Wealth Dynamics in Communities

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Abstract

This paper explores how favor exchange in communities influences investment and wealth dynamics. Our main result identifies a key obstacle to wealth accumulation: wealth crowds out favor exchange. Low-wealth households are forced to choose between growing their wealth and maintaining access to the support of their communities. The result is that some communities are "left behind," with wealth disparities persisting, and sometimes growing, over time. Using numerical simulations, we show that "place-based" policies encourage both favor exchange *and* wealth accumulation and so have the potential to especially benefit left-behind communities.

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1 Introduction

Rising economic tides do not lift all households equally. Many economies suffer from severe spatial inequality, as some communities are excluded from economic opportunities and remain "left behind" (Economist 2017; Austin et al. 2018). Such left-behind communities are prevalent in both rich and developing countries, and in both rural and urban areas.[1]

Faced with limited economic prospects, members of left-behind communities rely on one another for practical support. Neighbors engage in all kinds of favor exchange, from the trade of food, lodging, and childcare among poor communities in Milwaukee (Desmond 2012, 2016) and immigrant groups in Miami (Portes and Sensenbrenner 1993), to the exchange of rice and kerosene among villagers in India (Jackson et al. 2012). Yet, in sharp contrast to its role in improving consumption, community support plays only a limited role in improving wealth (Stack 1975). Instead, households tend to merely "get by" (Warren et al. 2001), even in the presence of seemingly high-return investments like paying off short-term debts (Ananth et al. 2007, Stegman 2007, Mel et al. 2008). For all of its benefits, why doesn't community support translate to growing wealth?

This paper studies wealth accumulation in left-behind communities. We develop a model of favor exchange, investment dynamics, and mobility across locations. Using this model, we identify a key constraint that prevents households in left-behind communities from accumulating wealth: wealth crowds out favor exchange. Households that rely on favor exchange therefore face sharp costs to investment, since growing their wealth entails losing access to community support. The outcome is a persistent wealth gap between left-behind communities and the rest of the economy.

The key to these results is what we call the "too big for their boots" effect, which is that wealth undermines trust. Neighbors are willing to support a household only if they trust it to reciprocate in the future. Rather than reciprocate, however, that household can instead

[1] In the United States, Desmond [2012] and Hendrickson et al. [2018] give examples from cities and small towns, respectively. For examples in developing countries, see, e.g., Hoff and Sen [2006], Jakiela and Ozier [2016], and Munshi and Rosenzweig [2016].

forego favor exchange and rely on monetary exchange to meet its needs. Because wealthier households are less reliant on favor exchange, they are less willing to repay past support and so are less able to exchange favors with their neighbors.

This "too big for their boots" effect resonates with classic ethnographic work on left-behind communities. In a seminal exploration of favor exchange in a low-wealth United States community, Stack [1975] notices that the *wealthiest* members of the community are most at risk of being excluded from exchange:

> "Members of second-generation welfare families have calculated the risk of giving. As people say, 'The poorer you are, the more likely you are to pay back.' This criterion often determines which kin and friends are actively recruited into exchange networks." - *p. 43*

Stack's observation is that wealthier members are excluded from favor exchange because they can more easily leave the community.

Our main result identifies how this effect shapes investment dynamics andwealth accumulation. We show that households in the community are forced to trade off future wealth against current favor exchange. This trade-off results in sharply limited wealth accumulation, and in some cases, even strictly decreasing wealth. This result explains why favor exchange is ubiquitous inside communities but does not translate to significant wealth accumulation. It is also consistent with recent empirical evidence documenting that one-time transfers typically have limited to no impact on their recipient's long-term wealth (Ananth et al. 2007, Karlan et al. 2019, Balboni et al. 2020).

This result implies that broader economic changes will impact left-behind communities in nuanced and sometimes counter-productive ways. Over the past two decades, productivity in the most productive metropolitan areas has grown relative to the rest of the country (Parilla and Muro 2017). Using numerical simulations, we show that increasing productivity outside of a left-behind community undermines favor exchange within it, resulting in lower consumption *and* lower wealth. Expanding investment opportunities can similarly undermine favor exchange and so decrease long-term wealth in these communities.

We apply this framework to shed light on the policy debate around how to help left-behind communities. Two widely discussed policies are (i) "place-based" policies that provide benefits *within* the community, and (ii) "mobility-based" policies that encourage households to *leave* the community.[2] Using numerical simulations, we show that place-based policies mitigate the "too big for their boots" effect and so have the potential to simultaneously encourage favor exchange *and* wealth accumulation. In contrast, mobility-based policies help those who leave the community, but at the cost of discouraging investment among those who remain.

The contribution of this paper is to explore how favor exchange influences wealth dynamics. Those dynamics occur in the shadow of anonymous monetary exchange, so we are related to papers that study the interaction between formal and informal markets (Kranton 1996, Banerjee and Newman 1998, Gagnon and Goyal 2017, Banerjee et al. 2018, Jackson and Xing 2019). Relative to this literature, we explicitly model wealth accumulation and, by doing so, uncover a trade off between *future* wealth and *current* favor exchange that limits wealth accumulation in communities. This focus on wealth dynamics separates us from other papers that study how communities distort decision-making (Austen-Smith and Fryer 2005, Hoff and Sen 2006).

