# Project Choice from a Verifiable Proposal

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**Abstract**

An agent observes the set of available projects and proposes some, but not necessarily all, of them. A principal chooses one or none from the proposed set. We solve for a mechanism that minimizes the principal's worst-case regret. If the agent can propose only one project, it is chosen for sure if the principal's payoff exceeds a threshold; otherwise, the probability that it is chosen decreases in the agent's payoff. If the agent can propose multiple projects, his payoff from a multiproject proposal equals the maximal payoff from proposing each project alone. Our results highlight the benefits from randomization and from the ability to propose multiple projects.

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# 1 Introduction

Project choice is one of the most important functions of an organization. The process often involves two parties: (i) a party at a lower hierarchical level who has expertise and

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proposes projects, and (ii) a part at a higher hierarchical level who evaluates the proposed projects and makes the choice. This describes the relationship between a division and the headquarters when the division has a chance to choose a factory location or to choose an office building. It also applies to the relationship between a department and the university when the department has a hiring slot open.

This process of project choice is naturally a principal-agent problem. The agent privately observes which projects are available and proposes a subset of the available projects. The principal chooses one from the proposed projects or rejects them all. If the two parties had identical preferences over projects, the agent would propose the project that is their shared favorite among the available ones, and the principal would always automatically approve the agent's proposal. In many applications, however, the two parties do not share the same preferences. For instance, the division may fail to internalize each project's externalities on other divisions; the department and the university may put different weights on candidates' research and nonresearch abilities. Armed with the proposal-setting power, the agent has a tendency to propose his favorite project and hide his less preferred ones, even if those projects are "superstars" for the principal. How shall the principal encourage the agent to propose the principal's preferred projects? What is the principal's optimal mechanism for choosing a project?

It is easy to see that no mechanism can guarantee that the principal's favorite project among the available ones will always be chosen. We define the principal's *regret* as the difference between his payoff from his favorite project and his expected payoff from the project chosen under the mechanism. We look for a mechanism that works fairly well for the principal in all circumstances, i.e., a mechanism that minimizes the principal's worst-case regret. This worst-case regret approach to uncertainty can be traced back to Wald (1950) and Savage (1951). It has since been used widely in game theory, mechanism design, and machine learning. A decision theoretical axiomatization for the minimax regret criterion can be found in Milnor (1954) and Stoye (2011).

Depending on the principal's verification capacity, we distinguish two environments. In the *multiproject* environment, the agent can propose any subset of the available projects. In the *single-project* environment, the agent can propose only one available project. Besides project choice within organizations, the single-project environment also applies to antitrust regulation: a firm chooses a merger from available merger opportunities to propose and the regulator decides whether to approve or reject the firm's proposal (e.g., Lyons (2003), Neven and Röller (2005), Armstrong and Vickers (2010), Ottaviani and Wickelgren (2011), Nocke and Whinston (2013)).

We take the environment as exogenous and derive the optimal mechanisms in both environments. In the single-project environment, the only way for the principal to incentivize the agent is to reject his proposal with positive probability. The multiproject environment, however, allows the principal to "spend" this rejection probability on other proposed projects. Therefore, even though the principal chooses at most one project, he expects to do better in the multiproject environment than in the single-project one. Comparing the two environments will also allow us to quantify the principal's gain from higher verification capacity.

We begin with the single-project environment. A mechanism specifies for each proposed single project the probability that it will be approved. In the optimal mechanism, if the proposed project gives the principal a sufficiently high payoff, it is approved for sure. We call such projects *good* projects for the principal. If, on the contrary, the proposed project is *mediocre* for the principal, it is approved only with some probability. The probability that a mediocre project is approved decreases in its payoff to the agent, in order to deter the agent from hiding projects that are more valuable for the principal. This mechanism aligns the incentives of the agent with those of the principal in the following ways. First, if the agent has at least one good project for the principal, he will propose a good project. Second, if all his projects are mediocre for the principal, he will propose the principal's favorite one.

In the multiproject environment, a mechanism specifies for each proposed set of projects a randomization over the proposed projects and "no project." If the agent proposes only one project, the optimal mechanism takes a form similar to the one in the single-project environment. In particular, if the proposed project is sufficiently good for the principal, it is chosen for sure. Otherwise, the project is chosen with some probability that decreases in its payoff to the agent. If the agent proposes more than one project, the randomization maximizes the principal's expected payoff, subject to the constraint that the agent is promised the maximal expected payoff he would get from proposing each project alone. Under this mechanism, the more projects the agent proposes, the weakly higher his expected payoff is, so the agent is willing to propose all available projects.

Since the agent gets the maximal expected payoff from proposing each project alone, we call this mechanism the *project-wide maximal-payoff mechanism*. This mechanism implements a compromise between the two parties in the multiproject environment: with some probability the choice favors the agent and with some probability it favors the principal. We also show that randomization is crucial for the principal's minimal worst-case regret to be lower in the multiproject environment than in the single-project one. In other words, if the principal is restricted to deterministic mechanisms, his minimal worst-case regret is the same in both the single-project and multiproject environments.

**Related literature.** Our paper is closely related to Armstrong and Vickers (2010) and Nocke and Whinston (2013), which study the project choice problem using the Bayesian approach. Armstrong and Vickers (2010) characterize the optimal deterministic mechanism in the single-project environment and show through examples that the principal does strictly better if randomization or multiproject proposals are allowed. Nocke and Whinston (2013) focus on mergers (i.e., projects) that are ex ante different and further incorporate the bargaining process among firms. They show that a tougher standard is imposed on mergers involving larger partners. We take the worst-case regret approach to this multidimensional screening problem. This more tractable approach allows us to explore questions which are intractable under the Bayesian approach, including how much the principal benefits from randomization, from higher verification capacity and from a smaller project domain.

Goel and Hann-Caruthers (2020) considers the project choice problem where the number of available projects is public information. The projects are only partially verifiable, since the agent's only constraint is not to overreport projects' payoffs to the principal. Because their agent cannot hide projects like our agent does, he loses the proposal-setting power. The resulting incentive schemes are thus quite different.

Since in our model the agent can propose only those projects that are available, the agent's proposal is some evidence about his private information. Hence, our paper is closely related to research on verifiable disclosure (e.g., Grossman and Hart (1980), Grossman (1981), Milgrom (1981), Dye (1985)) and, more broadly, the evidence literature (see Dekel (2016) for a survey). We discuss the relation to this literature in more detail after we introduce the model.

Our result relates to a theme in Aghion and Tirole (1997), namely, that the principal has formal authority, but the agent shares real authority due to his private information. We take this theme one step further. Our agent's real authority has two sources: he knows which projects are available, and he determines the proposal from which the principal chooses a project. The idea of striking a compromise is related to Bonatti and Rantakari (2016). They examine the compromise between two symmetric, competing agents whose efforts are crucial for discovering projects. We instead focus on the compromise between an agent who proposes projects and a principal who chooses one or none from the proposed projects.

Finally, our paper contributes to the literature on mechanism design in which the designer minimizes his worst-case regret. Hurwicz and Shapiro (1978) examine a moral hazard problem. Bergemann and Schlag (2008, 2011) examine monopoly pricing. Renou and Schlag

(2011) apply the solution concept of $\varepsilon$-minimax regret to the problem of implementing social choice correspondences. Beviá and Corchón (2019) examine the contest which minimizes the designer's worst-case regret. Guo and Shmaya (2019) study the optimal mechanism for monopoly regulation and Malladi (2020) studies the optimal approval rules for innovation. More broadly, we contribute to the growing literature of mechanism design with worst-case objectives. For a survey on robustness in mechanism design, see Carroll (2019).

# 2 Model and mechanism

Let $D$ be the domain of all possible *verifiable projects*. Let $u : D \to \mathbf{R}_+$ be the agent's payoff function, so his payoff is $u(a)$ if project $a$ is chosen. If no project is chosen, the agent's payoff is zero.

