.$$

Therefore, an uninformed player $i$ is indeed indifferent among all effort levels. The first term is the incremental payoff from exerting effort. However, exerting effort increase the probability of obtaining a success or a signal. In both cases, the continuation payoff decreases from $V$ to $U$, as captured by the second term.[31]

We now analyze an uninformed player $i$'s incentive to exert effort in the no-exit phase,

[31] Note that $V$ must be weakly greater than $U$, because an uninformed player always has the option to exit immediately and obtain $U$.

when $t < t^*$. Let $V(t)$ denote the (normalized) continuation payoff of player $i$ if he is uninformed:

$$V(t) = \max_{\tilde{k}_i \in [0,1]} \left\{ r\left(Q_1^N h - c\tilde{k}_i \mathrm{d}t\right) + e^{-rt}\left[Q_1^N U + Q_2^N W(t+\mathrm{d}t) + \left(1 - Q_1^N - Q_2^N\right) V(t+\mathrm{d}t)\right] \right\},$$

where $Q_1^N$ is the probability to obtain a success, and $Q_2^N$ is the probability to obtain a signal conditional of no success, as given in (27). Here, $W(t)$ is player $i$'s continuation payoff if he is informed at time $t$. From the analysis above, we know that the choice of $\tilde{k}_i$ does not affect $V(t+\mathrm{d}t)$ or $W(t+\mathrm{d}t)$.[32] Substituting $V(t+\mathrm{d}t) = V(t) + V'(t)\mathrm{d}t$ and rearranging, we obtain the Bellman equation:

$$V(t) = \max_{\tilde{k}_i \in [0,1]} \left\{ \left(Q_1^N h - c\tilde{k}_i \mathrm{d}t\right) + \frac{1}{r}\left[Q_1^N U + Q_2^N W(t) - (Q_1^N + Q_2^N) V(t) + V'(t)\right] \right\}. \quad (34)$$

We first calculate the value function $W(t)$. Recall that $q^u(t)$ is the probability that player $j$ is uninformed conditional on state $b$ and $k_j(t)$ is the effort rate.

$$W(t) = r q^u(t) k_j^N(t) \lambda_\ell h \mathrm{d}t + e^{-r\mathrm{d}t}\left[q^u(t) k_j^N(t) \lambda_\ell (U - W(t+\mathrm{d}t))\mathrm{d}t + W(t+\mathrm{d}t)\right].$$

Substituting the probabilities and the equilibrium effort level, we obtain an ODE of $W(t)$. The boundary condition $W(t^*) = U$ allows us to determine the unique solution for $W(t)$:

$$W(t) = \frac{e^{(t-t^*)(C_1+r)}\left(U(C_1+r)(C_2 - \lambda_\ell + \lambda_h)e^{C_1 t^*} + \lambda_\ell r(U - C_1 h)\right) + C_1 \lambda_\ell (hr + U)}{(C_1+r)\left(e^{C_1 t}(C_2 - \lambda_\ell + \lambda_h) + \lambda_\ell\right)}.$$

From the FOC with respect to $\tilde{k}_i$, we can solve for $V(t)$ in terms of $W(t)$:

$$V(t) = \frac{-cr + (\lambda_h - \lambda_\ell)\left[p^g(t)(hr+U) + (1 - p^g(t))W(t)\right] + \lambda_\ell (hr + U)}{\lambda_h}.$$

Substituting the value function $V(t), W(t)$ and the equilibrium effort into the Bellman equation (34), we can easily verify that the Bellman equation is satisfied. Moreover, $\lim_{t \uparrow t^*} V(t)$ is equal to the stationary value as in (33).

Notice, that the lower bound on the prior belief $p_0 \geq \overline{p}^I$ implies $f^{G,*} \geq 0$. The exit rate is positive and hence well-defined. Moreover, if one of the conditions in Lemma 2 holds, then the effort level in the no-exit phase is interior and equal to (8).

**Drop in effort levels at** $t^*$ We next show that at $t^*$, the effort level decreases discontinu-

[32]Note that time always moves forward, so only the right-hand derivative of the value functions $V, W$ matter here. It turns out that $V, W$ are not of class $C^1$.

ously. That is, $\lim_{t \uparrow t^*} k_i^{N,*}(t) > k^{G,*}$. Solving $p_0$ as a function of $k^{G,*}$ and substituting $p_0$ into $\lim_{t \uparrow t^*} k_i^{N,*}(t) > k^{G,*}$, the inequality is equivalent to

$$(ck^{G,*}\lambda_h + cr - \lambda_h(hr + U)) \left(hk^{G,*}\lambda_\ell(c - h\lambda_\ell)(k^{G,*}\lambda_h + r) - U(r(c - h\lambda_h) + ck^{G,*}\lambda_h) + \lambda_h U^2\right) < 0.$$

This can be shown to be true, based on the observation that $ck^{G,*}\lambda_h + cr - \lambda_h(hr + U) < 0$. □

*Proof of Proposition 3.* From the Bellman equation (18), we can solve for the value function and effort level and obtain (19).

It is easy to check for each of the cases $(i)$ – $(iii)$ that for effort levels given by (19) it holds that $k_j^I(t)\lambda_\ell h < U$, and hence it is optimal for informed players to exit immediately.