Our model draws on the relational contracting literature (Macaulay 1963, Bull 1987, Levin 2003, Malcomson 2013), especially papers that consider the role of outside options (Baker et al. 1994, Kovrijnykh 2013). We contribute to this literature by introducing an endogenous state variable, wealth, and by modeling anonymous monetary exhange as an alternative to long-term relationships. Our focus on favor exchange is related to papers that study cooperation in networks (Wolitzky 2013, Ali and Miller 2016, 2018, Miller and Tan 2018, Jackson et al. 2012), although we abstract from questions of network structure to instead focus on wealth dynamics.

[2]See, e.g., Austin et al. [2018] and Bartik [2020] for recent overviews of place-based policies, and Katz et al. [2001] and Chetty et al. [2016] for detailed studies of a mobility-based policy, the "Moving to Opportunity" program.

Our main result is that wealth crowds out favor exchange, therefore discouraging investment. The classic explanation for under-investment is that investments entail fixed costs and so are inaccessible to low-wealth households (e.g., Nelson 1956). Even community support might not be enough to overcome these fixed costs, since commitment problems can prevent households from pooling their resources (Advani 2019). While this explanation is compelling when investment entails fixed costs, recent work has documented under-investment even without such fixed costs (Karlan et al. 2019; Balboni et al. 2020). Existing explanations for under-investment in the absence of fixed costs rely on either behavioral preferences (Bernheim et al. 2015; Banerjee and Mullainathan 2010) or incomplete and monopolistic capital markets (Mookherjee and Ray 2002; Liu and Roth 2019). We offer a different explanation that requires neither behavioral preferences nor capital-market imperfections, which is that investment entails substantial hidden costs because it undermines favor exchange. We show that once we account for these hidden costs, even seemingly high-return investments can have negative returns.

The idea that money eases commitment problems dates to Jevons [1875]'s argument that money solves the "double coincidence of wants." Prendergast and Stole [1999, 2000] build on this idea to compare market and barter economies. We build on a related idea to explore wealth dynamics.

2 Model

A long-lived **household** ("it") has discount factor $\delta \in (0,1)$ and initial wealth $w_0 \geqslant 0$. The household starts in the community. At the beginning of each period $t \in \{0, 1, ...\}$, if the household still lives in the community, it can choose to stay or to move to a city. A household in the city remains there forever.

If the household is in the community in period t, then it plays the following **community game** with a short-lived **neighbor** t ("she"), who represents another member of the community:

1. The household requests a consumption level $c_t \geqslant 0$ and offers a payment $p_t \in [0, w_t]$ in exchange. The payment cannot exceed the household's wealth, $p_t \leq w_t$.

2. Neighbor t accepts or rejects this exchange, $d_t \in \{1, 0\}$. If she accepts ($d_t = 1$), then she receives p_t and incurs the cost of providing c_t. If she rejects ($d_t = 0$), then no trade occurs.

3. The household decides how much of a favor, $f_t \geqslant 0$, to perform for neighbor t. The household incurs the cost of providing f_t.

4. The household invests its remaining wealth, $w_t - p_t d_t$, to generate w_{t+1}. Let $R(\cdot)$ give the return on investment, so that

$$w_{t+1} = R(w_t - p_t d_t).$$

Let $U(\cdot)$ be the household's consumption utility in the community. The household's period-t payoff is $\pi_t = U(c_t d_t) - f_t$. Neighbor t's payoff is $(p_t - c_t)\, d_t + f_t$. The community is tight-knit and so interactions are observed by all neighbors.

We assume that consumption utility $U(\cdot)$ and investment returns $R(\cdot)$ are strictly increasing and strictly concave, with $U''(\cdot)$ and $R'(\cdot)$ continuous, $R(0) = U(0) = 0$, $\lim_{c \downarrow 0} U'(c) = \infty$, and $\lim_{c \to \infty} U'(c) = 0$. We say that investment *generates positive returns* at wealth w if $R'(w) \geqslant \frac{1}{\delta}$. We assume that $R'(w) > \frac{1}{\delta}$ for $w < \bar{w}$ and $R'(w) = \frac{1}{\delta}$ for $w \geqslant \bar{w}$, so that investment generates positive returns at all wealth levels and strictly so below a threshold $\bar{w} > 0$.

If the household has moved to the city by period t, then it plays the **city game** with a short-lived **vendor** t ("she"), who has the same actions and payoff as neighbor t. The city game is identical to the community game in all but two ways. First, each vendor observes only her own interaction with the household, so that interactions are anonymous in the city.

Second, the household's consumption utility in the city is $\hat{U}(\cdot)$ instead of $U(\cdot)$, so that the household's period-t payoff is $\pi_t = \hat{U}(c_t d_t) - f_t$. Our result holds as long as consumption in the city has weakly higher marginal utility than that in the community. Thus, we assume that $\hat{U}(\cdot)$ satisfies the same regularity conditions as $U(\cdot)$, with $\hat{U}'(c) \geq U'(c)$ for all $c > 0$.