The agent's private *type* $A \subseteq D$ is a finite set of available projects. The agent proposes a set $P$ of projects, and the principal can choose one project from this set. The set $P$ is called the agent's *proposal*. It must satisfy two conditions. First, the agent can propose only available projects. Hence, the agent's proposal must be a subset of his type, $P \subseteq A$. This is what we meant earlier when we said that projects are verifiable. Second, $P \in \mathcal{E}$ for some fixed set $\mathcal{E}$ of subsets of $D$. The set $\mathcal{E}$ captures all the exogenous restrictions on the proposal. For instance, in the setting of antitrust regulation, the agent is restricted to proposing at most one project. In many organizations, the principal have limited verification capacity or limited attention, so the agent can propose at most a certain number of projects.

We begin with two environments which are natural first steps: *single-project* and *multiproject*. In the single-project environment, the agent can propose at most one available project, so $\mathcal{E} = \{P \subseteq D : |P| \leqslant 1\}$. In the multiproject environment, the agent can propose any set of available projects so $\mathcal{E} = 2^D$, the power set of $D$. In subsection 6.1, we discuss the intermediate environments in which the agent can propose up to $k$ projects for some fixed number $k \geqslant 2$.

The agent's proposal $P$ serves two roles. First, if we view a proposal as a message, then different types have access to different messages. Hence, the agent's proposal is some evidence about his type, as in Green and Laffont (1986). We explore the implication of this evidence role in section 3. Second, the proposal determines the set of projects from which the principal can choose. This second role is a key difference between our paper and the evidence literature. Once the agent puts his proposal on the table, there is no relevant information asymmetry left. This implies that cheap-talk communication will not help. We elaborate on this point in subsection 6.2.

A *subprobability measure over $D$ with a finite support* is given by $\pi : D \to [0,1]$ such that

$$\text{support}(\pi) = \{a \in D : \pi(a) > 0\}$$

is finite, and $\sum_a \pi(a) \leqslant 1$. When we say that a project *is chosen from* a subprobability measure $\pi$ with finite support, we mean that project $a$ is chosen with probability $\pi(a)$, and that no project is chosen with probability $1 - \sum_a \pi(a)$.

The principal's ability to reject all proposed projects (or equivalently, to choose no project) is crucial for him to retain some "bargaining power." If, on the contrary, the principal must choose a project as long as the agent has proposed some, then the agent effectively has all the bargaining power. The agent will propose only his favorite project which will be chosen for sure.

A *mechanism* $\rho$ attaches to each proposal $P \in \mathcal{E}$ a subprobability measure $\rho(\cdot|P)$ such that $\text{support}(\rho(\cdot|P)) \subseteq P$. The interpretation is that, if the agent proposes $P$, then a project is chosen from the subprobability measure $\rho(\cdot|P)$. Thus, the agent's expected payoff under the mechanism $\rho$ if he proposes $P$ is $U(\rho, P) = \sum_{a \in P} u(a)\rho(a|P)$.

A *choice function* $f$ attaches to each type $A$ of the agent a subprobability measure $f(\cdot|A)$

such that support$(f(\cdot|A)) \subseteq A$. The interpretation is that, if the set of available projects is $A$, then a project is chosen from the subprobability measure $f(\cdot|A)$.

A choice function $f$ is *implemented* by a mechanism $\rho$ if, for every type $A$ of the agent, there exists a probability measure $\mu$ with support over $\text{argmax}_{P\subseteq A, P\in\mathcal{E}} U(\rho, P)$ such that $f(a|A) = \sum_P \mu(P)\rho(a|P)$. The interpretation is that the agent selects only proposals that give him the highest expected payoff among the proposals that he can make, and that, if the agent has multiple optimal proposals, then he can randomize among them.

# 3 The evidence structure

When the agent proposes a set $P$ of projects, he provides evidence that his type $A$ satisfies $P \subseteq A$. In this section, we discuss the implication of this role of the agent's proposal as well as the relation to the evidence literature.

## 3.1 Normality in the multiproject environment

In our multiproject environment, where $\mathcal{E} = 2^D$, the agent has the ability to provide the maximal evidence for his type. This property is called *normality* in the literature (Lipman and Seppi (1995), Bull and Watson (2007), Ben-Porath, Dekel and Lipman (2019)). Another interpretation of the multiproject environment is to view an agent who proposes a set $P$ as an agent who claims that his type is $P$. The relation that "type $A$ can claim to be type $B$" between types is reflexive and transitive, by the corresponding properties of the inclusion relation between sets. Transitivity is called the nested range condition in Green and Laffont (1986) and is also assumed in Hart, Kremer and Perry (2017).

In our single-project environment, where $\mathcal{E} = \{P \subseteq D : |P| \leqslant 1\}$, normality does not hold. The single-project environment is the main focus in Armstrong and Vickers (2010) and Nocke and Whinston (2013), and is similar to the assumption in Glazer and Rubinstein

(2006) and Sher (2014) that the speaker can make one and only one of the statements he has access to.

## 3.2 Revelation principle in the multiproject environment

Consider the multiproject environment $\mathcal{E} = 2^D$. A mechanism $\rho$ is *incentive-compatible (IC)* if the agent finds it optimal to propose his type $A$ truthfully. That is, $U(\rho, A) \geqslant U(\rho, P)$ for every finite set $A \subseteq D$ and every subset $P \subseteq A$. Equivalently, a mechanism $\rho$ is IC if and only if $U(\rho, P)$ weakly increases in $P$ with respect to set inclusion. The following proposition states the revelation principle in the multiproject environment.

**Proposition 3.1.** *Assume $\mathcal{E} = 2^D$. If a choice function $f$ is implemented by some mechanism, then the mechanism $f$ is IC and implements the choice function $f$.*

As we explained in subsection 3.1, the multiproject environment satisfies normality and the nested range condition. Previous papers (e.g., Green and Laffont (1986), Bull and Watson (2007)) have shown that the revelation principle holds under these assumptions. Our proposition 3.1 does not follow directly from their theorems, however, because the agent's proposal $P$ serves two roles in our model. In addition to providing evidence, the proposal also determines the set of projects from which the principal can choose. Nonetheless, a similar argument for the revelation principle can be made within our model as well.

*Proof of Proposition 3.1.* Assume that the mechanism $\rho$ implements the choice function $f$. Then for every finite set $A \subseteq D$ and every subset $P \subseteq A$, we have:

$$U(f, A) = \max_{Q \subseteq A} U(\rho, Q) \geqslant \max_{Q \subseteq P} U(\rho, Q) = U(f, P),$$

where the inequality follows from the fact that $Q \subseteq P$ implies $Q \subseteq A$, and the two equalities follow from the fact that $\rho$ implements $f$. Hence, the mechanism $f$ is IC. Also, by definition, if the mechanism $f$ is IC, then it implements the choice function $f$. $\square$

Since an implementable choice function is itself an IC mechanism and vice versa, we will use both terms interchangeably whenever we discuss the multiproject environment.

# 4 The principal's problem

Let $v : D \to \mathbf{R}_+$ be the principal's payoff function, so his payoff is $v(a)$ if project $a$ is chosen. If no project is chosen, the principal's payoff is zero.

The principal's *regret* from a choice function $f$ when the set of available projects is $A$ is:

$$\mathrm{RGRT}(f, A) = \max_{a \in A} v(a) - \sum_{a \in A} v(a) f(a|A).$$

The regret is the difference between what the principal could have achieved if he knew the set $A$ of available projects and what he actually achieves. Savage (1951) calls this difference *loss*. We instead call it regret, by following the more recent game theory and computer science literature. Wald (1950) and Savage (1972) propose to consider only *admissible* choice functions (i.e., choice functions that are not weakly dominated). A choice function $f$ is *admissible* if there exists no other $f'$ such that the principal's regret is weakly higher under $f$ than under $f'$ for every type of the agent and strictly higher for some type. For the rest of the paper, we focus on admissible choice functions.