(i) For $U \leq \lambda_\ell h$ and $p_0 < \overline{p}^I$, the effort level $k_j^I(t)$ given by (19) is interior. If informed players exit immediately, on-path players attach probability 1 to their opponent being uninformed.

(ii) If $U < c$, one obtains that for equilibrium efforts given by (19) to be interior, it must hold that

$$p_0 \leq \frac{c - h\lambda_\ell + (c - U)\lambda_h / r}{h(\lambda_h - \lambda_\ell)},$$

which is equivalent to

$$r\left[\lambda^s(p_0)\left(h + \frac{U}{r}\right) + \beta(1 - p_0)\frac{U}{r} - c\right] \leq c - (1 - p_0)\left(\lambda_\ell + \beta\right)U.$$

(iii) For $U \geq c$, is holds that $k_j^I(t) \geq 1$, and hence uninformed players exert maximal effort in equilibrium. In order to guarantee that uninformed players want to stay with the project and exert effort, it must be that the value function $V$ given by (19) is greater than $U$. This is the case, if and only if

$$p_0 \geq \frac{c - 2\lambda_\ell h + U}{2h(\lambda_h - \lambda_\ell)} = p^{c,*}, \tag{35}$$

that is, if the prior belief is higher than the cooperative threshold.

□

*Proof of Proposition 4 and Lemma 3.*

We show that under the conditions in Proposition 4, there exist a three-phase equilibrium.

We do that, by verifying that the strategy and belief profile identified in Proposition 4 is an equilibrium.

**Step 1:**

When the discount rate $r$ is sufficiently small, that is $r \leq \frac{U(\lambda_h-\lambda_\ell-\beta)(c-h\lambda_\ell)}{h^2\lambda_\ell(\lambda_h-\lambda_\ell)}$, the threshold belief $\overline{p}^I$ is negative. It is readily verified that, for any $p_0 \in \left(\frac{c-\lambda_\ell h}{\lambda_h h-\lambda_\ell h}, 1\right)$, the effort level based on the immediate-exit formula (23) satisfies the condition that $k_j^{I,*}\lambda_\ell h < U$. There exists an equilibrium consisting of just one immediate-exit phase. From now on, we focus on the parameter region such that $r > \frac{U(\lambda_h-\lambda_\ell-\beta)(c-h\lambda_\ell)}{h^2\lambda_\ell(\lambda_h-\lambda_\ell)}$, in which case the threshold belief $\overline{p}^I$ is strictly positive.

If the prior belief $p_0$ is below $\overline{p}^I$, there exists an equilibrium consisting of just one immediate-exit phase. If there exists $p_0 \in (0,1)$ such that $p_0 > \overline{p}^I$, it must be true that $\overline{p}^I < 1$. The constraint $\overline{p}^I < 1$ is equivalent to

$$r > \frac{\lambda_h U(c-h\lambda_\ell)}{h\lambda_\ell(h\lambda_h-c)}. \tag{36}$$

For all lower discount rate $r$, $\overline{p}^I \geq 1$. Therefore, for any $p_0 \in \left(\frac{c-\lambda_\ell h}{\lambda_h h-\lambda_\ell h}, 1\right)$, the effort level based on the immediate-exit formula (23) satisfies the condition that $k_j^{I,*}\lambda_\ell h < U$. There exists an equilibrium consisting of just one immediate-exit phase. From now on, we focus on the parameter region such that (36) holds and $p_0 > \overline{p}^I$.

**Step 2: Verifying equilibrium effort levels and exit rates**

Suppose player $j$ follows the equilibrium strategy $(k_j^*, f_j^*)$. We want to show that player $i$ finds it optimal to choose the equilibrium effort and exit levels. To do so, we formulate player $i$'s problem as a control problem with free endpoint: The uninformed player $i$ chooses his effort level and when to exit.

Conditional on no success and no exit of the opponent, let $p^g(t)$, $p^{b,ii}(t)$, $p^{b,iu}(t)$, $p^{b,ui}(t)$, $p^{b,uu}(t)$ denote the following probabilities: (i) the state is good; (ii) the state is bad and both are informed; (iii) the state is bad and only $i$ is informed; (iv) the state is bad and only $j$ is informed; (v) the state is bad and no player is informed. With the complementary probability, either a success has occurred or the opponent has exited. In both cases, player $i$ has switched the outside option. According to the construction of the equilibrium strategy $(k_j^*, f_j^*)$ (cf. Section 5), an informed player prefers to stay with the project in the no-exit phase, is indifferent between staying and exiting in the gradual-exit phase, and strictly prefers to take the outside option in the immediate-exit phase. If player $j$ exerts effort $k_j(t)$, then an uninformed player $i$'s flow payoff (net of $U$) at time $t$ is given by

$$\begin{aligned} f_0(t) =& h\left[(\lambda_h p^g(t) + \lambda_\ell p^{b,uu}(t))\left(\tilde{k}_i(t) + k_j(t)\right) + \lambda_\ell\left(\tilde{k}_i(t)p^{b,ui}(t) + k_j(t)p^{b,iu}(t)\right)\right] \\ & - \left(c\tilde{k}_i(t) + U\right)(p^g(t) + p^{b,ui}(t) + p^{b,uu}(t)) \\ & + (p^{b,ii}(t) + p^{b,iu}(t)) \max\left\{0, \frac{p^{b,iu}(t)}{p^{b,ii}(t) + p^{b,iu}(t)} k_j(t)\lambda_\ell h - U\right\}. \end{aligned}$$