The household's continuation payoff in period t is

$$\Pi_t \equiv (1-\delta) \sum_{s=t}^{\infty} \delta^{s-t} \pi_s.$$

We characterize household-optimal equilibria, which are the Perfect Bayesian Equilibria that maximize the household's *ex ante* expected payoff. Without loss of generality, we assume that the household leaves the community if it is indifferent between staying and leaving.

The following assumption ensures that in equilibrium, households that stay in the community have access to strictly positive-return investments.

Assumption 1 *Define $\bar{c} > 0$ as the solution to $U'(\bar{c}) = 1$. Then, $R(\bar{w} - \bar{c}) > \bar{w}$.*

In the context of the low-income midwestern community described by Stack [1975], the household and neighbors are members of a left-behind community called "the Flats." Members of the Flats exchange food, clothing, childcare, and other goods and services (c_t). To compensate one another, households can pay one another with money (p_t), but they can also promise to repay a past favor with a future favor. For example, the recipient of childcare ($c_t > 0$) might promise to reciprocate with future childcare ($f_t > 0$). As in our model, while these favors are costly to the provider, they are in-kind and so do not require any wealth. Households accumulate wealth ($R(\cdot)$) by repaying high-interest debt or making other high-return investments. The Flats is a tight-knit community and "everyone knows who is working, when welfare checks arrive, and when additional resources are available" (p.37), as well as who has failed to repay past favors. Households in the Flats can move to a nearby city, Chicago, which harbors greater opportunities ($\hat{U}' \geq U'$) but separates households from their favor-exchange networks.

In Online Appendix B, we show that our main takeaways are robust to relaxing either of the following two assumptions. First, rather than assuming that moving to the city is irreversible, we allow the household to return to the community after leaving. Second, we allow the household's cost of providing favor f_t to be convex rather than linear.

In our model, the city serves as an alternative to the community. We include the city to match our applications, which typically include mobility across locations, and to study place-based and mobility-based policies. However, similar wealth dynamics would arise without a city; wealth would still crowd out favor exchange, because even in the community, wealthier households rely more on monetary exchange and less on favor exchange. In particular, our main result holds if we eliminate the city and instead assume that any deviation is punished by reversion to a Markov Perfect Equilibrium in the community.

3 Life in the City

We first characterize wealth dynamics in the city. A household in the city faces a standard consumption-investment problem, takes full advantage of investment opportunities, and accumulates wealth.

Interactions are anonymous in the city, so $f_t = 0$ in equilibrium. Vendor t is therefore willing to accept an offer only if the payment covers her costs (i.e., $p_t \geq c_t$), and strictly prefers to do so if $p_t > c_t$. Consequently, every equilibrium entails $p_t = c_t$ in every $t \geqslant 0$, so that $w_{t+1} = R(w_t - c_t)$. For a household with wealth w, the resulting optimal consumption and payoff are given by:

$$\hat{C}(w) \in \arg\max_{c\in[0,w]} \Big((1-\delta)\hat{U}(c) + \delta\hat{\Pi}\left(R\left(w-c\right)\right)\Big) \text{ and}$$

$$\hat{\Pi}(w) = \max_{c\in[0,w]} \Big((1-\delta)\hat{U}(c) + \delta\hat{\Pi}\left(R\left(w-c\right)\right)\Big).$$

Our first result shows that $\hat{\Pi}(w)$ and $\hat{C}(w)$ are the unique equilibrium outcome in the city.

Proposition 1 *Both $\hat{\Pi}(\cdot)$ and $\hat{C}(\cdot)$ are strictly increasing, with $\hat{\Pi}(\cdot)$ continuous. In any equilibrium, $\Pi_t = \hat{\Pi}(w_t)$ and $c_t = \hat{C}(w_t)$ in any $t \geq 0$, with $(w_t)_{t=0}^{\infty}$ increasing and*

$$\lim_{t\to\infty} w_t > \bar{w}$$

on the equilibrium path.

The proof of Proposition 1 is routine and relegated to Online Appendix A. Since $R'(w) > \frac{1}{\delta}$ for $w < \bar{w}$, the standard Euler equation,

$$\hat{U}'(\hat{C}(w_t)) = \delta R'(w_t - \hat{C}(w_t))\hat{U}'(\hat{C}(w_{t+1})),\ \forall\, t, \quad \text{(Euler)}$$

implies that the household's long-term wealth is strictly above $\bar{w}$ in the city. Figure 1 simulates equilibrium outcomes in the city.[3]