The *worst-case regret* (WCR) from a choice function $f$ is:

$$\mathrm{WCR}(f) = \sup_{A \subseteq D, |A| < \infty} \mathrm{RGRT}(f, A),$$

where the supremum ranges over all possible types of the agent (i.e., all possible finite sets of available projects). The principal's problem is to minimize $\mathrm{WCR}(f)$ over all implementable choice functions $f$. This step is our only departure from the Bayesian approach. The Bayesian approach will instead assign a prior belief over the number and the characteristics of the available projects. The principal's problem, then, is to minimize the *expected* regret instead of the *worst-case* regret.

Note that, while our principal takes the worst-case regret approach to uncertainty about the agent's type, he calculates the expected payoff with respect to his own objective randomization. The same assumption is made by Savage (1972) when he discusses the use of randomized acts under the worst-case regret approach (Savage, 1972, Chapter 9.3). A similar assumption is made in the ambiguity aversion literature. For example, in Gilboa and Schmeidler (1989), the decision maker calculates his expected payoff with respect to random outcomes (i.e., "roulette lotteries") but evaluates acts using the maxmin approach with non-unique priors. If we make the alternative assumption that the principal takes the worst-case regret approach even towards his own randomization, we effectively restrict the principal to deterministic mechanism.

From now on, we assume that the set $D$ of all possible verifiable projects is $[\underline{u}, 1] \times [\underline{v}, 1]$ for some parameters $\underline{u}, \underline{v} \in [0, 1]$, and that the functions $u(\cdot)$ and $v(\cdot)$ are projections over the first and second coordinates. Abusing notation, we denote a project $a \in D$ also by $a = (u, v)$, where $u$ and $v$ are the agent's and the principal's payoffs, respectively, if project $a$ is chosen.

The parameters $\underline{u}$ and $\underline{v}$ quantify the uncertainty faced by the principal: the higher they are, the smaller the uncertainty. They also measure players' preference intensity over projects. As $\underline{u}$ increases, the agent's preferences over projects become less strong, so it becomes easier to align the incentives of the agent with those of the principal. As $\underline{v}$ increases, the principal's preferences over projects become less strong, so the agent's tendency to propose his own favorite project becomes less costly for the principal.

# 5 Optimal mechanisms

## 5.1 Preliminary intuition

We now use an example to illustrate the fundamental trade-off faced by the principal, as well as the intuition behind the optimal mechanisms. We first explain how randomization helps to reduce the WCR in the single-project environment. We then explain how the multiproject environment can further reduce the WCR. For this illustration, we assume that $\underline{v} = 0$ so $D = [\underline{u}, 1] \times [0, 1]$.

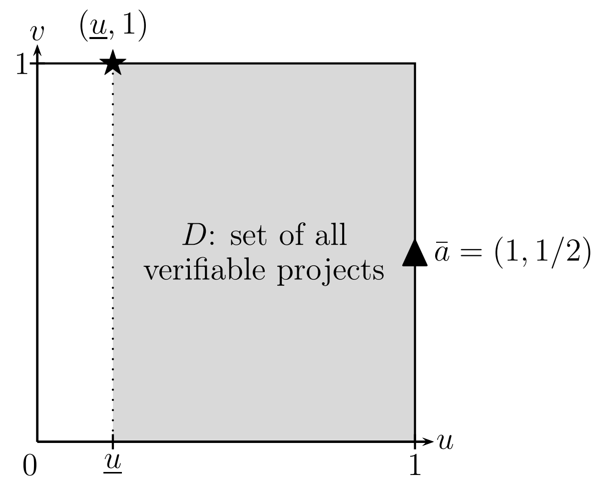


Figure 1: Preliminary intuition, $\underline{v} = 0$

Consider the single-project environment and assume first that the principal is restricted to deterministic mechanisms. In this case, a mechanism is a set of projects that the principal approves for sure, and all other projects are rejected outright. For each such mechanism, the principal has two fears. First, if the agent has multiple projects which will be approved, then he will propose what he likes the most, even if projects are available that are more valuable to the principal. Second, if the agent has only projects which will be rejected, then the principal loses the payoff from these projects. Applied to the project $\bar{a} = (1, 1/2)$, these two fears imply that no matter how the principal designs the deterministic mechanism, his

WCR is at least 1/2. As shown in figure 1, this project $\bar{a}$ gives the agent his highest payoff 1, while giving the principal only a moderate payoff 1/2. If the mechanism approves $\bar{a}$ and the set of available projects is $\{\bar{a}, (\underline{u}, 1)\}$, then the agent will propose $\bar{a}$ rather than $(\underline{u}, 1)$, so the principal suffers regret 1/2. If the mechanism rejects $\bar{a}$ but $\bar{a}$ is the only available project, then the principal also suffers regret 1/2. Thus, the WCR under any deterministic mechanism is at least 1/2. On the other hand, the deterministic mechanism that approves project $(u, v)$ if and only if $v \geqslant 1/2$ achieves the WCR of 1/2, so it is optimal among all the deterministic mechanisms.

We now explain how randomization can reduce the WCR in the single-project environment. We first note that, if $\underline{u} = 0$, then, even with randomized mechanisms, the principal cannot reduce his WCR below 1/2. This is because the only way to incentivize the agent to propose the project $(\underline{u}, 1) = (0, 1)$ when the set of available projects is $\{\bar{a}, (0, 1)\}$ is still to reject the project $\bar{a}$ outright if $\bar{a}$ is proposed. However, if $\underline{u} > 0$, then the principal can do better. He can approve the project $\bar{a}$ with probability $\underline{u}$, while still maintaining the agent's incentive to propose the principal's preferred project $(\underline{u}, 1)$ when the set of available projects is $\{\bar{a}, (\underline{u}, 1)\}$. We carry out this idea in Theorem 5.1 in subsection 5.2.

Let us now consider the multiproject environment. We again begin with deterministic mechanisms. Under deterministic mechanisms, more choice functions can be implemented in the multiproject environment than in the single-project one.[1] However, when restricted to deterministic mechanisms, the principal has the same minimal WCR in the multiproject environment as in the single-project one. This is because, if the principal wants to choose $(\underline{u}, 1)$ when the set of available projects is $\{\bar{a}, (\underline{u}, 1)\}$, then the only way to incentivize the agent to include $(\underline{u}, 1)$ in his proposal is to reject the project $\bar{a}$ when $\bar{a}$ is proposed alone.

We now explain how randomization can help in the multiproject environment, even when

[1] For example, the principal can implement the choice function that chooses (i) the agent's favorite project, if there are at least two available projects, and (ii) nothing, if there is at most one available project.

$\underline{u} = 0$. While a deterministic mechanism must pick either $\bar{a}$ or $(0, 1)$ or nothing when the agent proposes $\{\bar{a}, (0, 1)\}$, a randomized mechanism can reach a compromise by choosing each project with probability 1/2. On the other hand, if the agent proposes only $\bar{a}$, the principal chooses $\bar{a}$ with probability 1/2, so the agent of type $\{\bar{a}, (0, 1)\}$ is willing to propose $\{\bar{a}, (0, 1)\}$ instead of just $\bar{a}$. The regret is 1/4 both when the agent's type is $\{\bar{a}, (0, 1)\}$ and when his type is $\{\bar{a}\}$. We carry out this idea of reaching a compromise in Theorem 5.2 in subsection 5.3. Specifically, when the agent proposes $P$, the principal gives the agent the maximal payoff he can offer, subject to the constraint that he can give the agent this same payoff if the agent proposes $P \cup \{(\underline{u}, 1)\}$ and can still keep his regret under control.

## 5.2 Optimal mechanism in the single-project environment

Since the agent can propose at most one project, a mechanism specifies the approval probability for each proposed project. Instead of using our previous notation $\rho(a|\{a\})$, we let $\alpha(u, v) \in [0, 1]$ denote the approval probability if the agent proposes the project $(u, v)$.

