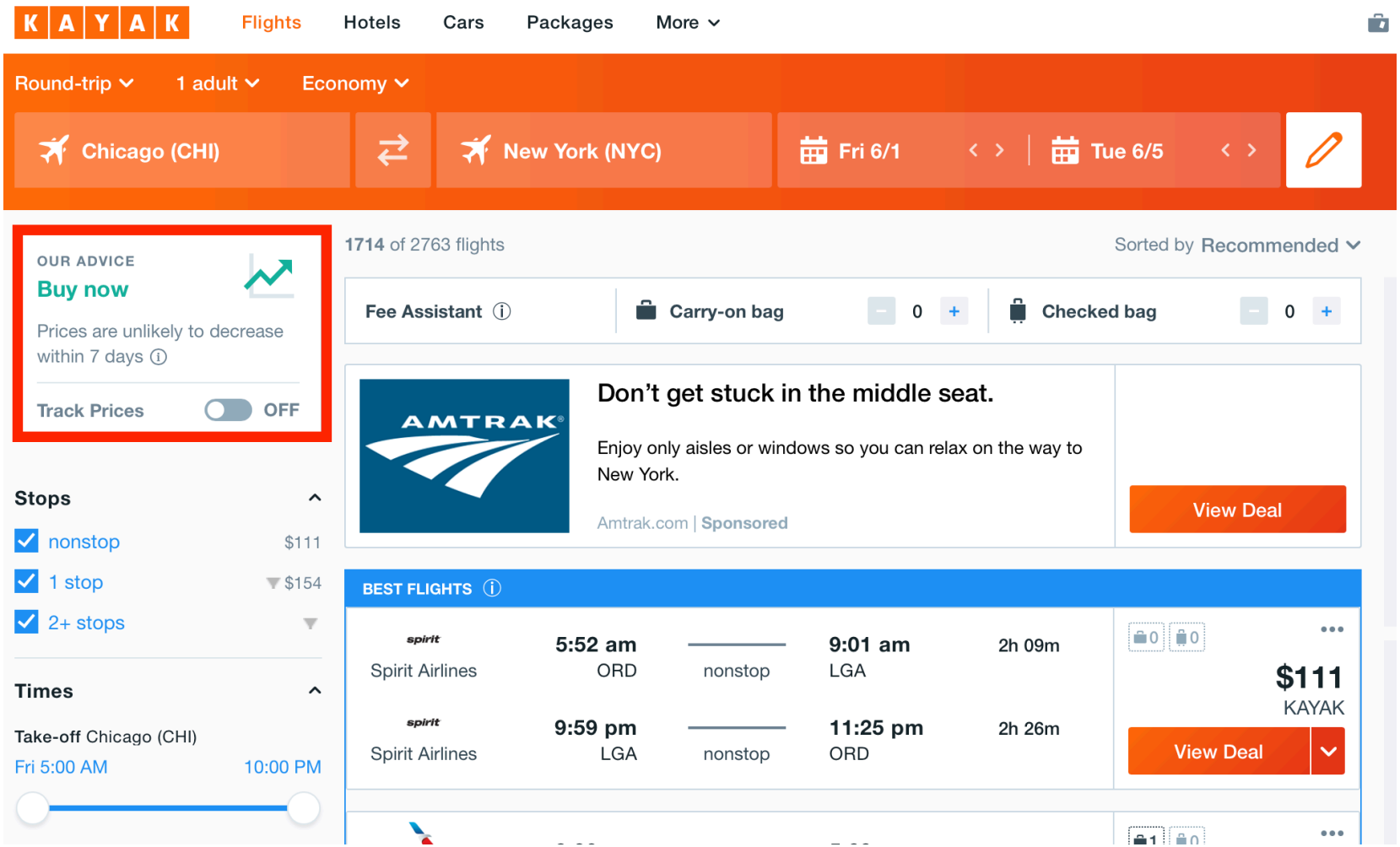
Costly Miscalibration

Yingni Guo, Eran Shmaya

Seminar Series on Communication and Persuasion

E-commerce platforms often provide information to customers about their products.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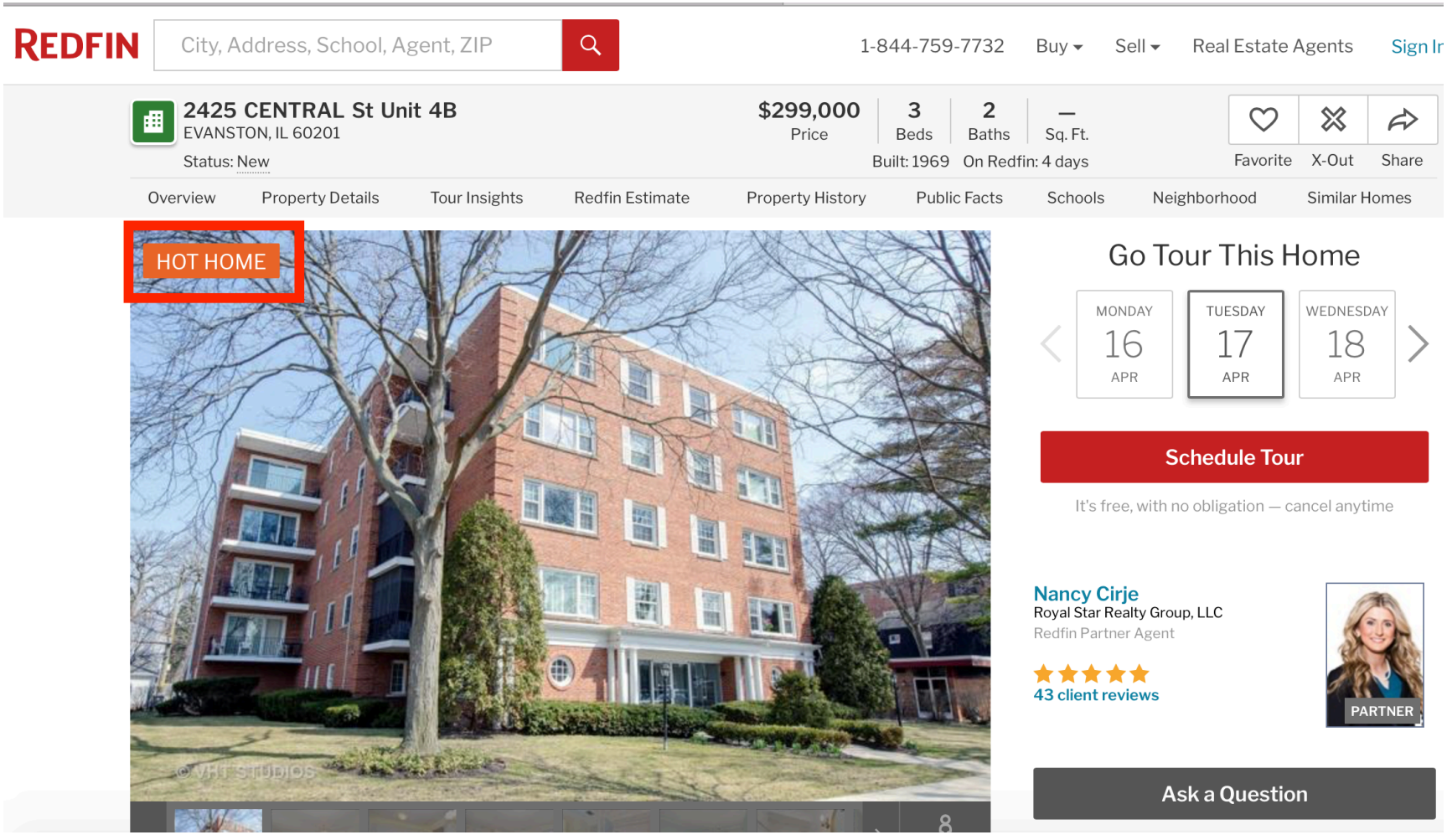

A hot home has a 70% chance or higher of having an accepted offer within two weeks of its debut.

What we do

- We model the communication between an online platform and its customers.
- We introduce a concept of noncheap talk called costly miscalibration.

Roadmap

1. Basic setup
2. Our model
3. High cost-intensity results
   - Rationalizable communication
4. Promotion games

Basic setup

- Two players: Sender and Receiver.
- State space: S.
- Action space: A.
- Sender's payoff: $v(s, a)$.
- Receiver's payoff: $u(s, a)$.
- The prior p has full support.

Basic setup

- Sender chooses $\sigma : S \to \Delta(M)$.
- Receiver chooses $\rho : M \to \Delta(A)$.
- A strategy profile (σ, ρ) induces a distribution $\pi_{\sigma,\rho} \in \Delta(S \times A)$.
- Sender's payoff from (σ, ρ) is

$$V(\sigma, \rho) = \int v \, \mathrm{d}\pi_{\sigma,\rho}$$

Receiver's payoff is

$$U(\sigma, \rho) = \int u \, \mathrm{d}\pi_{\sigma,\rho}.$$

Cheap-talk equilibrium

- Payoff functions V, U define a normal-form game between Sender and Receiver.
- This is a cheap-talk game as in Crawford and Sobel (1982).
- A BNE for the cheap-talk game is a Nash equilibrium in the normal form game defined by V, U:

$$V(\sigma^*, \rho^*) \geqslant V(\sigma, \rho^*), \ \forall \sigma$$
$$U(\sigma^*, \rho^*) \geqslant U(\sigma^*, \rho), \ \forall \rho.$$

Commitment payoff

- The commitment solution occurs when Sender's strategy σ is observable (e.g., Rayo and Segal (2010), Kamenica and Gentzkow (2011)).
- The only requirement is Receiver's incentive constraints.
- The commitment payoff for Sender is:

$$CP = max_{\sigma,\rho} V(\sigma, \rho)$$

over all profiles (σ, ρ) such that ρ is a best response to σ.