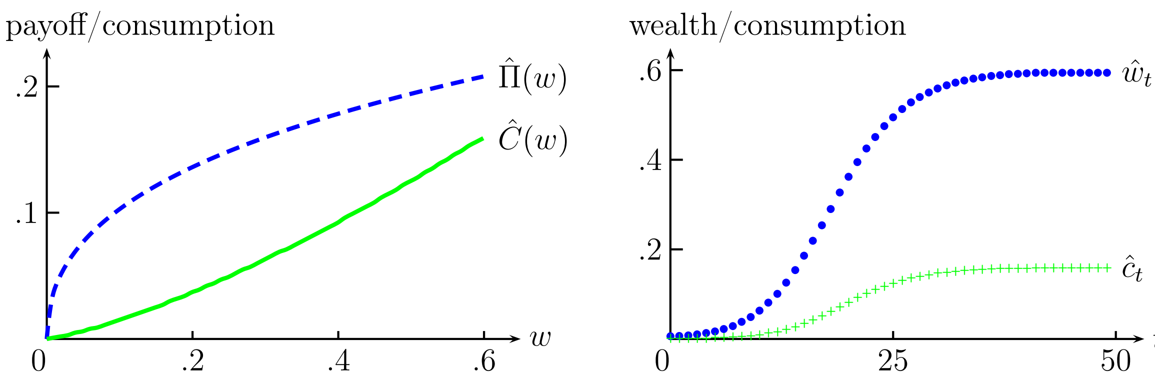

Figure 1: Left panel: the household's equilibrium payoff and consumption as a function of w; Right panel: consumption and wealth over time, starting at $w = 0.006$

[3]Parameter values are $\delta = \frac{8}{10}, U(c) = \frac{\sqrt{c}}{2}, \hat{U}(c) = \frac{13\sqrt{c}}{25}$, and

$$R(w) = \begin{cases} 3\left(\sqrt{w+1} - 1\right) & w \leqslant \frac{11}{25} \\ \frac{5w}{4} + \frac{1}{20} & \text{otherwise.} \end{cases}$$

4 Household-Optimal Wealth Dynamics

We now present our main result, which characterizes household-optimal equilibria in the community. Section 4.1 states this result and discusses its intuition. Section 4.2 draws out the implications of this result with numerical comparative statics. Section 4.3 gives the proof.

4.1 Life in the Community

Our main result identifies two reasons why wealth in the community remains substantially below wealth in the city. First, there is a *selection margin:* sufficiently wealthy households leave the community, whereas poorer households remain. Second, there is a *treatment effect:* the "too big for their boots" effect constitutes an extra cost of investment for households in the community, resulting in sharply limited long-term wealth for those households.

To understand our main result, note that the community's sole advantage over the city is that neighbors can observe and punish a household which reneges on $f_t > 0$. Therefore, the household can credibly promise $f_t > 0$ to repay neighbor t for providing a consumption level, c_t, that strictly exceeds the payment, p_t. Consequently, favor exchange can augment consumption only in the community.

The opportunity to engage in favor exchange is most attractive to low-wealth households, which would otherwise have low consumption and a high marginal utility of consumption. Conversely, wealthy households already consume a lot, so their marginal utility from further increasing c_t is low. Consequently, wealthy households leave the community while low-wealth households stay, giving us our selection margin.

To understand the treatment effect, consider a household with wealth w_t that stays in the community, consumes c_t, and invests $I_t \equiv w_t - p_t$. As in the standard Euler equation (Euler), increasing investment I_t allows for higher future consumption, which has marginal benefit $U'(c_{t+1})R'(I_t)$, at the cost of lower current consumption, which has marginal cost

$U'(c_t)$.

In the community, this familiar trade-off is augmented by a second, indirect cost of investment. Since the household can always renege on f_t, leave, and earn $\hat{\Pi}(R(I_t))$ in the city, the *maximum* favor that can be sustained in equilibrium depends on I_t. Denoting the household's on-path continuation payoff, from staying in the community next period, by $\Pi^*(R(I_t))$, f_t must satisfy the following **dynamic enforcement constraint***:*

$$f_t \leq \bar{f}(I_t) \equiv \frac{\delta}{1-\delta}(\Pi^*(R(I_t)) - \hat{\Pi}(R(I_t))). \tag{DE}$$

The dynamic enforcement constraint ensures that the household prefers to do the favor f_t and earn continuation surplus $\Pi^*(R(I_t))$, rather than reneging on f_t and earning punishment payoff $\hat{\Pi}(R(I_t))$. Investment I_t therefore determines the maximum favor that can be sustained in equilibrium, $\bar{f}(I_t)$. If (DE) binds, then changing I_t affects the size of the favor, f_t, which affects period-t consumption because $c_t = p_t + f_t$. Assuming that $\bar{f}(\cdot)$ is differentiable and (DE) binds, the marginal *indirect* cost of investment is $\bar{f}'(I_t)\,(U'(c_t) - 1)$, where the first term represents how a change in investment, I_t, affects the favor, f_t, and the second term represents how a change in f_t affects the household's period-t payoff, $U(c_t) - f_t$.

In a household-optimal equilibrium, the marginal costs of investment must equal its marginal benefit, resulting in the following *modified Euler equation:*

$$U'(c_t) = \delta R'(I_t)U'(c_{t+1}) + \bar{f}'(I_t)\,(U'(c_t) - 1)\,. \tag{modEuler}$$

The "too big for their boots" effect holds whenever $\bar{f}'(I_t) < 0$, so that investment crowds out favor exchange. We will show that $U'(c_t) - 1 > 0$ for any household in the community. Consequently, whenever the "too big for their boots" effect holds, investment in the community is strictly below the investment that would satisfy the standard Euler equation, (Euler). This is the sense in which the household under-invests.

This intuition elides a key complication: $\bar{f}(\cdot)$ depends on both $\Pi^*(\cdot)$ and $\hat{\Pi}(\cdot)$, which in turn depend on the household's future consumption, favor-exchange, and investment decisions. Wealth affects all of these decisions, rendering a full characterization of household-optimal equilibria intractable.