**Theorem 5.1** (Single-project environment)**.** *Assume* $\mathcal{E} = \{P \subseteq D : |P| \leqslant 1\}$. *Let*

$$R^s = \max_{v \in [\underline{v}, 1]} \min\left((1 - \underline{u})v, 1 - v\right) = \min\left(\frac{1 - \underline{u}}{2 - \underline{u}}, 1 - \underline{v}\right).$$

1. *The* WCR *under any mechanism is at least* $R^s$.
2. *The mechanism* $\alpha^s$ *is given by:*

$$\alpha^s(u, v) = \begin{cases} 1, & \text{if } v \geqslant 1 - R^s \text{ or } u = 0, \\ \underline{u}/u, & \text{if } v < 1 - R^s \text{ and } u > 0. \end{cases}$$

   *It implements a choice function that has the* WCR *of* $R^s$ *and is admissible.*

3. *If a mechanism $\alpha$ implements a choice function that has the* WCR *of $R^s$, then $\alpha(u, v) \leqslant \alpha^s(u, v)$ for every $(u, v) \in D$.*

The mechanism $\alpha^s$ consists of an *automatic-approval* region and a *chance* region. If the proposed project is sufficiently good for the principal (i.e., $v \geqslant 1 - R^s$), then it is automatically approved. If the project is mediocre for the principal (i.e., $v < 1 - R^s$), then the approval probability equals $\underline{u}/u$, so the agent expects a payoff $\underline{u}$ from proposing a mediocre project.

The agent will propose a project in the automatic-approval region if he has at least one such project. If all his projects are in the chance region, he will propose a project that gives the principal the highest payoff. The principal still suffers regret from two sources. First, if the agent has multiple projects that will be automatically approved, he will propose what he favors instead of what the principal favors. Second, if the agent has only projects in the chance region, his proposal is rejected with positive probability. The threshold for the automatic-approval region, $1 - R^s$, is chosen to keep the regret from both sources under control.

The approval probability $\alpha^s(u, v)$ increases in $v$ (the principal's payoff) and decreases in $u$ (the agent's payoff). This monotonicity in $v$ and $u$ is natural. In particularly, the principal is less likely to approve projects that give the agent high payoffs in order to deter the agent from hiding projects that give the principal high payoffs. It is interesting to compare our optimal mechanism $\alpha^s$ in the single-project environment to that in Armstrong and Vickers (2010). They characterize the optimal deterministic mechanism in a Bayesian setting. Under the assumptions that (i) projects are i.i.d. and (ii) the number of available projects is independent of their characteristics, they show that the optimal deterministic mechanism $\alpha(u, v)$ increases in $v$: a project $(u, v)$ is approved if and only if $v \geqslant r(u)$ for some function $r(u)$. They also characterize the optimal $r(u)$ explicitly. Their argument can be generalized to show that the optimal randomized mechanism $\alpha(u, v)$ also increases in $v$, but it is not clear how to solve for the optimal $\alpha(u, v)$. It is an open problem under which assumptions on the prior belief the optimal randomized mechanism $\alpha(u, v)$ in the Bayesian setting decreases in $u$.

The typical situation under the worst-case regret approach to uncertainty is that multiple mechanisms can achieve the minimal WCR. Assertion 3 in Theorem 5.1 says that the mechanism $\alpha^s$ is uniformly more generous in approving the agent's proposal than any other mechanism that can have the WCR of $R^s$. This assertion has two implications. First, among all mechanisms that can have the WCR of $R^s$, the mechanism $\alpha^s$ is the agent's most preferred one. Second, compared to any mechanism that can have the WCR of $R^s$, the mechanism $\alpha^s$ gives the principal a higher payoff (or equivalently, a lower regret) for every singleton $A$ and a strictly higher payoff for some singleton $A$.

## 5.3 Optimal mechanism in the multiproject environment

We now present the optimal mechanism in the multiproject environment. Let $\alpha : [\underline{u}, 1] \times [\underline{v}, 1] \to [0, 1]$ be a function and consider the following *project-wide maximal-payoff mechanism* (PMP mechanism) induced by the function $\alpha$:

1. If the proposal $P$ includes only one project $(u, v)$, it is approved with probability $\alpha(u, v)$.

2. If the proposal $P$ includes multiple projects, the mechanism randomizes over the proposed projects and no project to maximize the principal's expected payoff, while promising the agent an expected payoff of $\max_{(u,v)\in P} \alpha(u, v)u$. This is the maximal expected payoff the agent could get from proposing each project alone.

By the definition of a PMP mechanism, the more projects the agent proposes, the weakly higher his expected payoff will be. The agent is therefore willing to propose his type truthfully. In other words, PMP mechanisms are IC. Note that for a mechanism to be IC, the agent's payoff from a multiproject proposal must be at least his payoff from proposing each project alone. A PMP mechanism has the feature that the agent is promised exactly the maximal payoff from proposing each project alone, but not more.

Our next theorem shows that there exists an optimal PMP mechanism.

**Theorem 5.2** (Multiproject environment)**.** *Assume* $\mathcal{E} = 2^D$*. For every* $u \in [\underline{u}, 1]$ *and* $p \in [0,1]$*, let* $\gamma(p,u)$ *be*

$$\gamma(p,u) = \min\{q \in [0,1] : qu + (1-q)\underline{u} \geqslant pu\}. \tag{1}$$

*Let*

$$R^m = \max_{(u,v)\in D} \min_{p\in[0,1]} \max\bigl(v(1-p), (1-v)\gamma(p,u)\bigr). \tag{2}$$

1. *The* WCR *under any mechanism is at least* $R^m$*.*

2. *Let* $\rho^m$ *be the PMP mechanism induced by*

$$\alpha^m(u,v) = \max\{p \in [0,1] : (1-v)\gamma(p,u) \leqslant R^m\}. \tag{3}$$

   *It has the* WCR *of* $R^m$ *and is admissible.*

3. *If* $\rho$ *is an IC, admissible mechanism which has the* WCR *of* $R^m$*, then* $U(\rho, A) \leqslant U(\rho^m, A)$ *for every type* $A$*.*

The explicit expressions for $R^m$ and $\alpha^m(u,v)$ are presented at the end of this subsection.

It follows from (1) and (3) that $\alpha^m(u,v) = 1$ if $v \geqslant 1 - R^m$ and $\alpha^m(u,v) < 1$ otherwise. Like in the case of the single-project environment, when the agent proposes only one project, the project is approved for sure if its payoff to the principal is sufficiently high and approved with some probability otherwise. For this reason, we still call $v \geqslant 1-R^m$ and $v < 1-R^m$ the automatic-approval and the chance regions, respectively. Figure 2 depicts these two regions.

When the agent proposes more than one project, the principal promises the agent an expected payoff of $\max_{(u,v)\in P} \alpha^m(u,v)u$. In both panels of figure 2, each dotted curve connects all the projects that induce the same value of $\alpha^m(u,v)u$, so it can be interpreted as an "indifference curve" for the agent. For a project in the automatic-approval region, the principal is willing to compensate the agent his full payoff. In contrast, for a project in the chance region, the principal is willing to compensate the agent only a discounted payoff. The lower the project's payoff to the principal, the more severe the discounting. Hence, indifference curves are vertical in the automatic-approval region and tilt counterclockwise as the principal's payoff $v$ further decreases. The agent's expected payoff is determined by the project (among those proposed) that is on the highest indifference curve.