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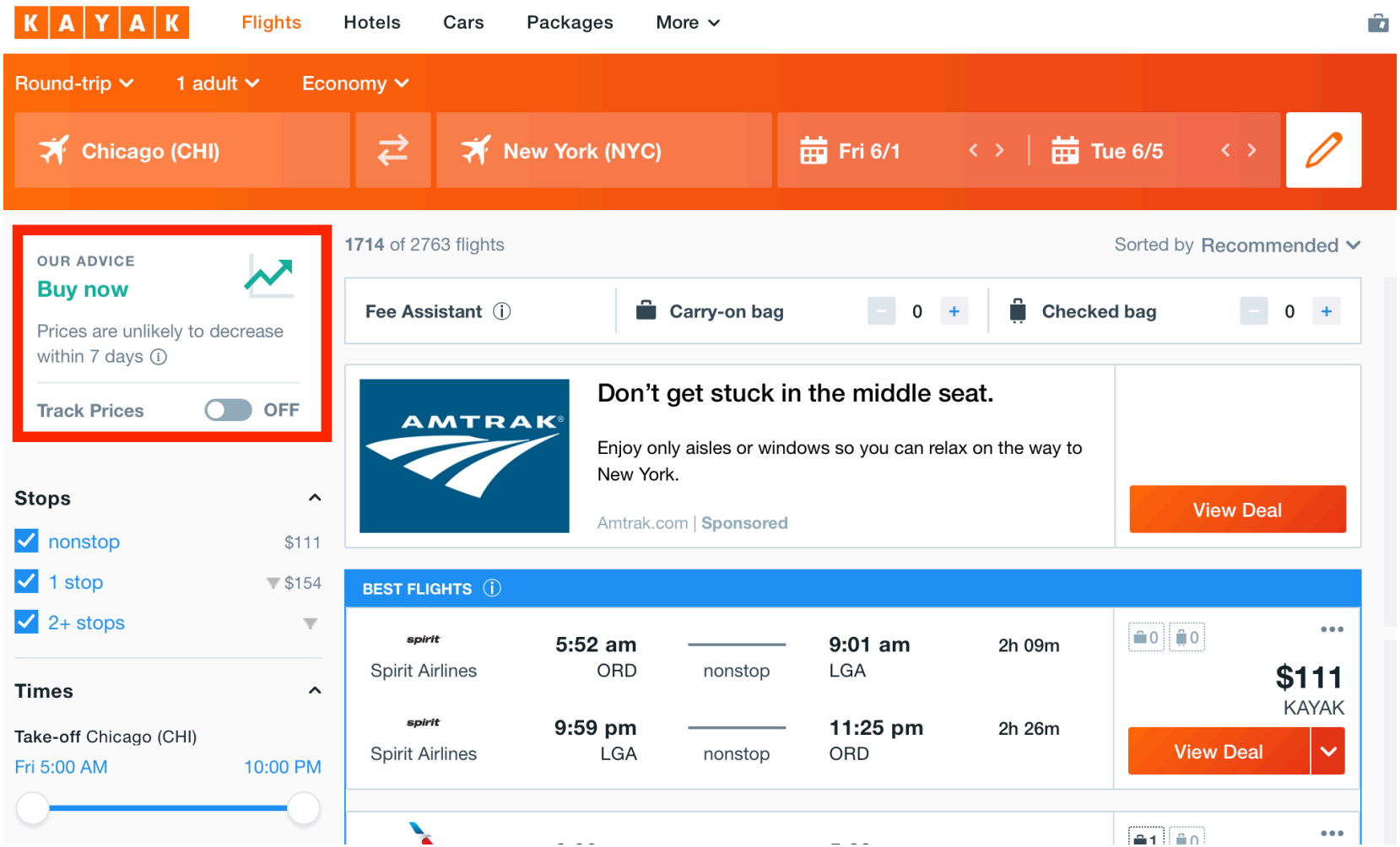
Our model

- The observability of Sender's strategy is a strong assumption in many applications.

  Kayak: *Our scientists develop these flight price trend forecasts using algorithms and mathematical models.*

  Redfin: *The hot homes algorithm automatically calculates the likelihood by analyzing more than 500 attributes of each home – including price, property type, and neighborhood – as well as buyer preferences in the area.*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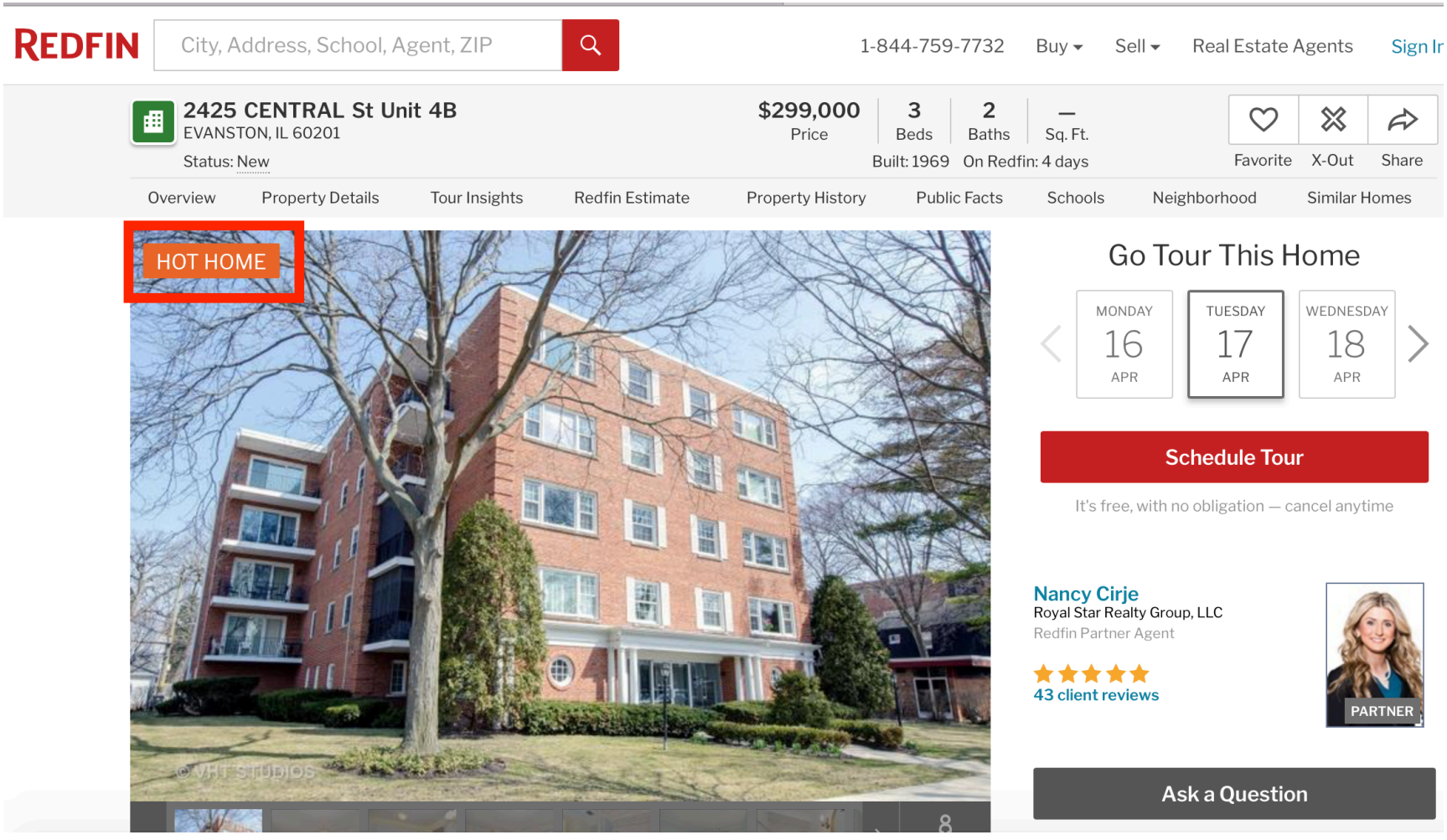

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- Sender's strategy is not observable. This is the same as cheap-talk models.
- Sender's talk is not cheap. Sender applies his algorithm on many products, which allows us to do some calibration test.

Calibration

- Let X be a S-valued r.v..
- Let Y be a $\Delta(S)$-valued r.v..
- Y is a probability assertion about X.
- Y is calibrated if $\Pr(X = s|Y) = Y(s)$ (e.g., Dawid (1982), Olszewski (2015)).
  - The weatherman makes forecasts for every district.
  - We look at those districts in which he says 30% rain, and indeed it was raining in 30% of those districts.
  - We look at those districts in which he says 70% rain, and indeed it was raining in 70% of those districts.

Calibrated Sender's strategies

- $M = \Delta(S)$ is exogenously given:
  - a message m is an asserted distribution over states.
- Let $\tau_\sigma(m)$ be the true distribution given a message m:

$$\tau_\sigma(s|m) = \frac{p(s)\sigma(m|s)}{\sum_{s'} p(s')\sigma(m|s')}, \quad \forall m \in \text{support}(\sigma).$$

- A Sender's strategy σ is calibrated if $\tau_\sigma(m) = m$, a.s..
- Sender's message can be taken at its face value.

Miscalibration

- Let $d : \Delta(S) \times \Delta(S) \to \mathbf{R}_+$ be a continuous function.
  - $d(q, m)$ measures the distance between a message $m \in \Delta(S)$ and a truth $q \in \Delta(S)$.
  - $d(q, m) = 0$ if and only if $m = q$.
- The expected distance between Sender's message and the truth is:

$$\kappa(\sigma) = \sum_s p(s) \int d(\tau_\sigma(m), m)\ \sigma(\mathrm{d}m|s)$$

- σ is calibrated if and only if $\kappa(\sigma) = 0$.