We define two state variables

$$w_1(t) = e^{-\lambda_h \int_0^t \tilde{k}_i(s)\mathrm{d}s}, \quad w_2(t) = e^{-(\lambda_\ell+\beta)\int_0^t \tilde{k}_i(s)\mathrm{d}s},$$

and let $\gamma_1(t)$ and $\gamma_2(t)$ be the associated costate variables. For ease of exposition, we also let

$$x_1(t) = e^{-\lambda_h \int_0^t k_j(s)\mathrm{d}s}, \quad x_2(t) = e^{-(\lambda_\ell+\beta)\int_0^t k_j(s)\mathrm{d}s}.$$

Since $k_j$ is given, $x_1(t)$ and $x_2(t)$ are given functions of time. Substituting $w_1' = -\lambda_h \tilde{k}_i w_1$ and $w_2' = -(\beta + \lambda_\ell)\tilde{k}_i w_2$, we obtain the Hamiltonian of this problem:

$$\mathcal{H}\left(\tilde{k}_i, w_1, w_2, \gamma_1, \gamma_2, t\right) = e^{-rt} f_0(t) - \tilde{k}_i(t)\left[(\beta + \lambda_\ell)\gamma_2(t)w_2(t) + \lambda_h \gamma_1(t)w_1(t)\right].$$

During the no-exit phase, the probabilities $p^g(t), p^{b,ui}(t), p^{b,uu}(t)$ are given as follows:

$$\begin{aligned} & p^g(t) = p_0 w_1(t)x_1(t), \quad p^{b,ui}(t) = (1-p_0)\frac{\beta w_2(t)(1-x_2(t))}{\beta+\lambda_\ell}, \text{ and} \\ & p^{b,uu}(t) = (1-p_0)w_2(t)x_2(t). \end{aligned} \tag{37}$$

The probability that player $i$ is informed is

$$p^{b,ii}(t) + p^{b,iu}(t) = \frac{\beta(1-p_0)(1-w_2(t))(\beta + \lambda_\ell x_2(t))}{(\beta+\lambda_\ell)^2},$$

and the probability that player $j$ is uninformed conditional on player $i$ being informed is

$$q^u(t) = \frac{p^{b,iu}(t)}{p^{b,ii}(t) + p^{b,iu}(t)} = \frac{(\beta+\lambda_\ell)x_2(t)}{\beta + \lambda_\ell x_2(t)}.$$

An informed player $i$ (weakly) prefers to take the outside option if $q^u(t)k_j(t)\lambda_\ell h - U \le 0$. Substituting the above probabilities into $\mathcal{H}\left(\tilde{k}_i, w_1, w_2, \gamma_1, \gamma_2, t\right)$, we obtain that the Hamiltonian is linear in the state variables $w_1, w_2$.[33] The derivative $\frac{\partial \mathcal{H}}{\partial \tilde{k}_i(t)}$ equals zero for all $t \in [0, t_1)$ if and

[33]It will turn out that the Hamiltonian is linear in $w_1, w_2$ during the gradual-exit and immediate-exit

only if both $\frac{\partial\left(\partial\mathcal{H}/\partial\tilde{k}_i(t)\right)}{\partial t}$ and $\frac{\partial\mathcal{H}}{\partial\tilde{k}_i(t)}$ equal zero for all $t\in[0,t_1)$. Substituting $\gamma_1',\gamma_2',w_1',w_2',x_1',x_2'$ into the equation $\frac{\partial\left(\partial\mathcal{H}/\partial\tilde{k}_i(t)\right)}{\partial t}=0$, we obtain the equilibrium effort

$$k_j(t)=\frac{p^g(t)(\lambda_h(hr+U)-cr)+(p^{b,ui}(t)+p^{b,uu}(t))(\lambda_\ell(hr+U)-cr)}{c(\lambda_\ell p^{b,uu}(t)+\lambda_h p^g(t))}.$$

This is the same formula that we obtain from the heuristic argument in Section 4. Notice that the equation defining the effort level is the same as in the stationary case. However, efforts in the general case generically differ, since the motion of beliefs in (4) depend on $\beta$. The condition $\frac{\partial\mathcal{H}}{\partial\tilde{k}_i(t)}=0$ requires that for all $t\in[0,t_1)$

$$(\beta+\lambda_\ell)\gamma_2(t)w_2(t)+\lambda_h\gamma_1(t)w_1(t)=e^{-rt}((h\lambda_\ell-c)(p^{b,ui}(t)+p^{b,uu}(t))+p^g(t)(h\lambda_h-c)).\quad(38)$$