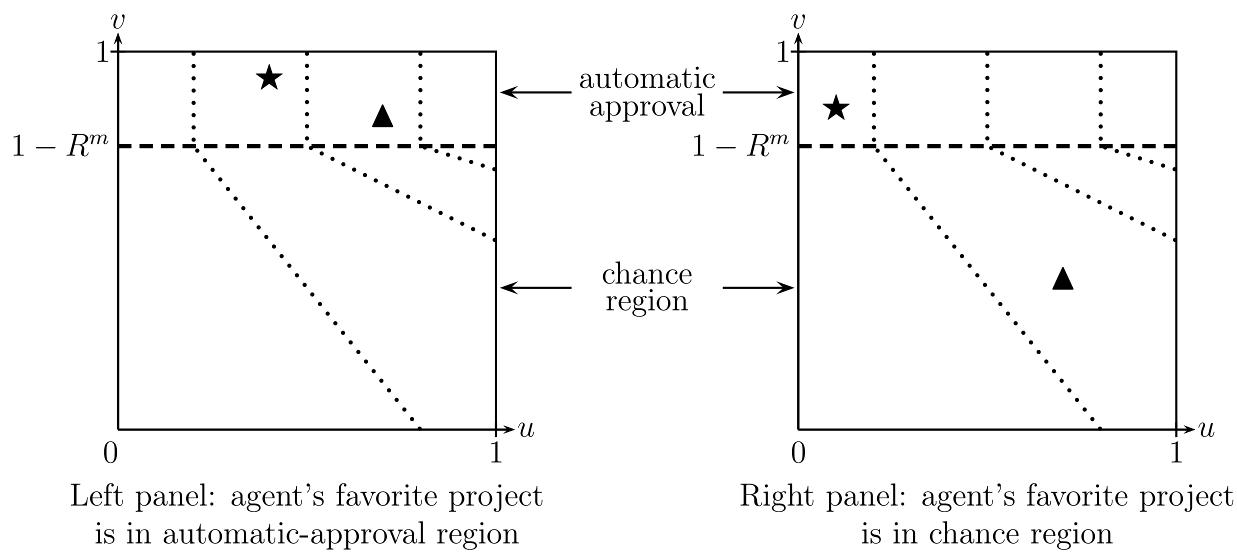


Figure 2: Reaching a compromise when agent's favorite project is in chance region, $\underline{u} = \underline{v} = 0$

Under the optimal mechanism $\rho^m$, if the agent's favorite project is in the automatic-approval region, then this project will be chosen for sure. In this case, there is no benefit to either party from proposing other available projects. The left panel of figure 2 gives such an example: ★ and ▲ denote the available projects and ▲ will be chosen for sure. In contrast, if the agent's favorite project is in the chance region, the benefit to the principal from the agent's proposing multiple projects can be significant. The right panel of figure 2 illustrates such an example. Instead of rejecting ▲ with positive probability, the mechanism randomizes between ▲ and ★ while promising the agent the same payoff he would get from proposing ▲ alone. In such cases, the optimal mechanism imposes a compromise between the two parties: sometimes the choice favors the agent, and at other times it favors the principal.

Lastly, the explicit expressions for $R^m$ and $\alpha^m$ are given by:

$$R^m = \begin{cases} \frac{(1-\underline{u})\left(2-\underline{u}-2\sqrt{1-\underline{u}}\right)}{\underline{u}^2} & \text{if } \underline{v} < \frac{1-\sqrt{1-\underline{u}}}{\underline{u}}, \\ \frac{(1-\underline{u})(1-\underline{v})\underline{v}}{1-\underline{u}\,\underline{v}} & \text{otherwise,} \end{cases}$$

and

$$\alpha^m(u,v) = \begin{cases} 1, & \text{if } v \geqslant 1 - R^m \text{ or } u = 0, \\ \left(1 - \frac{R^m}{1-v}\right)\frac{\underline{u}}{u} + \frac{R^m}{1-v}, & \text{if } v < 1 - R^m \text{ and } u > 0. \end{cases}$$

## 5.4 Comparing the WCR under two environments

Figure 3 compares the WCR under the single-project and the multiproject environments. The left panel depicts the WCR as a function of $\underline{u}$ for a fixed $\underline{v}$. The right panel depicts the WCR as a function of $\underline{v}$ for a fixed $\underline{u}$. Roughly speaking, the principal's gain from having the multiproject environment as compared to the single-project environment, measured by $R^m - R^s$, is larger when $\underline{u}$ or $\underline{v}$ is smaller (i.e., when the principal faces more uncertainty or when players can potentially have strong preferences over projects).

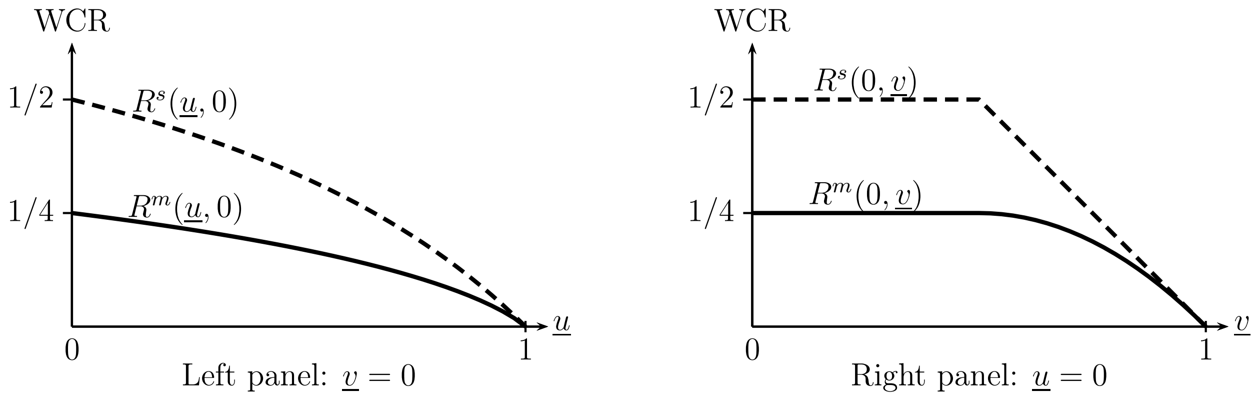


Figure 3: WCR: single-project (dashed curve) vs. multiproject (solid curve)

# 6 Discussion

## 6.1 Intermediate verification capacity

We have focused on the single-project and the multiproject environments, which are natural first steps for us to study. Nonetheless, there are intermediate environments in which the principal can verify up to $k$ projects for some fixed $k \geqslant 2$, so $\mathcal{E} = \{P \subseteq D : |P| \leqslant k\}$. We call this the $k$-project environment.

**Proposition 6.1** (Two are enough)**.** *For any $k \geqslant 2$, the PMP mechanism induced by $\alpha^m(u, v)$ is optimal in the $k$-project environment. The* WCR *under this mechanism is $R^m$.*

*Proof.* Let $A$ be the set of available projects. Let $(u_p, v_p) \in \text{argmax}\{v : (u, v) \in A\}$ and $(u_a, v_a) \in \text{argmax}\{\alpha^m(u, v)u : (u, v) \in A\}$. Let $P = \{(u_p, v_p), (u_a, v_a)\}$. Then under the PMP mechanism induced by $\alpha^m(u, v)$, the agent is willing to propose $P$ since this proposal gives him $\alpha^m(u_a, v_a)u_a$, the maximal payoff he can get under the mechanism. The principal's payoff given the proposal $P$ equals his payoff if the set of available projects was actually $P$. By Theorem 5.2 this payoff is at least $v_p - R^m$, so the principal's regret is at most $R^m$. ☐

Proposition 6.1 shows that having the full benefit of compromise does not require infinite or high verification capacity. A capacity of only two projects is sufficient. Furthermore, even if the principal can verify up to ten projects, it suffices to let the agent propose up to two, which provides a parsimonious way to get the full benefit of compromise.

## 6.2 Cheap-talk communication does not help for any $\mathcal{E}$

We could have started from a more general definition of a mechanism that chooses a project based on both the proposal $P$ and a cheap-talk message $m$ from the agent, as in Bull and Watson (2007) and Ben-Porath, Dekel and Lipman (2019). However, in our model cheap talk does not benefit the principal. This is because the principal can choose a project only from the proposed set $P$ and he knows the payoffs that each project in $P$ gives to both parties. Hence, no information asymmetry remains after the agent proposes $P$, and so there is no benefit to cheap talk.