Our main result, Proposition 2, focuses on the selection and treatment effects. *Selection* is summarized by a wealth level, $w^{se} < \bar{w}$, such that the household leaves the community whenever $w_t \geq w^{se}$, and a set $\mathcal{W} \subseteq [0, w^{se}]$ such that the household stays forever whenever $w_0 \in \mathcal{W}$. *Treatment* is summarized by a wealth level, $w^{tr} < w^{se}$, such that the long-term wealth of a household in the community is below w^{tr}.

Proposition 2 *Impose Assumption 1. There exist wealth levels $w^{tr} < w^{se} \in (0, \bar{w})$ and a positive-measure set $\mathcal{W} \subseteq [0, w^{se}]$ with $\sup \mathcal{W} = w^{se}$ such that in any household-optimal equilibrium:*

1. ***Selection.*** *The household stays in the community forever if $w_0 \in \mathcal{W}$, and otherwise leaves in $t = 0$.*

2. ***Treatment.*** *If the household stays in the community, then $(w_t)_{t=0}^{\infty}$ is monotone, with*

$$\lim_{t \to \infty} w_t \leqslant w^{tr}.$$

Moreover, $\mathcal{W} \cap [w^{tr}, w^{se}]$ has positive measure.

Section 4.3 gives the proof of this result. To see the intuition, we have already argued that wealthy households leave the community while some poorer households stay. Any household that stays, stays forever, since otherwise favor exchange would unravel from the household's last interaction within the community. This gives us a wealth level w^{se} above which households leave, and a set $\mathcal{W}$ of poorer households that stay forever.

Now, consider a household that stays with wealth *just below* w^{se}. Such a household is close to indifferent between leaving and staying. Thus, if the household's wealth always remains near w^{se}, then the right-hand side of (DE) is always close to 0, so $f_t \approx 0$ in every

$t \geq 0$. Since staying is optimal only if $f_t \gg 0$ in some t, this household prefers to stay only if it under-invests so severely that its wealth decreases. The proof of Proposition 2 strengthens this result by showing that $(w_t)_{t=0}^{\infty}$ is monotone and that a positive measure of households, $\mathcal{W} \cap [w^{tr}, w^{se}]$, stay in the community. For these households, wealth declines to a level below w^{tr}.

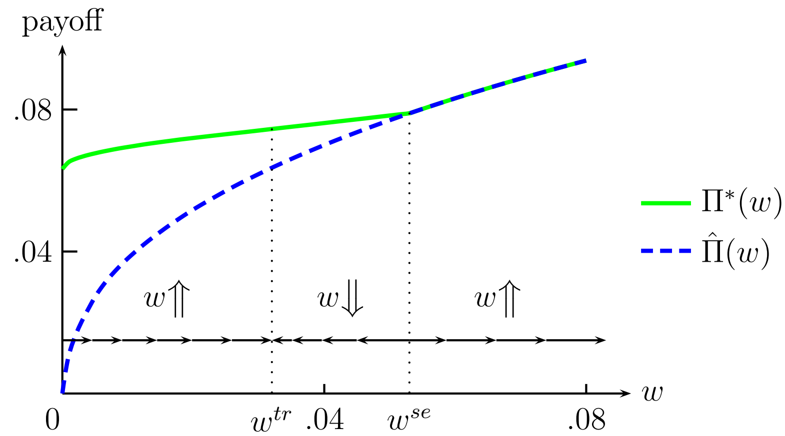

Figure 2: Simulated household-optimal equilibrium payoffs and wealth dynamics

Figure 2 summarizes Proposition 2. In this simulation, the household moves to the city if $w_0 \geqslant w^{se}$ and otherwise stays. Among those that stay, households with $w_0 \leq w^{tr}$ grow their wealth, but only to w^{tr}. Those with $w_0 \in (w^{tr}, w^{se})$ have declining wealth over time.

One policy implication of this result is that one-time transfers do not necessarily improve long-term wealth. Consider a policy that pays a one-time transfer to the household. If this transfer is small enough that $w_t < w^{se}$, then long-term wealth is completely unaffected; it remains w^{tr}. This result resonates with Karlan et al. [2019], which finds that temporary debt relief tends not to improve long-term solvency. In contrast, a transfer large enough to result in $w_t \geq w^{se}$ induces further investment, but only by spurring the household to leave the community.

4.2 Numerical Comparative Statics

This section builds on Proposition 2 to explore how changes in the economic context can have nuanced and non-monotonic effects on equilibrium wealth and welfare.