Let $t_1^*$ denote the transition time from the no-exit to the gradual-exit phase, and $t_2^*$ denote the transition time from the gradual-exit to the immediate-exit phase. During the gradual-exit phase, an informed player $i$ is indifferent between exiting and not. The last term in $f_0(t)$ is always zero. the evolution of $p^g(t),p^{b,uu}(t)$ is the same as in the no-exit phase. The evolution of $p^{b,ui}(t)$ incorporates player $j$'s exit behavior:

$$p^{b,ui}(t)=w_2(t)e^{F_j(t)}\left(\beta(1-p_0)\int_{t_1}^{t}e^{-F_j(s)}k_j(s)x_2(s)\mathrm{d}s+\frac{p^{b,ui}(t_1)}{w_2(t_1)}\right),$$

where $F_j(t)=-\int_{t_1}^{t}f_j(s)\mathrm{d}s$. The derivative $\frac{\partial\mathcal{H}}{\partial\tilde{k}_i(t)}$ equals zero for all $t\in[t_1^*,t_2^*)$ if and only if both $\frac{\partial\left(\partial\mathcal{H}/\partial\tilde{k}_i(t)\right)}{\partial t}$ and $\frac{\partial\mathcal{H}}{\partial\tilde{k}_i(t)}$ equal zero for all $t\in[t_1^*,t_2^*)$. Substituting $\gamma_1',\gamma_2',w_1',w_2',x_1',x_2'$ into the equation $\frac{\partial\left(\partial\mathcal{H}/\partial\tilde{k}_i(t)\right)}{\partial t}=0$, we obtain the equilibrium exit rate:

$$\begin{aligned}f_j(t)=&\frac{p^{b,uu}(t)(U(\beta+\lambda_\ell)+h\lambda_\ell r-\lambda_\ell k_j(t)(\beta h+c)-cr)+p^g(t)(\lambda_h(hr+U)-c(\lambda_h k_j(t)+r))}{p^{b,ui}(t)(c-h\lambda_\ell)}\\&+\frac{(U(\beta+\lambda_\ell)-cr+h\lambda_\ell r)}{(c-h\lambda_\ell)}.\end{aligned}$$

This is the same formula that we obtain from the heuristic argument. The condition $\frac{\partial\mathcal{H}}{\partial\tilde{k}_i(t)}=0$ requires that for all $t\in[t_1^*,t_2^*)$

$$(\beta+\lambda_\ell)\gamma_2(t)w_2(t)+\lambda_h\gamma_1(t)w_1(t)=e^{-rt}((h\lambda_\ell-c)(p^{b,ui}(t)+p^{b,uu}(t))+p^g(t)(h\lambda_h-c)).\quad(39)$$

phase as well.

From the construction of $(k_j^*, f_j^*)$, we know that $\lim_{t\uparrow t_2^*} p^{b,ui}(t) = 0$.[34] Informed players strictly prefer to exit immediately after $t_2^*$. Therefore, if an player $i$ observes no exit of his opponent, he believes that his opponent is uninformed. Thus, $p^{b,ui}(t)$ remains zero for all $t \geq t_2^*$. Player $i$'s flow payoff (net of $U$) at time $t$ is given by

$$h\left[(\lambda_h p^g(t) + \lambda_\ell p^{b,uu}(t))\left(\tilde{k}_i(t) + k_j(t)\right)\right] - \left(c\tilde{k}_i(t) + U\right)(p^g(t) + p^{b,uu}(t)).$$

The derivative $\frac{\partial \mathcal{H}}{\partial \tilde{k}_i(t)}$ equals zero for all $t \in [t_2^*, t_3^*)$ if and only if both $\frac{\partial\left(\partial \mathcal{H}/\partial \tilde{k}_i(t)\right)}{\partial t}$ and $\frac{\partial \mathcal{H}}{\partial \tilde{k}_i(t)}$ equal zero for all $t \in [t_2^*, t_3^*)$. Substituting $\gamma_1', \gamma_2', w_1', w_2', x_1', x_2'$ into the equation $\frac{\partial\left(\partial \mathcal{H}/\partial \tilde{k}_i(t)\right)}{\partial t} = 0$, we obtain the equilibrium effort level:

$$k_j(t) = \frac{p^{b,uu}(t)(U(\beta + \lambda_\ell) + r(h\lambda_\ell - c)) + p^g(t)(\lambda_h(hr + U) - cr)}{c(\beta + \lambda_\ell)p^{b,uu}(t) + c\lambda_h p^g(t)}.$$

This is the same formula that we obtain from the heuristic argument. The condition $\frac{\partial \mathcal{H}}{\partial \tilde{k}_i(t)} = 0$ requires that for $t \geq t_2^*$

$$(\beta + \lambda_\ell)\gamma_2(t)w_2(t) + \lambda_h \gamma_1(t) w_1(t_2) = e^{-rt}(p^{b,uu}(t)(h\lambda_\ell - c) + p^g(t)(h\lambda_h - c)).$$

This is consistent with (38) and (39) since $p^{b,ui}(t)$ equals zero for $t \geq t_2^*$. The exit time of an uninformed player is denoted by $t_3^*$. It is chosen such that the posterior belief that the state is good, that is $q^g(t) = \frac{p^g(t)}{p^g(t) + p^{b,uu}(t)}$, equals $\frac{c - \lambda_\ell h}{(\lambda_h - \lambda_\ell)h}$. In this case, the equilibrium effort at $t_3^*$ is equal to $U/c$. Player $i$'s flow payoff at $t_3^*$ is exactly zero.