More specifically, for any proposal $P$ and any cheap-talk messages $m_1, m_2$, we argue that it is without loss for the principal to choose the same subprobability measure over $P$ after $(P, m_1)$ and after $(P, m_2)$. Suppose otherwise that the principal chooses a subprobability measure $\pi_1$ after $(P, m_1)$ and chooses $\pi_2$ after $(P, m_2)$. If the agent strictly prefers $\pi_1$ to $\pi_2$, then he can profitably deviate to $(P, m_1)$ whenever he is supposed to say $(P, m_2)$. Hence, $(P, m_2)$ never occurs on the equilibrium path. If the agent is indifferent between $\pi_1$ and $\pi_2$, then the principal can pick his preferred measure between $\pi_1$ and $\pi_2$ after both $(P, m_1)$ and $(P, m_2)$, without affecting the agent's incentives. This argument does not depend on the exogenous restriction $\mathcal{E}$ on the agent's proposal $P$, so cheap-talk communication does not help for any $\mathcal{E}$.

## 6.3 The commitment assumption

Commitment is crucial for the principal to have some "bargaining power" in the project choice problem. If the principal has no commitment power, sequential rationality requires that he choose his favorite project among the proposed one(s). The agent now has all the bargaining power. He will propose only his favorite project which will be chosen for sure.

In the multiproject environment, the full-commitment solution involves two types of ex post suboptimality. First, no project is chosen despite that the agent has proposed some. Second, a worse project for the principal is chosen despite that a better project for him is also proposed. Some applications may fall between the full-commitment and the no-commitment settings: the principal can commit to choosing no project but cannot commit to choosing a worse project when a better project is also proposed. In such a partial-commitment setting, a multiproject proposal is effectively a single-project proposal with only the principal's favorite project among the proposed one. The optimal mechanism in this partial-commitment setting is then the same as that in the single-project environment characterized in Theorem 5.1.

# 7 Proofs

## 7.1 Proof of Theorem 5.1

**Claim 7.1.** *The* WCR *from any mechanism is at least* $R^s$*.*

*Proof.* Let $v \in [\underline{v}, 1]$. If $\alpha(1, v) > \underline{u}$, then, if the agent has two projects $(1, v)$ and $(\underline{u}, 1)$, the agent will propose $(1, v)$ and the regret will be $1 - \alpha(1, v)v \geqslant 1 - v$. If $\alpha(1, v) \leqslant \underline{u}$, then, if the agent has only the project $(1, v)$, the regret is $v - \alpha(1, v)v \geqslant v(1 - \underline{u})$. Therefore, WCR $\geqslant \min\left((1 - \underline{u})v, 1 - v\right)$ for every $v \in [\underline{v}, 1]$. □

**Claim 7.2.** *The* WCR *from* $\alpha^s$ *is* $R^s$*.*

*Proof.* We call a project $(u, v)$ *good* if $v \geqslant 1 - R^s$ and *mediocre* if $v < 1 - R^s$. From the definition of $R^s$ it follows that $(1 - \underline{u})v \leqslant R^s$ for every mediocre project.

According to $\alpha^s$, if the agent proposes a mediocre project, then his expected payoff is $\underline{u}$; if the agent proposes a good project $(u, v)$, then his expected payoff is $u \geqslant \underline{u}$. Therefore, if the agent has some good project, he will propose a good project $(u, v)$ and the regret is at most $1 - v \leqslant R^s$. If all projects are mediocre, then the agent will propose the project $(u, v)$ with the highest $v$, so the regret is at most $(1 - \alpha^s(u, v))v = (1 - \underline{u}/u)v \leqslant (1 - \underline{u})v \leqslant R^s$. □

**Claim 7.3.** *If* $\alpha$ *has the* WCR *of* $R^s$*, then* $\alpha(u, v) \leqslant \alpha^s(u, v)$ *for every* $(u, v) \in D$*. Hence,* $\alpha^s$ *is admissible.*

*Proof.* Fix a project $(u, v)$. If $v \geqslant 1 - R^s$ or $u = 0$, then $\alpha^s(u, v) = 1$ and therefore $\alpha(u, v) \leqslant \alpha^s(u, v)$. If $v < 1 - R^s$ and $u > 0$, then since the WCR under $\alpha$ is $R^s$, it must be the case that if $A = \{(u, v), (\underline{u}, 1)\}$, then the agent proposes the project $(\underline{u}, 1)$. Otherwise, the regret is at least $1 - v > R^s$. Therefore $\alpha(u, v)u \leqslant \alpha(\underline{u}, 1)\underline{u} \leqslant \underline{u}$, which implies $\alpha(u, v) \leqslant \underline{u}/u = \alpha^s(u, v)$, as desired.

Finally, if $\alpha$ has the WCR of $R^s$ and $\alpha \neq \alpha^s$, then there exists $(u, v) \in D$ such that $\alpha(u, v) < \alpha^s(u, v)$. The regret is strictly higher under $\alpha$ than under $\alpha^s$ if $A = \{(u, v)\}$, so $\alpha^s$ is admissible. □

## 7.2 Proof of Theorem 5.2

Let $a^* = (\underline{u}, 1)$. Let $\overline{U}(P)$ be the optimal value of the following linear programming with variables $\pi(u,v)$ for every $(u,v) \in P$:

$$\overline{U}(P) = \max_{\pi} \quad \underline{u} + \sum_{(u,v)\in P} \pi(u,v)(u - \underline{u}) \tag{4a}$$

$$\text{s.t.} \quad \pi(u,v) \geqslant 0, \ \forall (u,v) \in P, \tag{4b}$$

$$\sum_{(u,v)\in P} \pi(u,v) \leqslant 1, \tag{4c}$$

$$\sum_{(u,v)\in P} \pi(u,v)(1-v) \leqslant R^m. \tag{4d}$$

The following claim explains the role of $\overline{U}(P)$ in our argument: $\overline{U}(P)$ is the maximal payoff that the principal can give the agent for the proposal $P$ such that the principal can give the agent this same payoff if the agent proposed $P \cup \{a^*\}$, while still keeping regret below $R^m$.

**Claim 7.4.** *If $\rho$ is an IC mechanism which has the* WCR *of at most $R^m$, then $U(\rho, P) \leqslant \overline{U}(P)$ for every proposal $P$.*

*Proof.* Let $\tilde{P} = P \cup \{a^*\}$. Let $\pi = \rho(\cdot|\tilde{P})$. Since the regret under the mechanism $\rho$ when the set of available projects is $\tilde{P}$ is at most $R^m$, it follows that $\sum_{(u,v)\in P} \pi(u,v)(1-v) \leqslant R^m$. Therefore the restriction of $\pi$ to the set $P$ is a feasible point in problem (4). Moreover

$$U(\rho, \tilde{P}) = \pi(a^*)\underline{u} + \sum_{(u,v)\in P} \pi(u,v)u \leqslant \underline{u} + \sum_{(u,v)\in P} \pi(u,v)(u - \underline{u}), \tag{5}$$

where the inequality follows from $\pi(a^*) + \sum_{(u,v)\in P} \pi(u,v) \leqslant 1$. The right hand side of (5) is the objective function of (4) at $\pi$. Therefore, $U(\rho, \tilde{P}) \leqslant \overline{U}(P)$. Finally, since the mechanism $\rho$ is IC, it follows that $U(\rho, P) \leqslant U(\rho, \tilde{P})$. Therefore, $U(\rho, P) \leqslant \overline{U}(P)$, as desired. □

When $P$ is a singleton $\{(u,v)\}$, we also denote $\overline{U}(\{(u,v)\})$ by $\overline{U}(u,v)$. The following claim, which follows immediately from (1) and (3), explains the role of the function $\alpha^m(u,v)$ in our argument.

**Claim 7.5.** *When $P$ is a singleton $\{(u,v)\}$, $\overline{U}(u,v) = \alpha^m(u,v)u$.*

For a proposal $P$, let $\underline{U}(P) = \max_{(u,v)\in P} \alpha^m(u,v)u$. The following claim explains the role of $\underline{U}(P)$ in our argument.