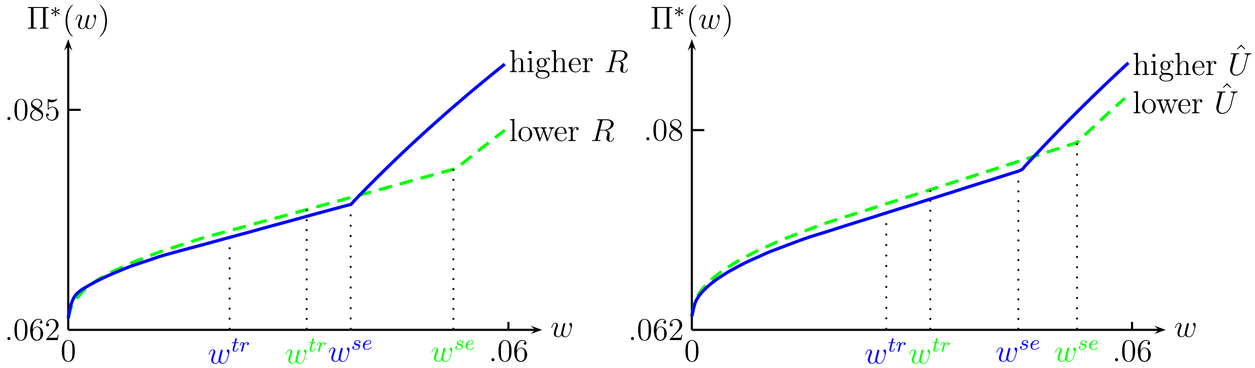

Figure 3: Simulated comparative statics with respect to $R(\cdot)$ and $\hat{U}(\cdot)$

The left panel of Figure 3 illustrates the effect of an increase in investment returns, $R(\cdot)$. In the city, a household benefits from higher $R(\cdot)$ regardless of its initial wealth. In contrast, the impact of increasing $R(\cdot)$ on the community can be unevenly distributed among its poorer and wealthier members. Increasing $R(\cdot)$ increases the household's payoff from leaving, which tightens the dynamic enforcement constraint, (DE), and so undermines favor exchange in the community. This negative impact is larger for wealthier households, for whom (DE) is more stringent. For less-wealthy households, this negative effect is smaller and so can be outweighed by the ability to grow wealth more quickly. Thus, poorer members of the community benefit, wealthier members can be harmed, and long-term wealth in the community, w^{tr}, decreases.

The right panel of Figure 3 considers the effect of an increase in productivity in the city, which we model as an increase in consumption utility in the city, $\hat{U}(\cdot)$. Over the past two decades, productivity has diverged across locations in the United States, with the most productive metropolitan areas growing even more productive relative to the rest of the country (Parilla and Muro 2017). This numerical simulation shows that such productivity improvements can reduce both welfare and long-term wealth in the community. While nothing material has changed within the community, increased productivity outside the community leads to a higher outside option, which depresses both favor exchange and wealth accumulation. The same simulation implies that households with especially bad outside options might be able to accumulate more wealth. Consistent with this implication, Portes and Sensenbrenner [1993] details significant wealth accumulation within the Dominican immigrant community in New York and argues that one reason for this success is that these households face limited outside opportunities.

4.3 The Proof of Proposition 2

Let $\Pi^*(w)$ be the maximum equilibrium payoff of a household with wealth w. Define

$$\begin{aligned} \Pi_c(w) \equiv \quad & \max_{c\geq 0, f\geq 0} \quad \{(1-\delta)(U(c)-f)+\delta\Pi^*(R(w+f-c))\} \\ & \text{s.t.} \qquad 0 \leq c-f \leq w \qquad (1) \\ & \qquad\qquad f \leq \frac{\delta}{1-\delta}\left(\Pi^*(R(w+f-c))-\hat{\Pi}(R(w+f-c))\right). \qquad (2) \end{aligned}$$

We show that $\Pi_c(w)$ is the household's maximum payoff conditional on staying in the community in the current period. Hence, the household's maximum equilibrium payoff, $\Pi^*(w)$, is the maximum of $\hat{\Pi}(w)$ and $\Pi_c(w)$.

Lemma 1 *The household's maximum equilibrium payoff is* $\Pi^*(w_0) = \max\left\{\hat{\Pi}(w_0), \Pi_c(w_0)\right\}$, *where* $\Pi_c(\cdot)$ *and* $\Pi^*(\cdot)$ *are strictly increasing.*

Proof of Lemma 1: We show that $\Pi_c(\cdot)$ is the household's maximum equilibrium payoff conditional on staying in the community in the current period. In any equilibrium, neighbor 0 accepts only if $c_0 \leq p_0 + f_0$. The household's continuation payoff is at most $\Pi^*(R(w_0-p_0))$ and at least $\hat{\Pi}(R(w_0-p_0))$. Hence, it is willing to do favor f_0 only if

$$f_0 \leq \frac{\delta}{1-\delta}\left(\Pi^*(R(w_0-p_0))-\hat{\Pi}(R(w_0-p_0))\right).$$

Setting $c_0 = p_0 + f_0$ yields $\Pi_c(w_0)$ as an upper bound on the household's payoff from staying.

This bound is tight. For any (c_0, f_0) that satisfies (1) and (2), it is an equilibrium to set $p_0 = c_0 - f_0 \geq 0$, play a household-optimal continuation equilibrium on-path, and respond to any deviation with the household leaving and $f_t = 0$ in all future periods.[4] Thus, $\Pi_c(\cdot)$ is the household's maximum equilibrium payoff conditional on staying. It follows that $\Pi^*(w) = \max\{\hat{\Pi}(w), \Pi_c(w)\}$. Since $\Pi_c(\cdot)$ is strictly increasing by inspection and $\hat{\Pi}(\cdot)$ is strictly increasing by Proposition 1, $\Pi^*(\cdot)$ is strictly increasing. $\square$

The next three lemmas characterize household-optimal equilibria in the community. First, we show that households that stay in the community, stay forever.