**Step 3: Transition times**

Next, we determine the transition times $t_1^*, t_2^*$. We show that there exists a unique pair $t_1^*, t_2^*$ such that, if the game proceeds to the gradual-exit phase at $t_1^*$, the probability $p^{b,ui}(t)$ approaches zero at $t = t_2^*$. We know moreover that, for informed players to be willing to exit immediately starting from $t_2^*$, the belief that the state is good at time $t_2^*$ must be below $\overline{p}^I$. Consider the two probabilities:

$$q^g(t) := \frac{p^g(t)}{p^g(t) + p^{b,uu}(t)}, \quad q^u(t) := \frac{p^{b,iu}(t)}{p^{b,ii}(t) + p^{b,iu}(t)}.$$

Here, $q^g(t)$ is the probability of state is good conditional on both players being uninformed; and $q^u(t)$ is the probability that the opponent is uninformed conditional on player $i$ being informed. Both $q^g(t)$ and $q^u(t)$ are in $[0, 1]$, and hence well-defined. The evolution of the beliefs $q^g(t), q^u(t)$ during the no-exit phase is given by (37). An informed player is willing to

[34]We will formally verify this in Step 3.

stay during the no-exit phase, if and only if $q^u(t)k_j^{N,*}(t)\lambda_\ell h \geq U$. Let $\hat{t}_1$ be the minimum time at which this inequality holds with equality. The transition time $t_1^*$ must satisfy $t_1^* \in [0, \hat{t}_1]$.

Substituting the equilibrium effort level and the exit rate, we obtain the evolution of the beliefs $q^g(t), q^u(t)$ during the gradual-exit phase:

$$q^{g\prime}(t) = -\frac{2(1-q^g(t))q^g(t)U(\lambda_h - \beta - \lambda_\ell)}{h\lambda_\ell q^u(t)}, \quad q^{u\prime}(t) = \frac{H_1(q^u(t))q^g(t) + H_2(q^u(t))}{h\lambda_\ell(1-q^g(t))(c-h\lambda_\ell)},$$

where the functions $H_1(\cdot)$ and $H_2(\cdot)$ are defined as follows:

$$H_1(q^u) = h\lambda_\ell(\lambda_h(hr+U) - cr)\,(q^u)^2 + (h\lambda_\ell r(c-h\lambda_\ell) - c\lambda_h U)q^u + U(\beta+\lambda_\ell)(c-h\lambda_\ell),$$
$$H_2(q^u) = (h\lambda_\ell - c)(U(\beta+\lambda_\ell) + h\lambda_\ell q^u r).$$

Note that if $q^u(t)$ ever reaches 1, it must be the case that $q^u(t)$ increases to 1 from below. This means that if $\lim_{t\to\hat{t}_2} q^u(t) = 1$, then $\exists\varepsilon > 0$ such that $q^{u\prime}(t) \geq 0$ for all $t \in \big(\hat{t}_2 - \varepsilon, \hat{t}_2\big)$. Substituting $q^u(t) = 1$ and $q^{u\prime}(t) \geq 0$, we obtain that $q^g(t) \geq \overline{p}^I$. On the other hand, it is required that when the game proceeds to the immediate-exit phase at $t_2^*$, $q^u(t_2^*) = 1$ and $q^g(t_2^*) \leq \overline{p}^I$. Therefore, if such a $t_2^*$ exists, it must be the case that $q^g(t_2^*) = \overline{p}^I$.

Notice that $q^{g\prime}(t)$ is always negative. The sign of $q^{u\prime}(t)$ depends on the location of the vector $(q^g(t), q^u(t)) \in [0,1]^2$. If the parameters are such that $H_1(q^u)$ is positive for all $q^u \in [0,1]$, then $q^{u\prime}(t)$ is positive if and only if $(q^g(t), q^u(t))$ lies above the line defined by

$$q^g(t) = -\frac{H_2(q^u(t))}{H_1(q^u(t))}.$$

If the parameters are such that $H_1(q^u)$ is negative for some $q^u \in (0,1)$, it is readily verified that the equation $H_1(q^u) = 0$ has two roots in $(0,1)$. For all $q^u \leq 1$ that are above the larger root, $q^{u\prime}(t)$ is positive if and only if $(q^g(t), q^u(t))$ lies above the line $q^g = -\frac{H_2(q^u)}{H_1(q^u)}$. Let us summarize some important observations:

(i) The beliefs at time zero are $(q^g(t), q^u(t)) = (p_0, 1)$. This vector lies above the line $q^g = -\frac{H_2(q^u)}{H_1(q^u)}$;

(ii) if the first phase ended at $\hat{t}_1$, the beliefs at $\hat{t}_1$, $(q^g(\hat{t}_1), q^u(\hat{t}_1))$, are such that $q^{u\prime}(t)$ is strictly negative;

(iii) The only legitimate belief at which the game can transition from the gradual-exit to the immediate-exit phase is $(q^g(t), q^u(t)) = (\overline{p}^I, 1)$. This vector is on the line $q^g = -\frac{H_2(q^u)}{H_1(q^u)}$.

This shows that there exists a $t_1^* \in (0, \hat{t}_1)$ such that if the game transitions from the first to the second phase at $t_1^*$, there exists a $t_2^* > t_1^*$ such that at $t_2^*$ it holds that $(q^g(t_2^*), q^u(t_2^*)) = (\overline{p}^I, 1)$, and the game moves to the immediate-exit phase.