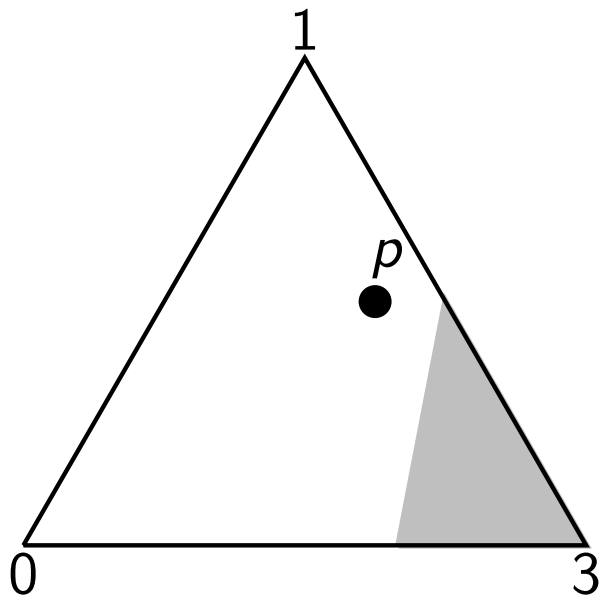
Calibrated strategy and miscalibration: an example

- $S = \{0, 1, 3\}$ with the prior $p = \left(\frac{1}{8}, \frac{1}{2}, \frac{3}{8}\right)$; $A = \{B, NB\}$.
- Sender's payoff is

$$v(s, B) = 1, \quad \text{and} \quad v(s, NB) = 0.$$

- Receiver's payoff is

$$u(s, B) = s - 2, \quad \text{and} \quad u(s, NB) = 0.$$



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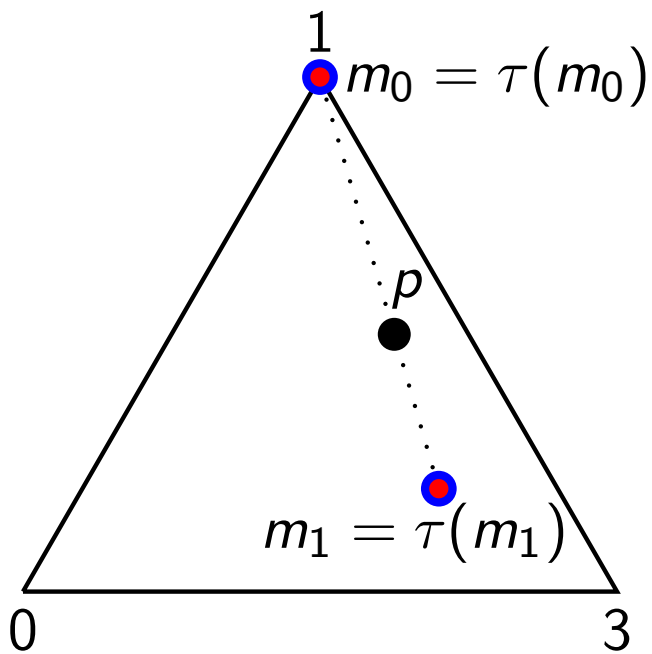
Calibrated strategy and miscalibration: an example

message \ state	0	1	3
$m_0 = (0, 1, 0)$	0	$\frac{3}{4}$	0
$m_1 = \left(\frac{1}{5}, \frac{1}{5}, \frac{3}{5}\right)$	1	$\frac{1}{4}$	1

A calibrated strategy



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A calibrated strategy

message \ state	0	1	3
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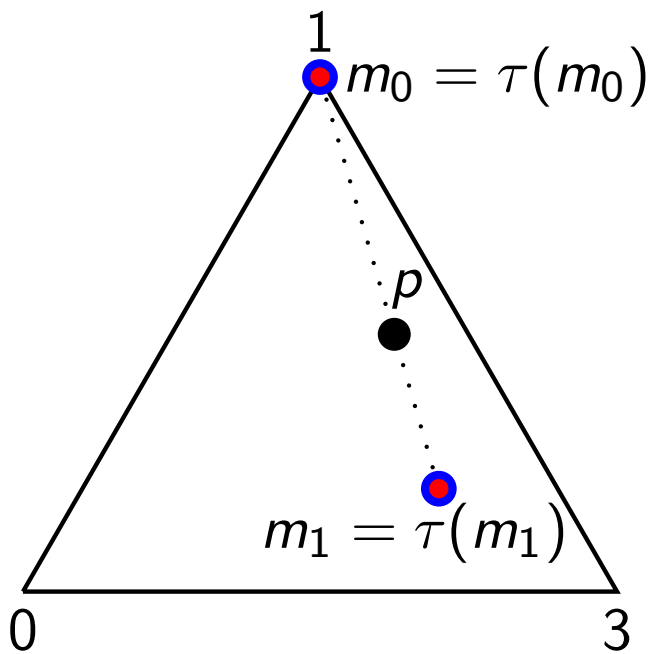
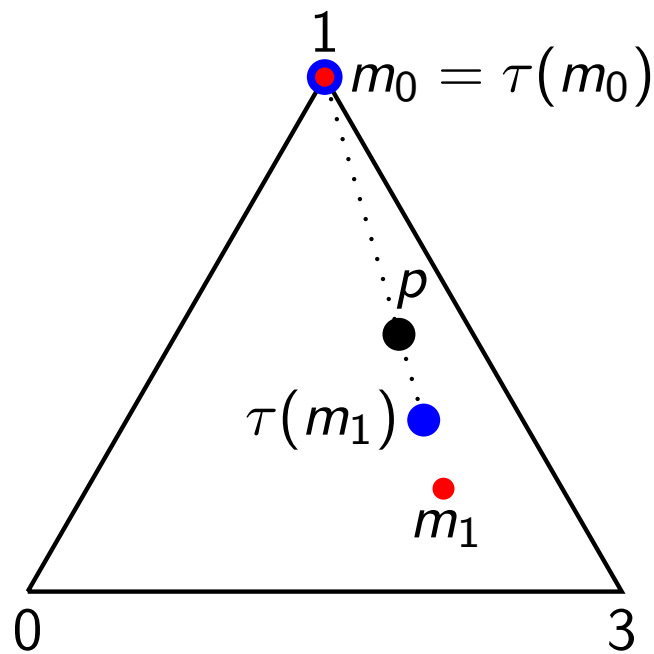
$m_1 = \left(\frac{1}{5}, \frac{1}{5}, \frac{3}{5}\right)$	1	$\frac{1}{2}$	1

A miscalibrated strategy





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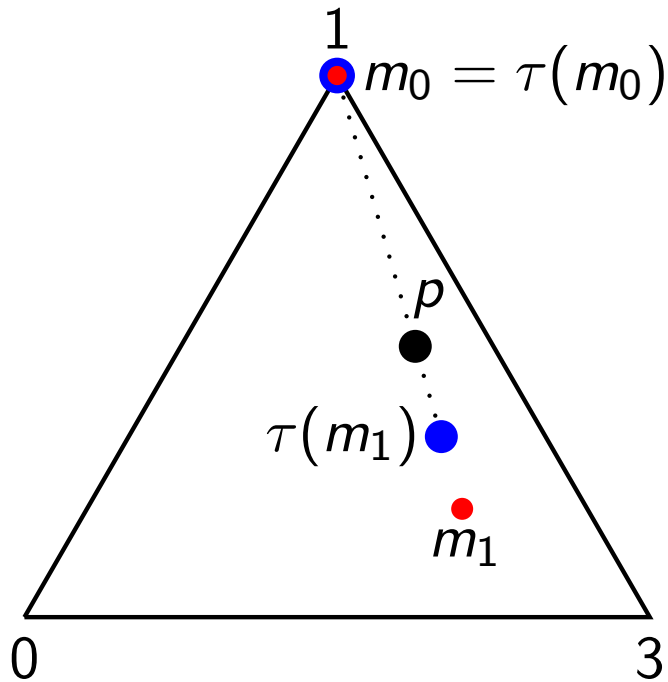
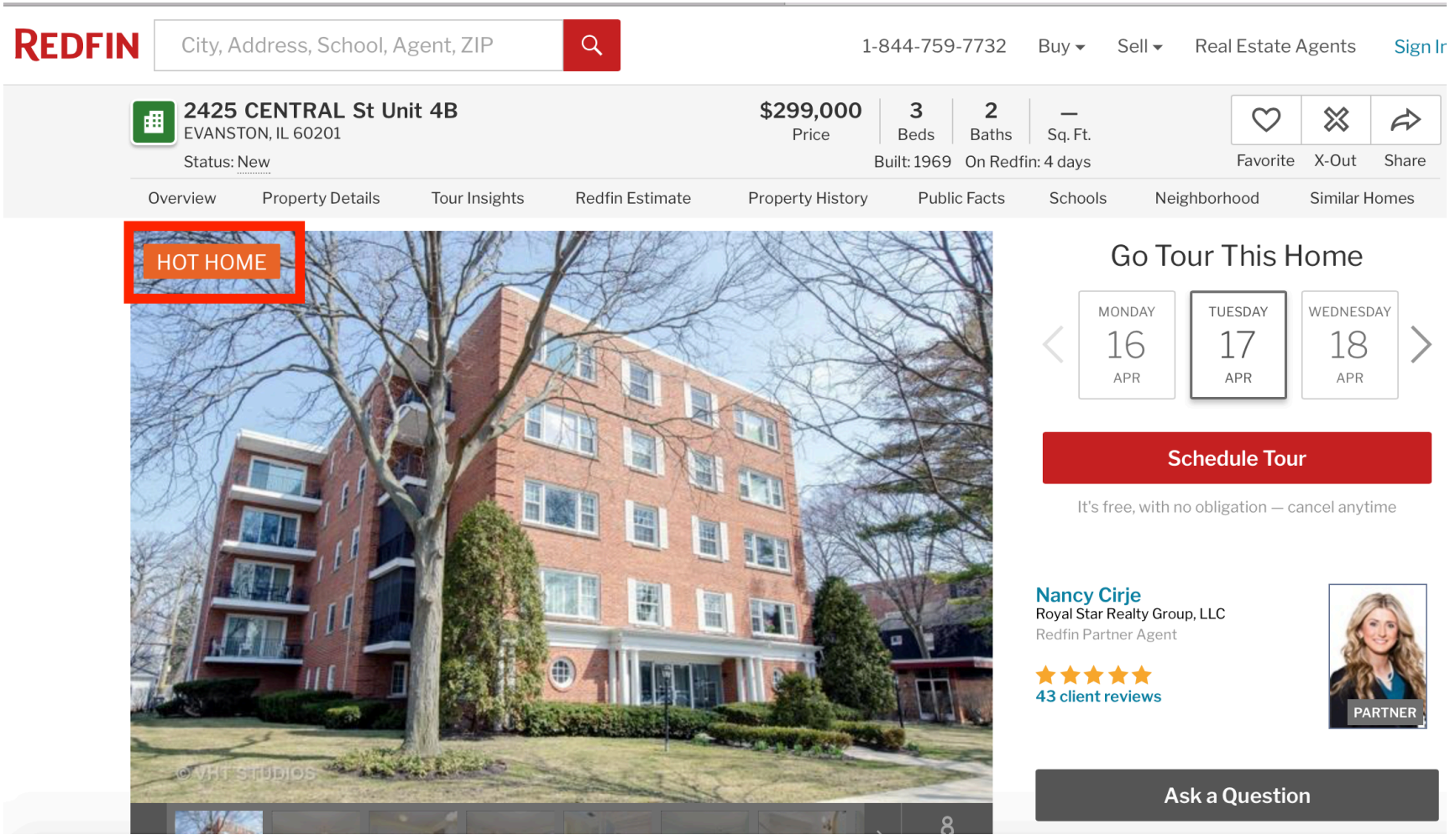