Lemma 2 *If $w_0 \geq 0$ is such that $\Pi^*(w_0) > \hat{\Pi}(w_0)$, then in any $t \geq 0$ of any household-optimal equilibrium, $\Pi^*(w_t) > \hat{\Pi}(w_t)$ on the equilibrium path.*

Proof of Lemma 2: Suppose $t > 0$ is the first period in which $\Pi^*(w_t) = \hat{\Pi}(w_t)$, so $\Pi^*(w_{t-1}) = \Pi_c(w_{t-1}) > \hat{\Pi}(w_{t-1})$. Let $\{c_{t-1}, f_{t-1}\}$ achieve $\Pi_c(w_{t-1})$. Since $\Pi^*(w_t) = \hat{\Pi}(w_t)$, (2) implies $f_{t-1} = 0$. If the household exits in $t-1$, its payoff $\hat{\Pi}(w_{t-1})$ from living in the city is at least $\Pi_c(w_{t-1})$, since it can choose the same c_{t-1} and earn continuation payoff $\hat{\Pi}(w_t) = \Pi^*(w_t)$. This contradicts the presumption that $\Pi_c(w_{t-1}) > \hat{\Pi}(w_{t-1})$. $\square$

Second, we show that wealthy households leave the community, while poorer households stay.

Lemma 3 *The set $\mathcal{W} \equiv \left\{w : \Pi^*(w) > \hat{\Pi}(w)\right\}$ has positive measure. Moreover, $w^{se} \equiv \sup\left\{w : \Pi^*(w) > \hat{\Pi}(w)\right\}$ satisfies $0 < w^{se} < \bar{w}$.*

Proof of Lemma 3: First, we show that $\Pi^*(0) > \hat{\Pi}(0) = 0$. Because $\lim_{c\downarrow 0} U'(c) = \infty$, there exists a $c > 0$ such that $c \leq \delta U(c)$. Suppose that in all $t \geq 0$, $f_t = c_t = c$ and $p_t = 0$ on the equilibrium path, so the household's equilibrium payoff is $U(c) - c$. Any deviation is punished by $f_t = 0$ in all future periods and the household immediately exiting. This

[4] If $f_t = 0$ in all $t \geq 0$, then the household is willing to leave because $U \leq \hat{U}$.

strategy delivers a strictly positive payoff. It is an equilibrium because $c \leq \delta U(c)$ implies (2). Thus, $\Pi^*(0) > 0$. Since $\Pi^*(w)$ is increasing and $\hat{\Pi}(w)$ is continuous, there exists an open interval around 0 such that $\Pi^*(w) > \hat{\Pi}(w)$. So $\left\{w : \Pi^*(w) > \hat{\Pi}(w)\right\}$ has positive measure.

Next, we show that $w^{se} < \bar{w}$. Let $\bar{c}$ satisfy $U'(\bar{c}) = 1$, and let w_0 be such that $\Pi^*(w_0) > \hat{\Pi}(w_0)$. By Lemma 2, $\Pi^*(w_t) > \hat{\Pi}(w_t)$ in any $t \geq 0$ of any household-optimal equilibrium. Suppose that $c_t > \bar{c}$ in period $t \geq 0$. If $f_t > 0$, then we can perturb the equilibrium by decreasing c_t and f_t by $\epsilon > 0$, which increases the household's payoff at rate $1 - U'(c_t) > 0$ as $\epsilon \to 0$. So, $f_t = 0$.

Let $\tau > t$ be the first period after t such that $f_\tau > 0$. Consider decreasing p_t and c_t by $\epsilon > 0$, increasing p_τ by $\chi(\epsilon)$, and decreasing f_τ by $\chi(\epsilon)$, where $\chi(\epsilon)$ is chosen so that $w_{\tau+1}$ remains constant. Then, $\chi(\epsilon) \geq \frac{\epsilon}{\delta^{\tau-t}}$ because $R'(\cdot) \geq \frac{1}{\delta}$. As $\epsilon \to 0$, this perturbation increases the household's payoff by at least $\delta^{\tau-t}\frac{1}{\delta^{\tau-t}} - U'(c_t) > 0$. It is an equilibrium because $f_s = 0$ for all $s \in [t, \tau - 1]$, so (2) still holds in these periods.

The above argument implies that if $c_t > \bar{c}$, then $f_\tau = 0$ for all $\tau \geq t$. But then $\Pi^*(w_t) \leq \hat{\Pi}(w_t)$, contradicting Lemma 2. Therefore, if $\Pi^*(w_0) > \hat{\Pi}(w_0)$, then $c_t \leq \bar{c}$ in every $t \geq 0$ and so $\Pi^*(w_0) \leq U(\bar{c})$. Since $R(\bar{w} - \bar{c}) > \bar{w}$ by Assumption 1, $\Pi^*(\bar{w}) \geq \hat{\Pi}(\bar{w}) > \hat{U}(\bar{c}) \geq U(\bar{c})$. By the definition of w^{se}, there exists a sequence of initial wealth levels in $\mathcal{W}$ which are arbitrarily close to w^{se} such that the household strictly prefer to stay in the community with those initial wealth levels. If $w^{se} \geq \bar{w}$, the equilibrium payoffs at those initial wealth levels would be strictly above $U(\bar{c})$ due to the continuity of $\hat{\Pi}(w)$. This leads to a contradiction so $w^{se} < \bar{w}$. $\square$

Finally, we show that household-optimal equilibria exhibit monotone wealth dynamics.