Figure 9 illustrates an example.[35] The solid line corresponds to the equation $q^g = -\frac{H_2(q^u)}{H_1(q^u)}$. The derivative $q^{u\prime}(t)$ is positive above it and negative below it. The dashed line illustrates how the beliefs $(q^g(t), q^u(t))$ evolve in the no-exit phase. The beliefs start at $(p_0, 1)$ and move toward the origin along the dashed line. The point $D$ corresponds to the belief at time $\hat{t}_1$ if the no-exit phase lasts until $\hat{t}_1$. We need to choose a point on the dashed line at which the game proceeds to the gradual-exit phase. If the game proceeded to the gradual-exit phase at point $B$, the beliefs would exit the triangle $ABC$ at point $B$ (because $q^{u\prime} > 0$). If the game proceeded to the gradual-exit phase at point $C$, the beliefs would exit the triangle $ABC$ at point $C$ (because at $C$, $q^{u\prime} = 0$ and $q^{g\prime} < 0$). By continuity, there exits a point between $B$ and $C$ such that the beliefs exit the triangle $ABC$ at point $A$.

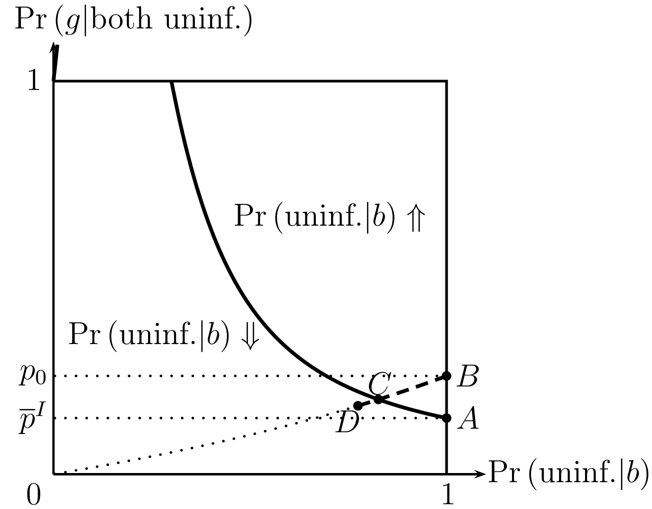


Figure 9: Evolution of the conditional beliefs

**Step 4:** (Proof of Lemma 3)
We find the sufficient conditions under which the boundary constraint $k_i \leq 1$ does not bind. The equilibrium effort in the no-exit phase is given by

$$k_i = \frac{cr((q^u - 1)q^g + 1) + (hr + U)(\lambda_\ell(q^g - 1) - \lambda_h q^u q^g)}{cq^u(\lambda_\ell(q^g - 1) - \lambda_h q^g)}.$$

The dependence of $k_i, q^u, q^g$ on $t$ is omitted. During the no-exit phase, both beliefs $q^u, q^g$ decrease in $t$. If $cr \geq \lambda_\ell(hr + U)$, both partial derivatives $\partial k_i/\partial q^g$ and $\partial k_i/\partial q^u$ are positive. Therefore, the highest effort during the no-exit phase occurs at time 0. The boundary

[35] Parameters are $\lambda_h = 1, \lambda_\ell = 1/3, \beta = 1/3, h = 1, c = 2/5, U = 1/20, r = 1/4, p_0 = 1/4$.

constraint $k_i \leq 1$ does not bind if and only if

$$p_0 \leq \frac{\frac{cr}{hr+U-c} - \lambda_\ell}{\lambda_h - \lambda_\ell}.$$

This is the same condition as in the stationary case.

If $cr < \lambda_\ell(hr + U)$, the partial derivative $\partial k_i/\partial q^u$ is negative. The partial derivative $\partial k_i/\partial q^g$ is positive if and only if

$$q^u \geq 1 - \frac{cr(\lambda_h - \lambda_\ell)}{\lambda_\ell(\lambda_h(hr + U) - cr)} := q^*.$$

Note that if $q^g = p_0$ and $q^u$ equals $q^*$ as defined above, the derivative $(q^u)'(t)$ is negative. Therefore, the beliefs $(q^u, q^g)$ during the no-exit phase is confined to the region $[q^*, 1] \times [0, p_0]$, and must satisfy the condition that $(q^u)'(t) > 0$. It is easily verified that when $cr < \lambda_\ell(hr+U)$, there exists a unique $q^{**} \in (q^*, 1)$ such that $(q^u)'(t) = 0$ when $(q^u, q^g) = (q^{**}, p_0)$. (Note that $q^{**}$ is a function of $p_0$.) The beliefs $(q^u, q^g)$ during the no-exit phase is further confined to the region $[q^{**}, 1] \times [0, p_0]$. The boundary constraint $k_i \leq 1$ does not bind if it does not bind when $(q^u, q^g) = (q^{**}, p_0)$. It is readily verified that $q^{**}$ decreases in $p_0$. Since $k_i$ increases in $q^g$ and decreases in $q^u$, $k_i$ at $(q^{**}, p_0)$ increases in $p_0$. When $p_0$ equals 1, $q^{**}$ and $k_i$ at $(q^{**}, p_0)$ are given by

$$q^{**} = \frac{c\lambda_h U}{h\lambda_\ell(\lambda_h(hr + U) - cr)}, \quad k_i = \frac{\lambda_h(hr + U) - cr}{c\lambda_h}.$$