- If d is the total-variation distance, the miscalibration measure is

$$\kappa(\sigma) = \frac{1}{4}0 + \frac{3}{4}d(\tau(m_1), m_1) = \frac{3}{4}\frac{2}{15} = \frac{1}{10}$$

The Redfin example



A hot home has a 70% chance or higher of having an accepted offer within two weeks of its debut.

The Redfin example

- Area: Cook county, 165 zip codes.

The Redfin example

- Period: 03/14/2018 – 03/27/2018.
- We collect 2,150 hot homes, and track their status up to 04/13/2018.
- When a home's status become "contingent/pending/sold," it has an accepted offer.
- The ratio of hot homes having an accepted offer within two weeks of its debut:

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- We collect 2,150 hot homes, and track their status up to 04/13/2018.
- When a home's status become "contingent/pending/sold," it has an accepted offer.
- The ratio of hot homes having an accepted offer within two weeks of its debut: 53.1% (p-value < 0.0001)

Sender-Receiver game

- Sender's payoff is
$$V(\sigma, \rho) - \lambda\kappa(\sigma).$$
- λ is the cost intensity: how severely Sender is punished for miscalibration.
  - Reputation damage for making incorrect assertions
  - Mayzlin, Dover and Chevalier (2014), Luca and Zervas (2016)
- Receiver's payoff is $U(\sigma, \rho)$.
- When $\lambda = 0$, it's the cheap-talk model.

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High cost-intensity results

$\exists \delta > 0$ such that
$d(q, m) \geqslant \delta |q - m|_1$:

e.g., TV, Euclidean
$d(q, m) = \frac{1}{2}|q - m|_1$

General cost function
d convex in q:

e.g., Kullback-Leibler distance
$d(q, m) = \sum_s q(s) \log \frac{q(s)}{m(s)}$

Theorem 1: for every game, $\exists \bar{\lambda}$, for every $\lambda > \bar{\lambda}$, Sender's best equilibrium gives him CP.

Theorem 2: in a generic game, $\forall \varepsilon > 0$, $\exists \bar{\lambda}$, for every $\lambda > \bar{\lambda}$, $\exists$ a BNE which gives Sender at least $CP - \varepsilon$.

Theorem 3: in a generic game, $\exists \bar{\lambda}$, for every $\lambda > \bar{\lambda}$, Sender's rationalizable strategies are his calibrated commitment strategies.

nongeneric costly lying end

Lemma 1:

Assume $\exists \delta > 0$ such that $d(q, m) \geqslant \delta |q - m|_1$.

When λ is sufficiently high, Sender's best response to any Receiver's strategy is calibrated.

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There exists a constant B such that for any $\rho(\cdot), q, q'$:

$$\sum_{a,s} q(s)\rho(a)v(s,a) - \sum_{a,s} q'(s)\rho(a)v(s,a) \leqslant B|q - q'|_1.$$

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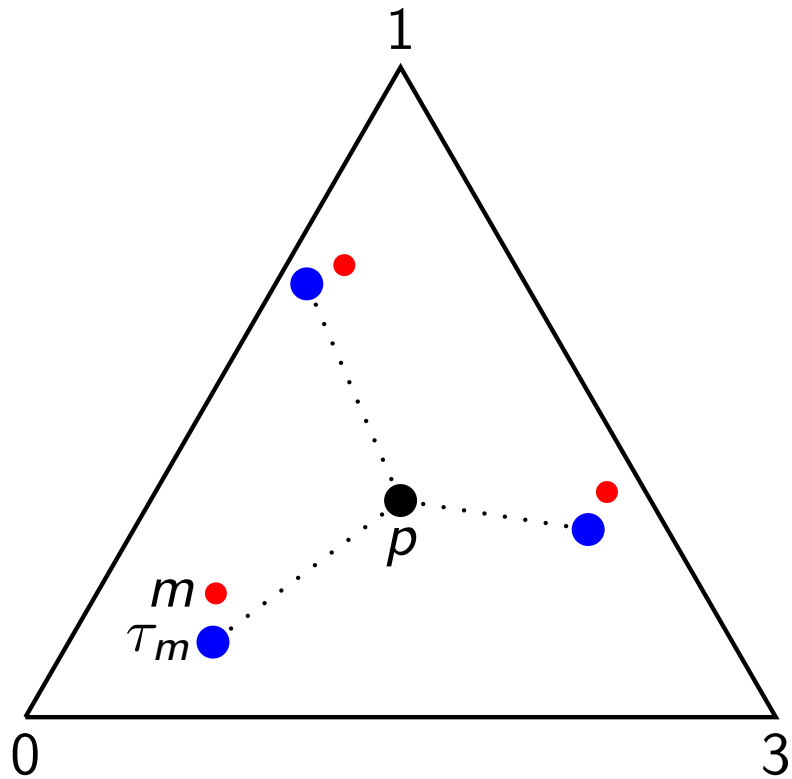

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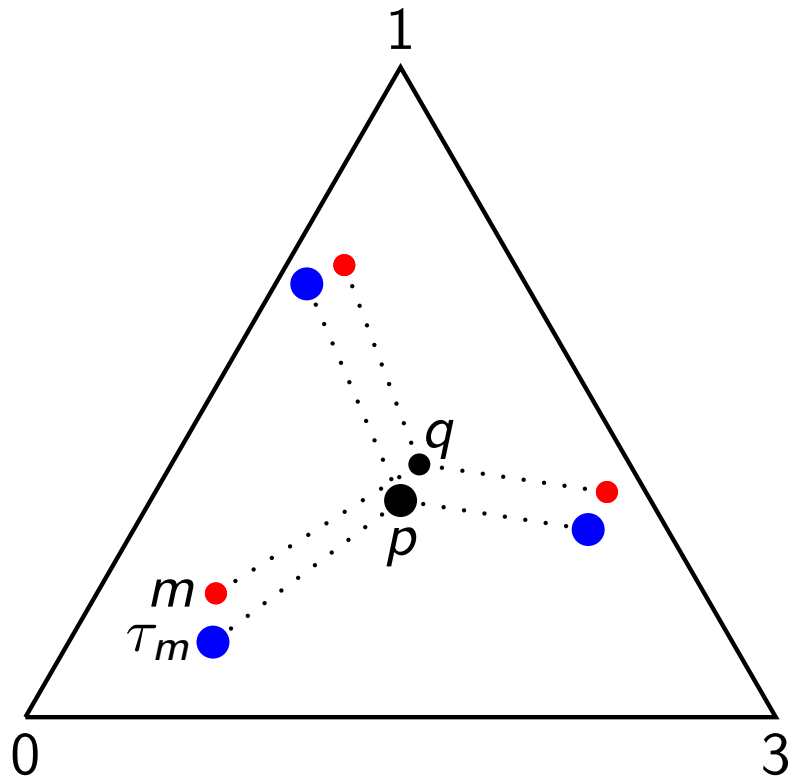