**Claim 7.6.** *If $\rho$ is an IC mechanism that accepts the singleton proposal $\{(u,v)\}$ with probability $\alpha^m(u,v)$, then $U(\rho,P) \geqslant \underline{U}(P)$.*

*Proof.* Since $\rho$ is IC, we have that $U(\rho,P) \geqslant U(\{(u,v)\},\rho) = \alpha^m(u,v)u$ for every $(u,v) \in P$. □

Claims 7.4 bounds from above the agent's expected payoff in an IC mechanism which has the WCR of at most $R^m$. Claim 7.6 bounds from below the agent's expected payoff in an IC mechanism which approves the singleton proposal $\{(u,v)\}$ with probability $\alpha^m(u,v)$. The following claim shows that the definition of $R^m$ is such that both bounds can be satisfied.

**Claim 7.7.** *$\underline{U}(P) \leqslant \overline{U}(P)$ for every $P$.*

*Proof.* The function $\overline{U}(P)$ defined in (4) is increasing in $P$. Therefore, from Claim 7.5 we have:

$$\alpha^m(u,v)u = \overline{U}(u,v) \leqslant \overline{U}(P),\ \forall (u,v) \in P.$$

It follows that:

$$\underline{U}(P) = \max_{(u,v)\in P} \alpha^m(u,v)u \leqslant \overline{U}(P).$$

□

By definition, the mechanism $\rho^m$ solves the following linear programming:

$$\rho^m(\cdot|P) \in \arg\max_{\pi} \quad \sum_{(u,v)\in P} \pi(u,v)v \tag{6a}$$

$$\text{s.t.} \qquad \pi(u,v) \geqslant 0,\ \forall (u,v) \in P, \tag{6b}$$

$$\sum_{(u,v)\in P} \pi(u,v) \leqslant 1, \tag{6c}$$

$$\sum_{(u,v)\in P} \pi(u,v)u = \underline{U}(P). \tag{6d}$$

It is possible that (6) has multiple optimal solutions. Since all the optimal solutions are payoff-equivalent for both the principal and the agent, we do not distinguish among them. From now on, the notation $\rho(\cdot|P) \neq \rho^m(\cdot|P)$ means that $\rho(\cdot|P)$ is not among the optimal solutions to (6).

The following lemma is the core of the argument. It gives an equivalent characterization of the mechanism $\rho^m$.

**Lemma 7.8.** *The optimal solutions to* (6) *and those to the following problem coincide. Hence,* $\rho^m(\cdot|P)$ *is also given by the solution to the following problem:*

$$\rho(\cdot|P) \in \arg\max_{\pi} \quad \sum_{(u,v)\in P} \pi(u,v)v \tag{7a}$$

$$\text{s.t.} \qquad \pi(u,v) \geqslant 0,\ \forall (u,v) \in P, \tag{7b}$$

$$\sum_{(u,v)\in P} \pi(u,v) \leqslant 1, \tag{7c}$$

$$\sum_{(u,v)\in P} \pi(u,v)u \geqslant \underline{U}(P), \tag{7d}$$

$$\sum_{(u,v)\in P} \pi(u,v)u \leqslant \overline{U}(P). \tag{7e}$$

*Proof of Lemma 7.8.* We discuss two cases separately.

*Case* 1. Assume that there exists some $(u, v) \in P$ such that $v \geqslant 1 - R^m$. Consider the following linear programming which is a relaxation of both problem (6) and (7):

$$\begin{aligned} \rho(\cdot|P) \in \arg\max_{\pi} \quad & \sum_{(u,v)\in P} \pi(u,v)v && \text{(8a)} \\ \text{s.t.} \quad & \pi(u,v) \geqslant 0,\ \forall (u,v) \in P, && \text{(8b)} \\ & \sum_{(u,v)\in P} \pi(u,v) \leqslant 1, && \text{(8c)} \\ & \sum_{(u,v)\in P} \pi(u,v)u \geqslant \underline{U}(P). && \text{(8d)} \end{aligned}$$

We claim that the constraint (8d) holds with equality at every optimal solution. Indeed, if (8d) is not binding then an optimal solution to (8) is also an optimal solution to the following linear programming:

$$\begin{aligned} \rho(\cdot|P) \in \arg\max_{\pi} \quad & \sum_{(u,v)\in P} \pi(u,v)v && \text{(9a)} \\ \text{s.t.} \quad & \pi(u,v) \geqslant 0,\ \forall (u,v) \in P, && \text{(9b)} \\ & \sum_{(u,v)\in P} \pi(u,v) \leqslant 1, && \text{(9c)} \end{aligned}$$

which is derived from (8) by removing (8d). Let $v_p = \max_{(u,v)\in P} v$ and $u_p = \max_{(u,v_p)\in P} u$. By the definition of $\alpha^m$ in (3), $\alpha^m(u_p, v_p) = 1$ given that $v_p \geqslant 1 - R^m$. Every optimal solution $\pi^*$ to problem (9) satisfies support$(\pi^*) \subseteq \text{argmax}_{(u,v)\in P} v$, which implies that

$$\sum_{(u,v)\in P} \pi^*(u,v)u \leqslant u_p = \alpha^m(u_p, v_p)u_p \leqslant \underline{U}(P).$$

This implies that every optimal solution to (8) satisfies (8d) with equality, so it is a feasible point in both (6) and (7). Since problem (8) is a relaxation of both problem (6) and (7), the optimal values of (6), (7), and (8) coincide. Hence, every optimal solution to (6) or (7) is optimal in (8). This, combined with the fact that every optimal solution to (8) is optimal in (6) and (7), implies that the optimal solutions to (6) and (7) coincide.

*Case* 2. Assume now that $v < 1 - R^m$ for every $(u, v) \in P$. We claim that $\underline{U}(P) = \overline{U}(P)$ and therefore problems (6) and (7) coincide. Given that $v < 1 - R^m$ for every $(u, v) \in P$, the constraint (4c) in problem (4) must be slack since if it is satisfied with an equality then (4d) is violated. Therefore, in this case $\overline{U}(P)$ also satisfies

$$\begin{aligned} \overline{U}(P) = \max_{\pi} \quad & \underline{u} + \sum_{(u,v)\in P} \pi(u,v)(u - \underline{u}) \\ \text{s.t.} \quad & \pi(u,v) \geqslant 0, \ \forall (u,v) \in P, \\ & \sum_{(u,v)\in P} \pi(u,v)(1-v) \leqslant R^m, \end{aligned} \tag{10}$$

which is derived from problem (4) by removing (4c). Problem (10) admits a solution $\pi^*$ with the property that, for some $(u^*, v^*) \in P$, the only non-zero element of $\pi^*$ is $\pi^*(u^*, v^*)$. Therefore, by Claim 7.5,

$$\overline{U}(P) = \overline{U}(u^*, v^*) = \alpha^m(u^*, v^*)u^* \leqslant \underline{U}(P).$$

Therefore, by Claim 7.7 we get $\overline{U}(P) = \underline{U}(P)$, as desired.

□

We now show that, when the set of available projects is a singleton, the regret under the mechanism $\rho^m$ is at most $R^m$.

**Claim 7.9.** *For every singleton $A = \{(u, v)\}$, the regret under $\rho^m$ is at most $R^m$.*

*Proof.* In this case, $\rho^m$ accepts with probability $\alpha^m(u, v)$ so the regret is $v(1-\alpha^m(u, v))$. By the definition of $R^m$, there exists some $\bar{p} \in [0, 1]$ such that $\max(v(1-\bar{p}), (1-v)\gamma(\bar{p}, u)) \leqslant R^m$.