If $\frac{\lambda_h(hr+U)-cr}{c\lambda_h} > 1$, there exists a $\tilde{p}_0 < 1$ such that $k_i$ at $(q^{**}, p_0)$ equals 1 when $p_0 = \tilde{p}_0$. The boundary constraint $k_i \leq 1$ does not bind if $p_0 \leq \tilde{p}_0$. If $\frac{\lambda_h(hr+U)-cr}{c\lambda_h} \leq 1$, the boundary constraint $k_i \leq 1$ does not bind for all $p_0 \in (0, 1)$. We let $\tilde{p}_0$ equal 1 in this case. To sum, the boundary constraint $k_i \leq 1$ does not bind if $p_0 \leq \tilde{p}_0$.

□

*Proof of Proposition Proposition 5.*

(*i*) and (*ii*): Given that $k_j^I(t)\lambda_\ell h < U$, it is optimal for an informed player to exit immediately. Let $p^g(t)$ denote the probability that the state is good and no success, signal or exit has occurred by time $t$, and $p^b(t)$ the probability that the state is bad and no success, signal or exit has occurred by time $t$:

$$p^g(t) = p_0 e^{-\lambda_h \int_0^t (\tilde{k}_i(s)+k_j(s))\mathrm{d}s}, \quad p^g(t) = (1 - p_0)e^{-(\beta+\lambda_\ell)\int_0^t (\tilde{k}_i(s)+k_j(s))\mathrm{d}s}.$$

We define two state variables $w_1(t) = e^{-\lambda_h \int_0^t (\tilde{k}_i(s)+k_j(s))\mathrm{d}s}$ and $w_2(t) = e^{-(\beta+\lambda_\ell)\int_0^t (\tilde{k}_i(s)+k_j(s))\mathrm{d}s}$

with $\gamma_1(t)$ and $\gamma_2(t)$ being the corresponding costate variables. The Hamiltonian of this problem is given by

$$\begin{aligned}\mathcal{H}(\tilde{k}_i, w_1, w_2, \gamma_1, \gamma_2, t) = & e^{-rt}\Big[p_0 w_1(t)(\tilde{k}_i(t)(h\lambda_h - c) + h\lambda_h k_j(t) - U) \\ & +(1-p_0)w_2(t)(\tilde{k}_i(t)(h\lambda_\ell - c) + h\lambda_\ell k_j(t) - U)\Big] \\ & - (\tilde{k}_i(t) + k_j(t))\left[(\beta+\lambda_\ell)\gamma_2(t)w_2(t) + \lambda_h\gamma_1(t)w_1(t)\right].\end{aligned} \tag{40}$$

We want to show that the equilibrium effort is given by (23), where $p(t)$ is the belief that the state is good conditional on no success, signal or exit by time $t$, which is given by

$$p(t) = \frac{p_0 w_1(t)}{p_0 w_1(t) + (1-p_0)w_2(t)}.$$

Taking the derivative of $\partial\mathcal{H}/\partial\tilde{k}_i(t)$ with respect to $t$ and substituting $w_1'(t), w_2'(t), \gamma_1'(t), \gamma_2'(t)$, we obtain that the sign of $\frac{\partial\left(\partial\mathcal{H}/\partial\tilde{k}_i(t)\right)}{\partial t}$ is the same as the sign of

$$k_j(t) - H(p(t)), \quad \text{where } \; H(p(t)) := \frac{\frac{r(h\lambda(p(t))-c)}{\beta(1-p(t))+\lambda(p(t))} + U}{c}.$$

If $H(p_0)$ is smaller than 1, $k_j(t)$ is chosen such that $\frac{\partial\left(\partial\mathcal{H}/\partial\tilde{k}_i(t)\right)}{\partial t}$ equals zero throughout. Then, the derivative $\partial\mathcal{H}/\partial\tilde{k}_i(t)$ is also equal to zero throughout. If $H(p_0)$ is greater than 1, $k_j(t)$ is equal to 1 whenever $H(p(t)) > 1$ holds. Note that $H(p(t))$ increases in $p(t)$. Therefore, there exists a time when $p(t)$ is sufficiently low such that $H(p(t))$ drops below 1. From then on, $k_j(t)$ equals $H(p(t))$ until $p(t)$ drops to $\frac{c-h\lambda_\ell}{h(\lambda_h-\lambda_\ell)}$, at which point both players opt out. Given this choice of the effort level, it is easily verified that $\partial\mathcal{H}/\partial\tilde{k}_i(t)$ is positive when $H(p(t))$ is above 1, and is zero when $H(p(t))$ drops below 1. The first order condition with respect to the control $\tilde{k}_i(t)$ is satisfied.

*(iii)* Given that $U \geq c > \lambda_\ell h$, it is optimal for an informed player to exit immediately. Again, the Hamiltonian of this problem is given by (40). We want to show that the two players exert full effort until the equilibrium belief of state $g$ equals $p^{c,*}$. First, given $\tilde{k}_i(t) = k_j(t) = 1$, the posterior belief of state $g$ at time $t$ is

$$\frac{p_0 e^{-\lambda_h 2t}}{p_0 e^{-\lambda_h 2t} + (1-p_0)e^{-(\beta+\lambda_\ell)2t}},$$

which is a strictly decreasing function at $t$. Let $T^c$ denote the time at which the posterior belief drops to $p^{c,*}$. It is easily verified that the flow payoff at $T^c$ given $\tilde{k}_i(t) = k_j(t) = 1$ is 0.