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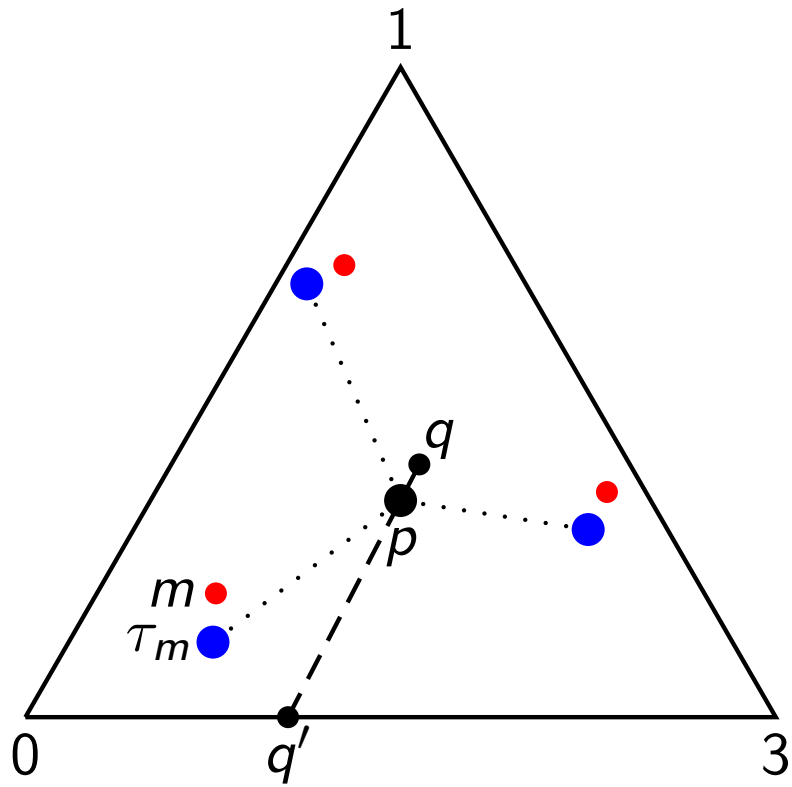
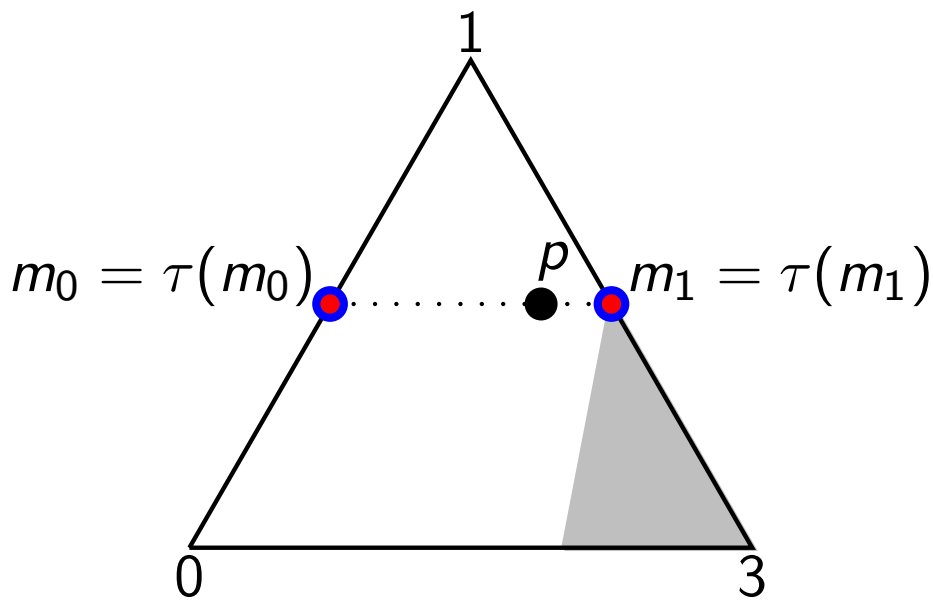


Theorem 1:

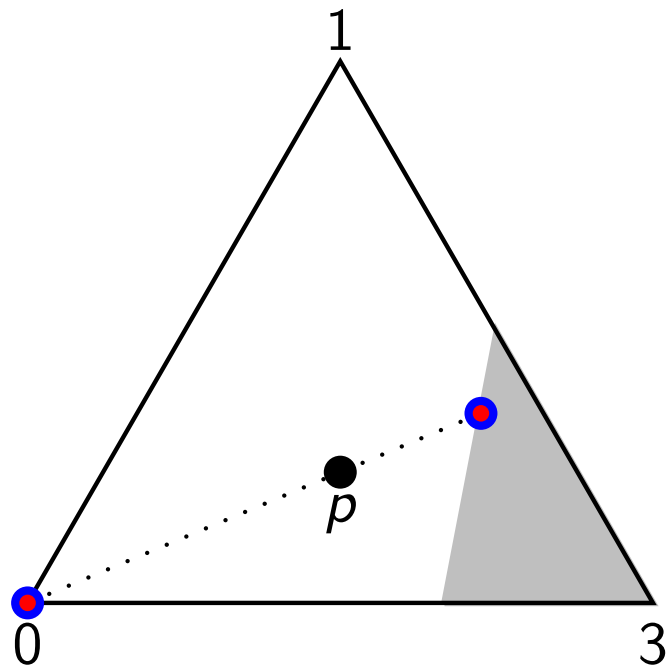
Assume $\exists \delta > 0$ such that $d(q, m) \geqslant \delta |q - m|_1$. For every game, $\exists \bar{\lambda}$, for every $\lambda > \bar{\lambda}$, Sender's best equilibrium gives him CP.

In the previous example, when $\lambda \geqslant 2$, Sender plays the following calibrated strategy. Receiver buys iff Sender announces m_1. This constitutes an equilibrium.



Theorem 1 does not hold for smooth cost function

- Suppose that $d(q, m) = \sum_s q(s) \log \frac{q(s)}{m(s)}$.
- For any λ, CP cannot be achieved in an equilibrium.



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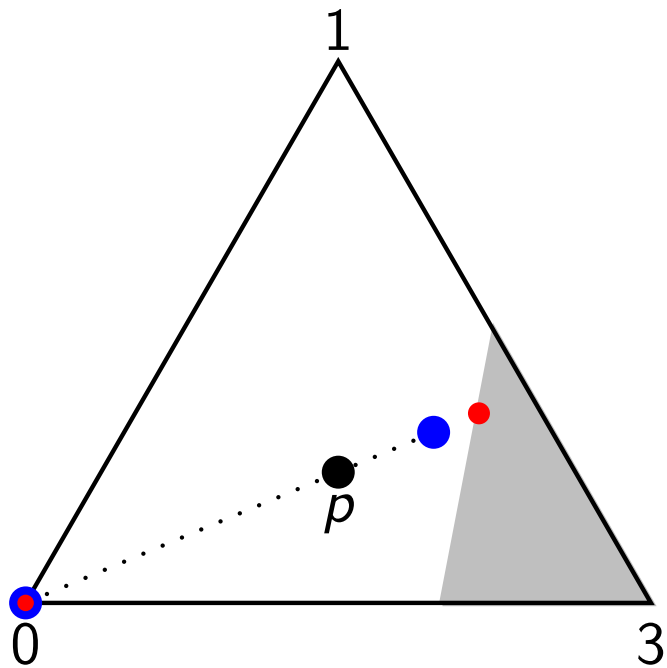

Theorem 2: Sender overshoots to gain commitment power

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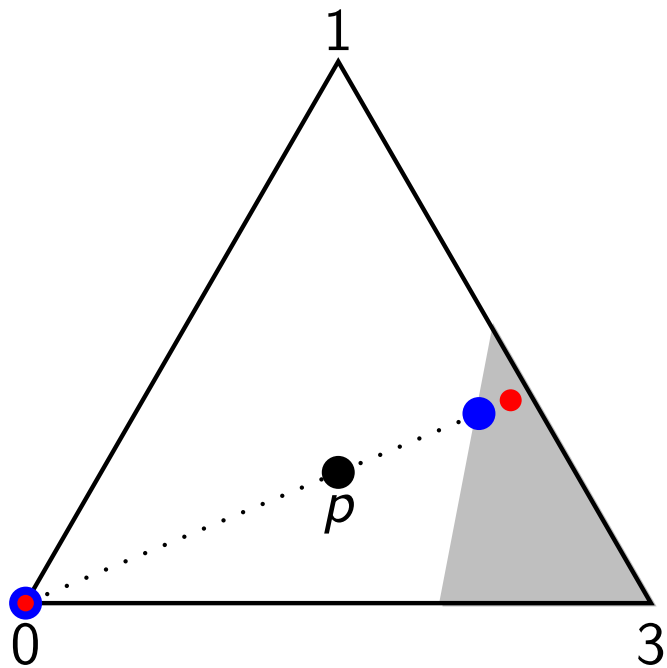


nongeneric | costly lying

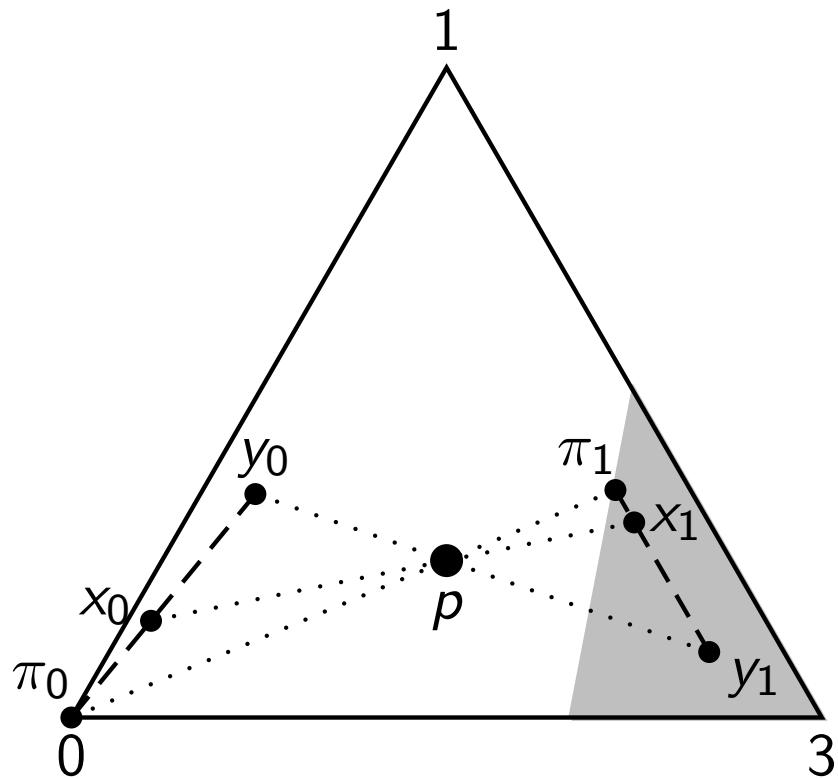
Lemma 2:

In a generic game, $\forall \varepsilon > 0$, $\exists x_a \in \Delta(S)$ for each $a \in A$ and a Sender's calibrated strategy σ, such that

(i) for each $a \in A$, a is Receiver's unique best response to x_a;

(ii) support$(\sigma) = \{x_a\}_{a \in A}$;

(iii) if ρ is Receiver's best response to σ, then $V(\sigma, \rho) > CP - \varepsilon$.