By (3), $\bar{p} \leqslant \alpha^m(u,v)$. Therefore, it also follows that $v(1-\alpha^m(u,v)) \leqslant v(1-\bar{p}) \leqslant R^m$. □

**Claim 7.10.** *The optimal value in problem* (7) *is at least* $\max_{(u,v)\in P} v - R^m$.

*Proof.* Since the constraints (7d) and (7e) cannot both be binding, it is sufficient to prove that the optimal value in the two problems derived from (7) by removing either (7d) or (7e) is at least $v_p - R^m$ where $v_p = \max_{(u,v)\in P} v$. Let $(u_p, v_p) \in P$ denote a principal's favorite project.

If we remove (7d) let $\pi$ be given by $\pi(u_p, v_p) = \alpha^m(u_p, v_p)$ and $\pi(u,v) = 0$ when $(u,v) \neq (u_p, v_p)$. Then $\sum_{(u,v)\in P} \pi(u,v)u = \alpha^m(u_p,v_p)u_p \leqslant \underline{U}(P) \leqslant \overline{U}(P)$ so (7e) is satisfied. Also $v_p(1-\alpha^m(u_p,v_p)) \leqslant R^m$ by Claim 7.9, which implies that the value of the objective function in (7) at $\pi$ is at least $v_p - R^m$, as desired.

If we remove (7e) let $\pi$ be the optimal solution to (4) and let $\pi'$ be the probability distribution over $P$ such that $\pi'(u,v) = \pi(u,v)$ when $(u,v) \neq (u_p, v_p)$ and $\pi'(u_p,v_p) = 1 - \sum_{(u,v)\in P\setminus\{(u_p,v_p)\}} \pi(u,v)$, so $\pi'$ is derived from $\pi$ by allocating the probability of choosing no project to $(u_p, v_p)$. Then

$$\sum_{(u,v)\in P} \pi'(u,v)u = u_p + \sum_{(u,v)\in P} \pi(u,v)(u - u_p) \geqslant \underline{u} + \sum_{(u,v)\in P} \pi(u,v)(u - \underline{u}) = \overline{U}(P) \geqslant \underline{U}(P),$$

where the last equality follows from the fact that $\pi$ is optimal in (4). Therefore, $\pi'$ satisfies (7d). Also

$$\sum_{(u,v)\in P} \pi'(v)(v_p - v) = \sum_{(u,v)\in P} \pi(v)(v_p - v) \leqslant \sum_{(u,v)\in P} \pi(v)(1-v) \leqslant R^m$$

where the last inequality follows from (4d), as desired. □

*Proof of Theorem 5.2.* 1. Fix $(u,v) \in D$ and let $P = \{(u,v)\}$ and $\tilde{P} = \{(u,v), (\underline{u}, 1)\}$.

Let $p$ be the probability that $\rho$ accepts $(u,v)$ when the proposal is $P$. So, $\mathrm{RGRT}(P,\rho) =$

$(1-p)v$. Since the mechanism is IC, the agent's expected payoff under $\tilde{P}$ must be at least $pu$. By definition of $\gamma(u,p)$, this implies that when the proposal is $\tilde{P}$ the mechanism accepts $(u,v)$ with probability at least $\gamma(u,p)$. So, $\mathrm{RGRT}(\tilde{P},\rho) \geqslant (1-v)\gamma(u,p)$. Therefore $\mathrm{WCR}(\rho) \geqslant \max((1-p)v,(1-v)\gamma(u,p))$.

2. The mechanism $\rho^m$ is IC, and it solves problem (7) by Lemma 7.8. By Claim 7.10, the optimal value in problem (7) is at least $\max_{(u,v)\in P} v - R^m$. Since the objective function in (7) is the principal's payoff under $\pi$, the principal's regret is at most $R^m$.

   We next argue that $\rho^m$ is admissible. Let $\rho$ be an IC mechanism which has the WCR of $R^m$ and let $\alpha(u,v)$ be the probability that $\rho$ accepts a singleton proposal $\{(u,v)\}$. Then, $\rho^m$ is not weakly dominated by $\rho$ based on the following two claims:

   (a) If the agent's type $A$ is a singleton $\{(u,v)\}$, then $\alpha(u,v) \leqslant \alpha^m(u,v)$ by claims 7.4 and 7.5. Hence, the principal's payoff is weakly higher under $\rho^m$ than under $\rho$ for singleton $A$.

   (b) Suppose that $\alpha(u,v) = \alpha^m(u,v)$ for every $(u,v)$. Fix a proposal $P$ and let $\pi = \rho(\cdot|P)$ so $U(\rho,P) = \sum_{(u,v)\in P} \pi(u,v)u$. Then, since $\rho$ is IC it follows from Claim 7.6 that $U(\rho,P) \geqslant \underline{U}(P)$, and, from Claim (7.4), that $U(\rho,P) \leqslant \overline{U}(P)$. Therefore $\pi$ is a feasible point in problem (7). Since $\rho^m(\cdot|P)$ is the optimal solution to (7), the principal's payoff is weakly higher under $\rho^m$ than under $\rho$.

3. Let $\rho$ be an IC, admissible mechanism which has the WCR of $R^m$ and which differs from $\rho$. We want to show that $U(\rho,P) \leqslant U(\rho^m,P)$ for every finite $P \subseteq D$. Recall that $U(\rho^m,P) = \underline{U}(P)$ for every $P$.

We first construct a new mechanism $\tilde{\rho}$ based on $\rho$ and $\rho^m$:

$$\tilde{\rho}(\cdot|P) = \begin{cases} \rho^m(\cdot|P), & \text{if } U(P,\rho) \geqslant \underline{U}(P) \\ \rho(\cdot|P), & \text{if } U(P,\rho) < \underline{U}(P). \end{cases}$$

By definition, $U(\tilde{\rho}, P) = \min(U(\rho, P), U(\rho^m, P))$. The functions $U(P, \rho)$ and $U(P, \rho^m)$ are increasing in $P$ since $\rho$ and $\rho^m$ are IC. Therefore $U(P, \tilde{\rho})$ is increasing in $P$, so $\tilde{\rho}$ is also IC. Moreover, for every $P$ either $\tilde{\rho}(\cdot|P) = \rho(\cdot|P)$ or $\tilde{\rho}(\cdot|P) = \rho^m(\cdot|P)$. Therefore the WCR under $\tilde{\rho}$ is also $R^m$.

We next argue that for every $P$, $\tilde{\rho}$ gives the principal a weakly higher payoff than $\rho$ does.

(a) Consider a set $P$ such that $U(\rho, P) < \underline{U}(P)$. Then $\tilde{\rho}(\cdot|P) = \rho(\cdot|P)$, so $\tilde{\rho}$ gives the principal the same payoff as $\rho$ does.

(b) Consider a set $P$ such that $U(\rho, P) \geqslant \underline{U}(P)$. From Claim 7.4 we know that $U(P, \rho) \leqslant \overline{U}(P)$ for every $P$. Therefore, $\rho(\cdot|P)$ is a feasible point in problem (7). It follows from Lemma 7.8 that $\rho^m$ gives the principal a weakly higher payoff than $\rho$ does. Moreover, if $\rho(\cdot|P) \neq \rho^m(\cdot|P)$, then $\rho^m$ gives the principal a strictly higher payoff than $\rho$ does.

    Since $\tilde{\rho}(\cdot|P) = \rho^m(\cdot|P)$ for every $P$ such that $U(\rho, P) \geqslant \underline{U}(P)$, $\tilde{\rho}$ gives the principal a weakly higher payoff than $\rho$ does for every such $P$.

We have argued that $\tilde{\rho}$ gives the principal a weakly higher payoff than $\rho$ does for every $P$. On the other hand, $\rho$ is admissible, so there cannot be a $P$ such that $\tilde{\rho}$ gives the principal a strictly higher payoff than $\rho$ does. This implies that for every $P$ such that $U(\rho, P) \geqslant \underline{U}(P)$, $\rho(\cdot|P) = \rho^m(\cdot|P)$, so $U(\rho, P)$ is equal to $\underline{U}(P)$. Hence, for every $P$, $U(\rho, P) \leqslant \underline{U}(P)$.

□

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