Second, the evolution of $\gamma_1, \gamma_2$ is given by

$$\begin{aligned}
\gamma_1'(t) &= p_0 e^{-rt}(\tilde{k}_i(t)(c - h\lambda_h) - h\lambda_h k_j(t) + U) + \lambda_h \gamma_1(t)(\tilde{k}_i(t) + k_j(t)) \\
&= p_0 e^{-rt}(c - 2h\lambda_h + U) + 2\lambda_h \gamma_1(t), \\
\gamma_2'(t) &= (\beta + \lambda_\ell)\gamma_2(t)(\tilde{k}_i(t) + k_j(t)) - (p_0 - 1)e^{-rt}(\tilde{k}_i(t)(c - h\lambda_\ell) - h\lambda_\ell k_j(t) + U) \\
&= 2(\beta + \lambda_\ell)\gamma_2(t) - (p_0 - 1)e^{-rt}(c - 2h\lambda_\ell + U).
\end{aligned}$$

and the boundary condition $\gamma_1(T^c) = \gamma_2(T^c) = 0$. Lastly, we show that the derivative of $\mathcal{H}$ with respect to $\tilde{k}_i(t)$ is positive. Substituting $\gamma_1(T^c) = \gamma_2(T^c) = 0$, we obtain the derivative $\frac{\partial \mathcal{H}}{\partial \tilde{k}_i}$ at time $T^c$:

$$e^{-rt}((p_0 - 1)w_2(t)(c - h\lambda_\ell) + p_0 w_1(t)(h\lambda_h - c)).$$

This is positive since $\frac{c - \lambda_\ell h}{h(\lambda_h - \lambda_\ell)} > p^{c,*}$. The derivative of $\frac{\partial \mathcal{H}}{\partial \tilde{k}_i}$ with respect to $t$ is

$$\begin{aligned}
\frac{\partial\left(\partial \mathcal{H}/\partial \tilde{k}_i\right)}{\partial t} =& e^{-rt}\left[p_0 e^{-2\lambda_h t}(c(\lambda_h + r) - \lambda_h(hr + U))\right. \\
&\left. + (1 - p_0)e^{-2t(\beta + \lambda_\ell)}(c(\beta + \lambda_\ell) - U(\beta + \lambda_\ell) + r(c - h\lambda_\ell))\right].
\end{aligned}$$

This is negative for all $t \leq T^c$. Therefore, $\partial \mathcal{H}/\partial \tilde{k}_i$ decreases in $t$, and is positive for all $t \geq T^c$. The Hamiltonian is concave in the state variables $w_1, w_2$, so these conditions are sufficient.

□

# B Formal discussion of the single-player optimal policy

As discussed in Subsection 3.1 when the belief that the state is good is above the single-player threshold $p^{s,*}$, the optimal strategy is to exert full effort until either a success or a signal arrives, and to then take the outside option. Here we provide a more formal argument of this result.

At any belief $p_t$, if a player exerts full effort $k_i(t) = 1$, his value function $V^s(p_t)$ must satisfy the following recursion.

$$\begin{aligned}
V^s(p_t) =& r\left(h(\lambda_\ell(1 - p_t) + \lambda_h p_t) - c\right) \mathrm{d}t \\
&+ e^{-r\mathrm{d}t}\left[\left((1 - p_t)\left(\beta + \lambda_\ell\right) + p_t \lambda_h\right) \mathrm{d}t(U - V^s(p_{t+\mathrm{d}t}) + V^s(p_{t+\mathrm{d}t})\right],
\end{aligned}$$

where $p_{t+\mathrm{d}t}$ is given by $p_t - (1 - p_t)p_t\left(\lambda_h - \lambda_\ell - \beta\right) \mathrm{d}t$ and $V^s(p_{t+\mathrm{d}t})$ is

$$V^s(p_{t+\mathrm{d}t}) = V^s(p_t) - (1 - p_t)p_t(\lambda_h - \lambda_\ell - \beta)(V^s)'(p_t)\mathrm{d}t.$$

Here, the first part captures the instantaneous benefits and costs from exerting effort, where $(\lambda_\ell(1-p_t)+\lambda_h p_t)$ is the probability of an instantaneous success. The second term captures the expected payoffs tomorrow. If a success or a signal arrives today, then tomorrow's payoff will be $U$, otherwise the player's payoff equals tomorrow's continuation payoff, $V^s(p_{t+\mathrm{d}t})$.

Let $p^{s,*}$ be the threshold at which the player takes the outside option. The value-matching condition $V^s(p^{s,*}) = U$ and the smooth-pasting condition $(V^s)'(p^{s,*}) = 0$ allow us to solve for the unique value function $V^s(p_t)$ and the threshold $p^{s,*}$. It is readily verified that $p^{s,*}$ is chosen such that the flow payoff from exerting full effort $h\lambda^s(p^{s,*}) - c$ equals $U$. If the prior belief is below $p^{s,*}$, it is optimal for a single player to take the outside option at time 0.

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