A nongeneric game

Theorem 2 does not hold for nongeneric games.

s \ a	safe	risky
0	$(0,0)$	$(1,0)$
1	$(0,0)$	$(1,-1)$

- In the commitment solution, Receiver chooses the risky action in state 0. Sender's payoff is $\Pr(s=0)$.
- For any smooth cost function, in every equilibrium Receiver chooses only the safe action.

promotion games end

Related literature

- Costly lying
  - Kartik, Ottaviani and Squintani (2007), Kartik (2009)
  - When $\lambda = 0$, it's the cheap-talk model. When $\lambda \to \infty$, full revelation results.
  - If $M = S$ and $d(q, m)$ is linear in q, our model reduces to costly-lying model.
- Persuasion with partial commitment or costly lying
  - Fréchette, Lizzeri and Perego (2017), Min (2017), Lipnowski, Ravid and Shishkin (2020)
  - Nguyen and Tan (2018)

promotion games end

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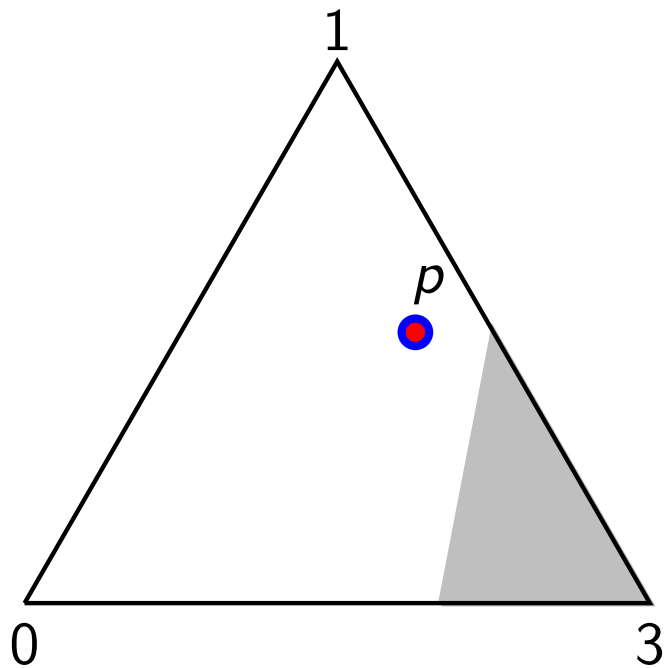
Babbling equilibrium always exits. But...

- After every message m, Receiver best responds to the prior.
- Sender announces the prior.



EFR result promotion games

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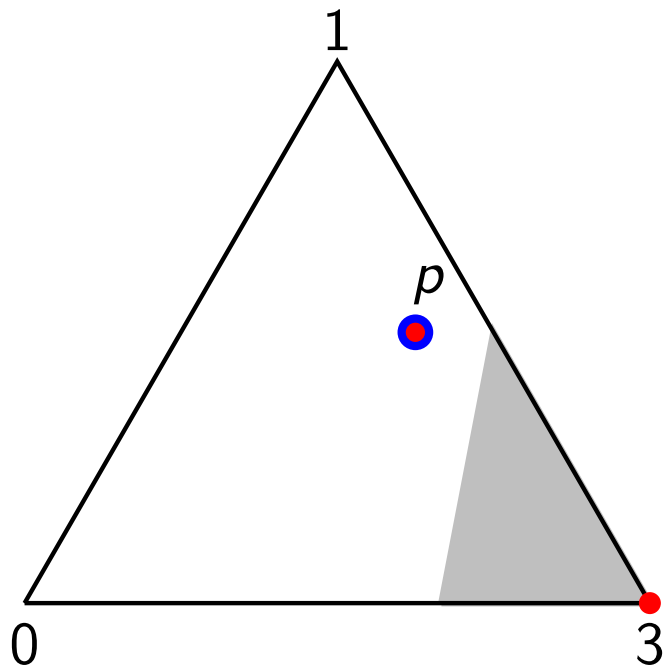

EFR result | promotion games

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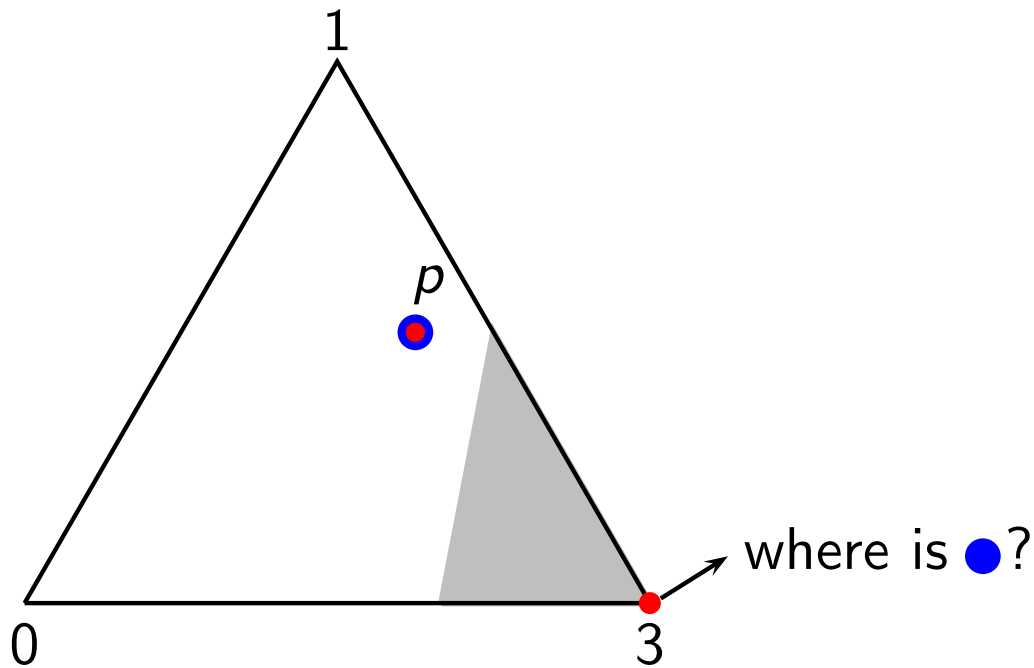

EFR result | promotion games

Extensive form rationalizability: definition

- A message m is an *atom* of σ if m is an atom of $\sigma(\cdot|s)$ for some s.
- Σ_n: set of Sender's strategies.
- R_n: correspondence from M to A of Receiver's actions after a message.
- $\{\Sigma_n, R_n\}$ are defined recursively as follows.

EFR: definition

1. Σ_0 is the set of all Sender's strategies.
2. $R_0(m) = A$ for every message $m \in M$.
3. For every $n \geqslant 1$, Σ_n is the set of all $\sigma \in \Sigma_{n-1}$ that are Sender's best responses to some strategy ρ of Receiver such that $\text{support}(\rho(\cdot|m)) \subseteq R_{n-1}(m)$.
4. For every $n \geqslant 1$ and every message m, if there exists some strategy $\sigma \in \Sigma_{n-1}$ such that m is an atom of σ, then $R_n(m)$ is the set of all actions a such that $a \in BR(\tau_\sigma(m))$ for such a strategy σ; If no such σ exists, then $R_n(m) = R_{n-1}(m)$.

Let $\Sigma_\infty = \cap_n \Sigma_n$ and $R_\infty(m) = \cap_n R_n(m)$. The strategies in Σ_∞ are *Sender's rationalizable strategies* and the actions in $R_\infty(m)$ are *Receiver's rationalizable actions after* m.

Theorem 3:

Assume $\exists \delta > 0$ such that $d(q, m) \geqslant \delta |q - m|_1$.
In a generic game, $\exists \bar{\lambda}$, for every $\lambda > \bar{\lambda}$, Sender's rationalizable strategies are his calibrated commitment strategies, i.e., calibrated strategies σ such that $V(\sigma, \rho) = CP$ for some best response ρ of Receiver to σ.

Lemma 1:

Assume $\exists \delta > 0$ such that $d(q, m) \geqslant \delta |q - m|_1$.
When λ is sufficiently high, Sender's best response to any Receiver's strategy is calibrated.

Proof of Theorem 3

Sender	Σ_0	Σ_1	Σ_2	Σ_3
	all σ	calibrated σ	calibrated σ	calibrated σ in commitment solution
Receiver	$R_0(m)$	$R_1(m)$	$R_2(m)$	$R_3(m)$
	A	A	Receiver's BR to belief m	Receiver's BR to belief m

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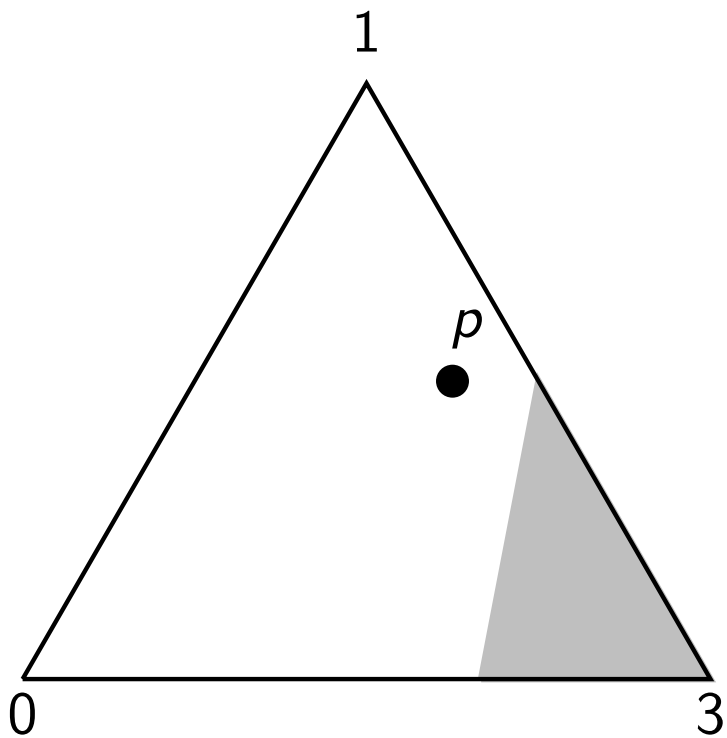
1. Basic setup
2. Our model
3. High cost-intensity results
   - Rationalizable communication
4. Promotion games

Sender-optimal equilibrium in promotion games, as λ varies

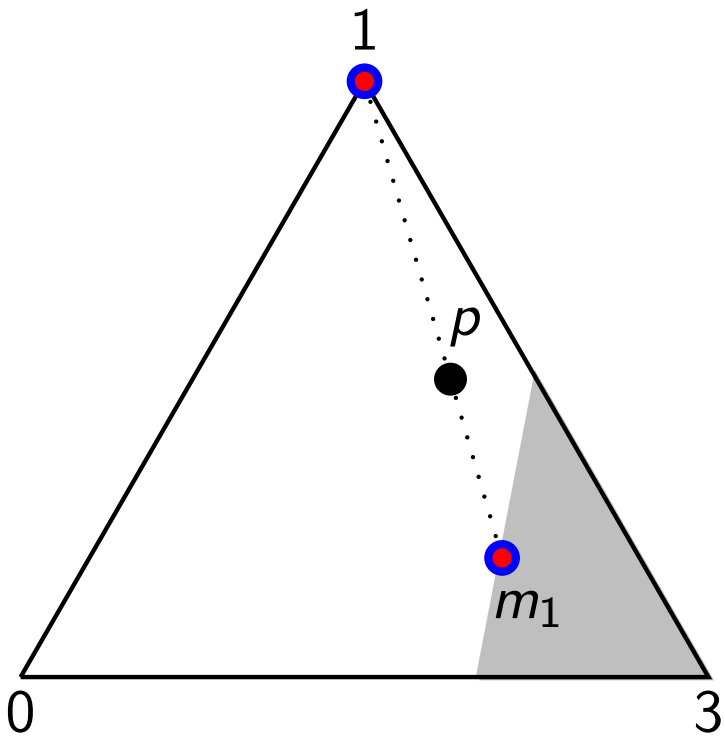
- Receiver has a binary action; Sender's payoff is state-independent.
- We characterize Sender-optimal equilibrium for all λ.



formal

Sender-optimal equilibrium in promotion games, as λ varies

- Receiver has a binary action; Sender's payoff is state-independent.
- We characterize Sender-optimal equilibrium for all λ.
- Sender uses messages that are distant for small λ.

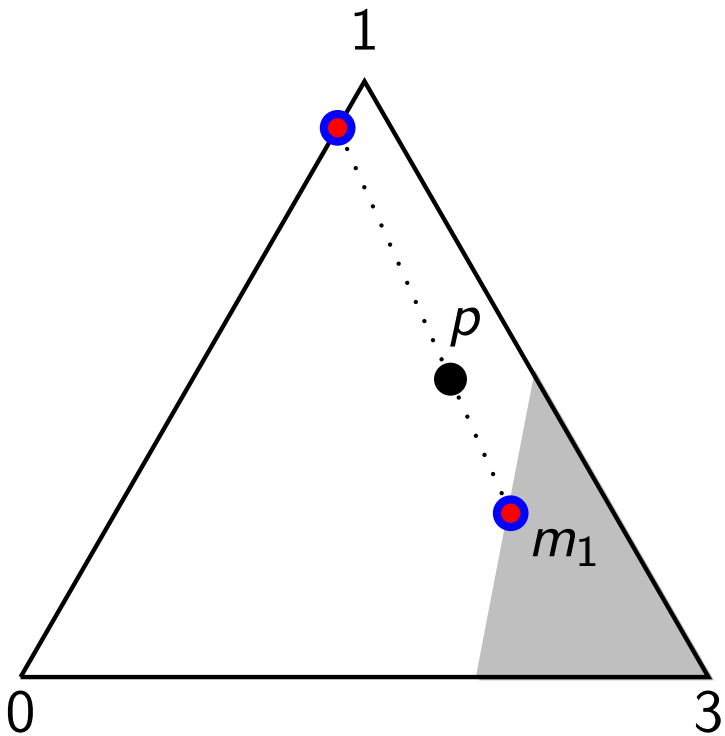


$\lambda \in \left(0, \frac{5}{4}\right]$:
Receiver buys w.p.
$\frac{4}{5}\lambda$ after m_1

formal

Sender-optimal equilibrium in promotion games, as λ varies

- Receiver has a binary action; Sender's payoff is state-independent.
- We characterize Sender-optimal equilibrium for all λ.
- Sender uses messages that are distant for small λ.
- As $\lambda \uparrow$, Sender's strategy shifts to his commitment strategy.



$\lambda = \frac{320}{207} \approx 1.5$:
Receiver buys w.p.
1 after m_1

formal

Sender-optimal equilibrium in promotion games, as λ varies

- Receiver has a binary action; Sender's payoff is state-independent.
- We characterize Sender-optimal equilibrium for all λ.
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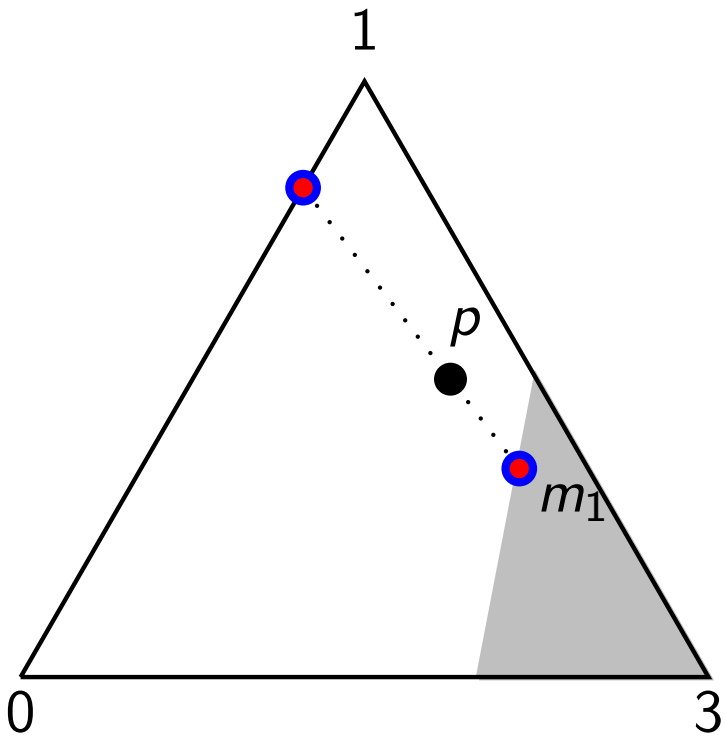
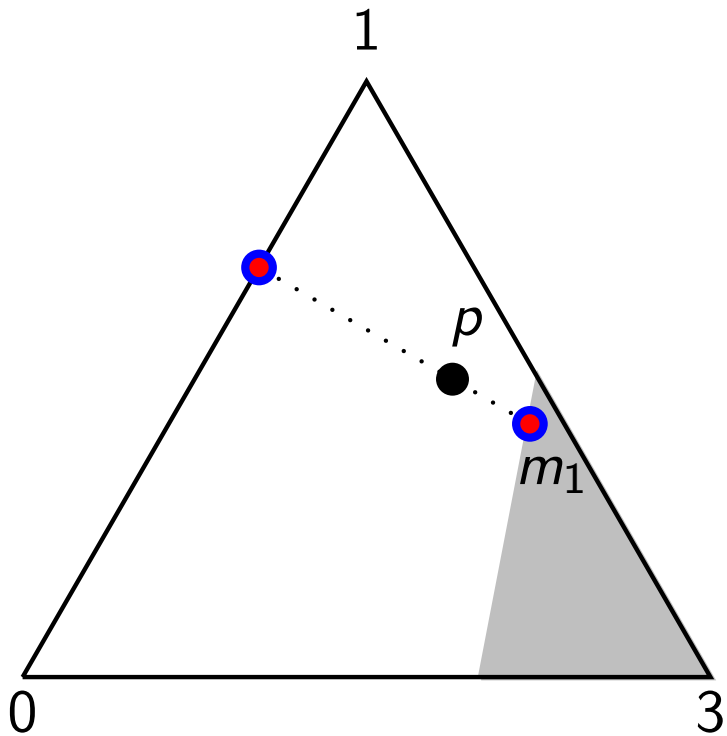


$\lambda = \frac{20}{11} \approx 1.8$:
Receiver buys w.p.
1 after m_1

formal

Sender-optimal equilibrium in promotion games, as λ varies

- Receiver has a binary action; Sender's payoff is state-independent.
- We characterize Sender-optimal equilibrium for all λ.
- Sender uses messages that are distant for small λ.
- As $\lambda \uparrow$, Sender's strategy shifts to his commitment strategy.

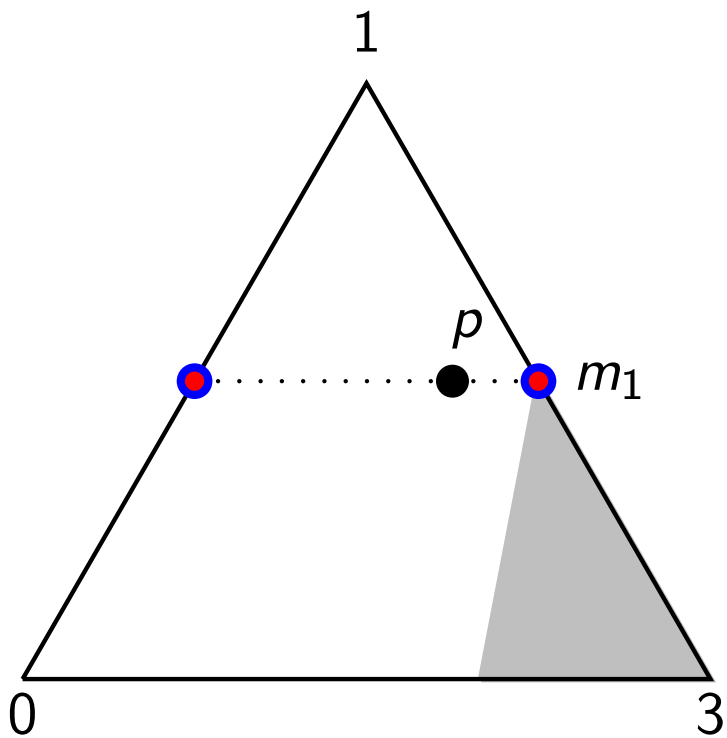


$\lambda = \frac{40}{21} \approx 1.9$:
Receiver buys w.p.
1 after m_1

formal

Sender-optimal equilibrium in promotion games, as λ varies

- Receiver has a binary action; Sender's payoff is state-independent.
- We characterize Sender-optimal equilibrium for all λ.
- Sender uses messages that are distant for small λ.
- As $\lambda \uparrow$, Sender's strategy shifts to his commitment strategy.



$\lambda \geqslant 2$:

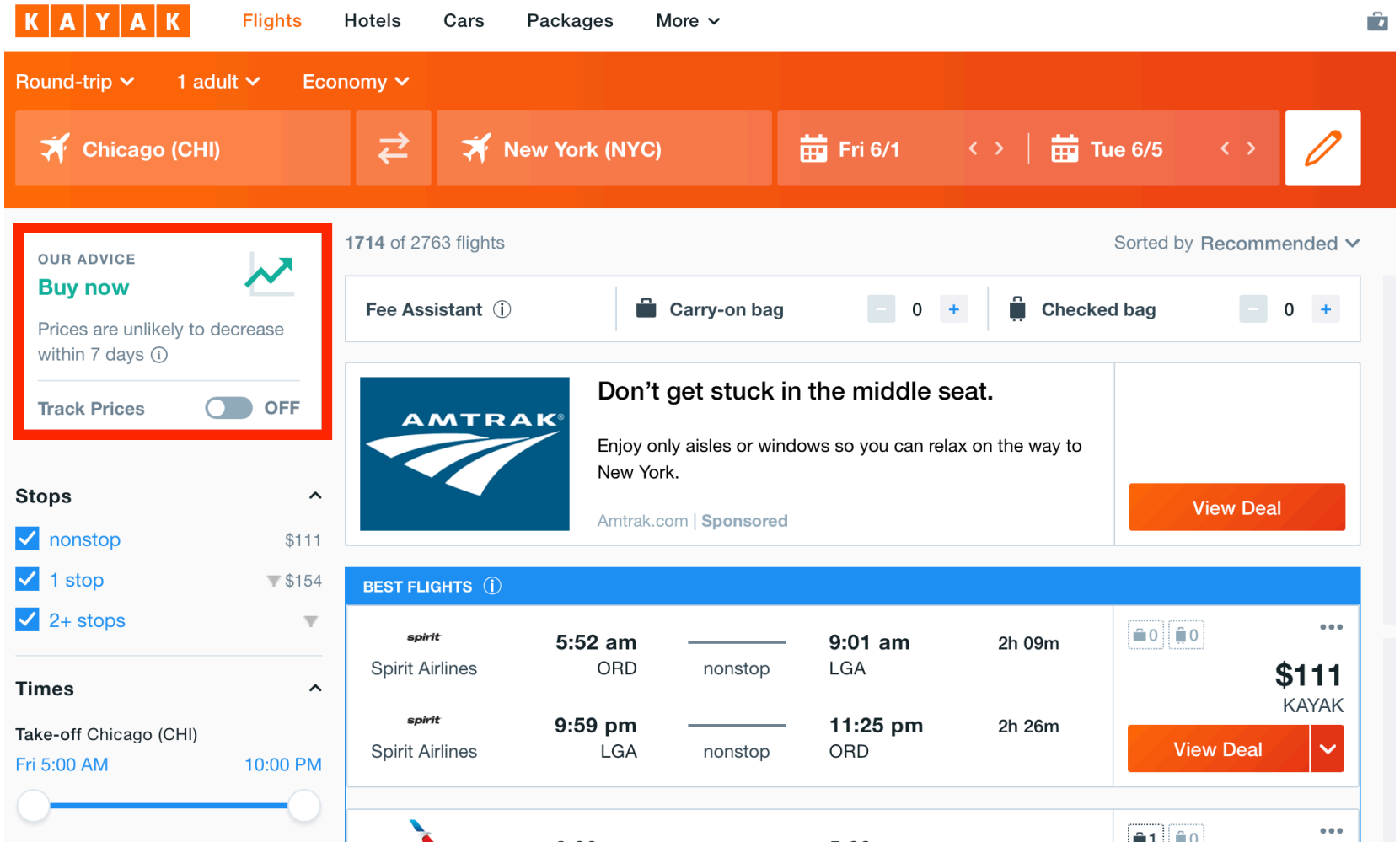
Receiver buys w.p. 1 after m_1

formal

Thank you!

Proof of Theorem 3

1. For $\lambda > \bar{\lambda}$, only calibrated strategies of Sender survive the first round of elimination. All calibrated strategies are best responses to Receiver's strategy that always chooses some fixed a. $\Longrightarrow \Sigma_1$ is the set of all calibrated strategies.
2. Any m is an atom of some calibrated strategy of Sender. $\Longrightarrow R_2(m)$ includes Receiver's best responses to belief m.
3. For every ρ that survived, Sender believes that he can get at least $CP - \varepsilon$ against ρ for every $\varepsilon > 0$.
4. Therefore, a rationalizable strategy of Sender must be calibrated and give him CP against some strategy ρ such that support$(\rho(\cdot|m))$ are Receiver's best responses to belief m.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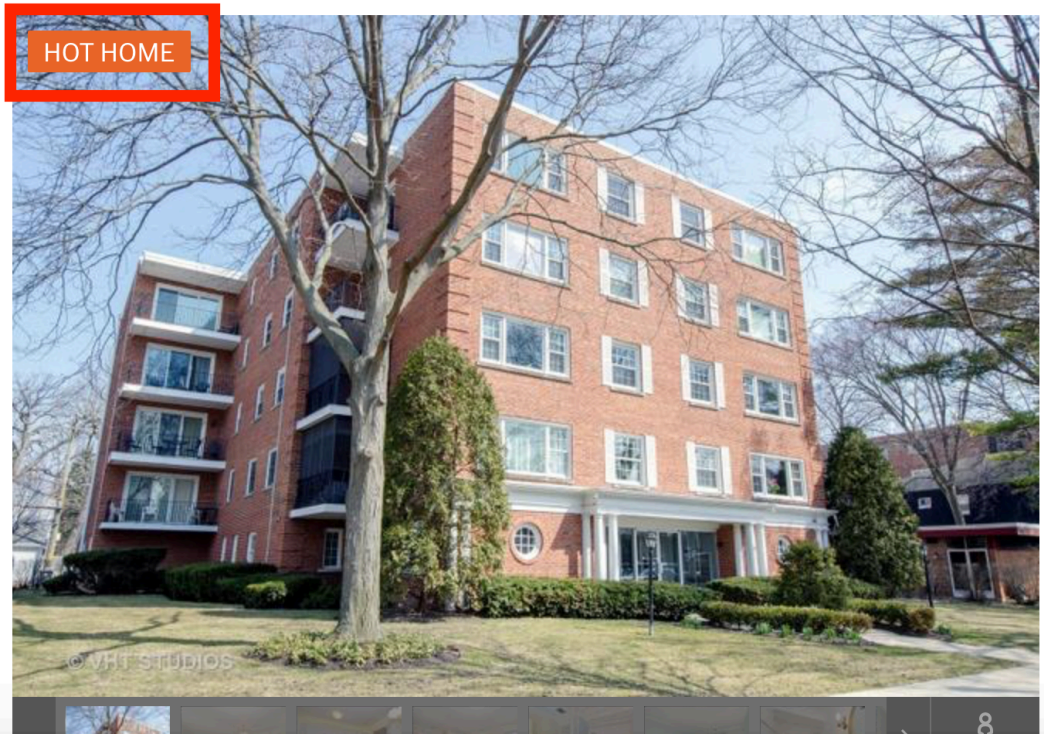

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Nancy Cirje
Royal Star Realty Group, LLC
Redfin Partner Agent

★★★★★
43 client reviews

Ask a Question

back kayak

The algorithm now identifies hot homes based on real estate conditions in each market, rather than at a national level as it did at launch. A hot home is one that has a 70% chance or higher of having an accepted offer within two weeks of its debut.

Redfin: *The hot homes algorithm automatically calculates the likelihood by analyzing more than 500 attributes of each home – including price, property type, and neighborhood – as well as buyer preferences in the area.*

back kayak

Proposition:

Let d be the TV distance. In a promotion game, there is a Sender's optimal equilibrium with the following properties:

(i) Sender's strategy is calibrated and uses two messages m_0 and m_1.

(ii) Receiver's strategy is such that Receiver buys with a positive probability when the message is m_1 and buys with probability 0 for any other message.

(iii) Receiver is indifferent when the distribution over states is m_1, and m_0 is on the boundary of the simplex $\Delta(S)$.

end