Lemma 4 *In any household-optimal equilibrium, $(w_t)_{t=0}^{\infty}$ is monotone.*

The (tedious) proof of Lemma 4 is relegated to Appendix A. The key step of this proof shows that household-optimal investment, $w_t - p_t$, increases in w_t. Thus, if $w_1 \geq w_0$, then $w_2 = R(w_1 - p_1) \geq R(w_0 - p_0) = w_1$ and so on, and similarly if $w_1 \leq w_0$.

We can now prove Proposition 2. **Selection** is implied by Lemma 2 and Lemma 3. For **treatment**, define

$$\tilde{c}(w) = w - R^{-1}(w)$$

and $\tilde{\Pi}(w) = U(\tilde{c}(w))$. Since $w^{se} < \bar{w}$ by Lemma 3, the consumption $\tilde{c}(w^{se})$ does not satisfy the Euler equation (Euler). Thus, there exists $K > 0$ such that

$$\tilde{\Pi}(w^{se}) + K < \hat{\Pi}(w^{se}).$$

Define

$$\bar{f}(w) = \frac{\delta}{1-\delta}\left(\hat{\Pi}(w^{se}) - \hat{\Pi}(w)\right)$$

and

$$\bar{p}(w) = w^{se} - R^{-1}(w).$$

Consider $w_0 < w^{se}$ such that $\Pi^*(w) > \hat{\Pi}(w)$, and suppose that there exists an equilibrium in which $(w_t)_{t=0}^{\infty}$ is increasing on the equilibrium path. We claim that $p_t \leq \bar{p}(w_0)$ and $f_t \leq \bar{f}(w_0)$ in every $t \geq 0$. Indeed,

$$f_t \leq \frac{\delta}{1-\delta}\left(\Pi^*(w_{t+1}) - \hat{\Pi}(w_{t+1})\right) \leq \frac{\delta}{1-\delta}\left(\Pi^*(w^{se}) - \hat{\Pi}(w_0)\right) = \bar{f}(w_0),$$

and

$$p_t = w_t - R^{-1}(w_{t+1}) \leq w^{se} - R^{-1}(w_0) = \bar{p}(w_0),$$

where the inequalities hold because (i) $w_t, w_{t+1} \leq w^{se}$ by Lemma 2, and (ii) $w_{t+1} \geq w_0$ by our presumption that $(w_t)_{t=0}^{\infty}$ is increasing.

Since $c_t \leq p_t + f_t$, the household's payoff satisfies

$$\Pi^*(w_0) \leq U\left(\bar{p}(w_0) + \bar{f}(w_0)\right) \equiv H(w_0).$$

The function $H(w_0)$ is continuous and decreasing in w_0, with $H(w^{se}) = \tilde{\Pi}(w^{se})$. Since $\tilde{\Pi}(w^{se}) + K < \hat{\Pi}(w^{se})$, there exists $w^{tr} < w^{se}$ such that for any $w_0 \in (w^{tr}, w^{se})$,

$$H(w_0) < \hat{\Pi}(w_0).$$

Therefore, for any $w_0 \in (w^{tr}, w^{se})$, if $w_0 \in \mathcal{W}$, then $(w_t)_{t=0}^{\infty}$ must be strictly decreasing, with $\lim_{t\to\infty} w_t \leq w^{tr}$.

By definition of w^{se}, there exists $w_0 \in \mathcal{W} \cap (w^{tr}, w^{se})$ such that $\Pi^*(w_0) > \hat{\Pi}(w_0)$. Since $\Pi^*(\cdot)$ and $\hat{\Pi}(\cdot)$ are increasing, with $\hat{\Pi}(\cdot)$ continuous, we conclude that $\Pi^*(w) > \hat{\Pi}(w)$ on a neighborhood around w_0. ■

5 Policy Simulations

This section considers policies to help left-behind communities. Our main takeaway is that **place-based policies**, which include local grants, tax incentives, infrastructure investments, and other policies that benefit those who stay in a community (Austin et al. 2018, Bartik 2020), have outsized benefits in left-behind communities. These policies can mitigate the "too big for their boots" effect and so encourage both favor exchange and investment within the community.

We contrast place-based policies with two alternative policy approaches. **Income-based policies** provide benefits that depend only on a household's income. Many assistance programs in the United States, including the Temporary Assistance for Needy Families program, are essentially income-based. The other alternative is **mobility-based policies**, which encourage households to leave left-behind communities. One example is the "Moving to Opportunity" program in the United States, which provides housing subsidies in higher-opportunity areas (Katz et al. 2001, Chetty et al. 2016).

Figure 4 presents simulations of these three policy approaches. To facilitate comparison, we model each policy as increasing the household's per-period utility, where mobility-, place-,

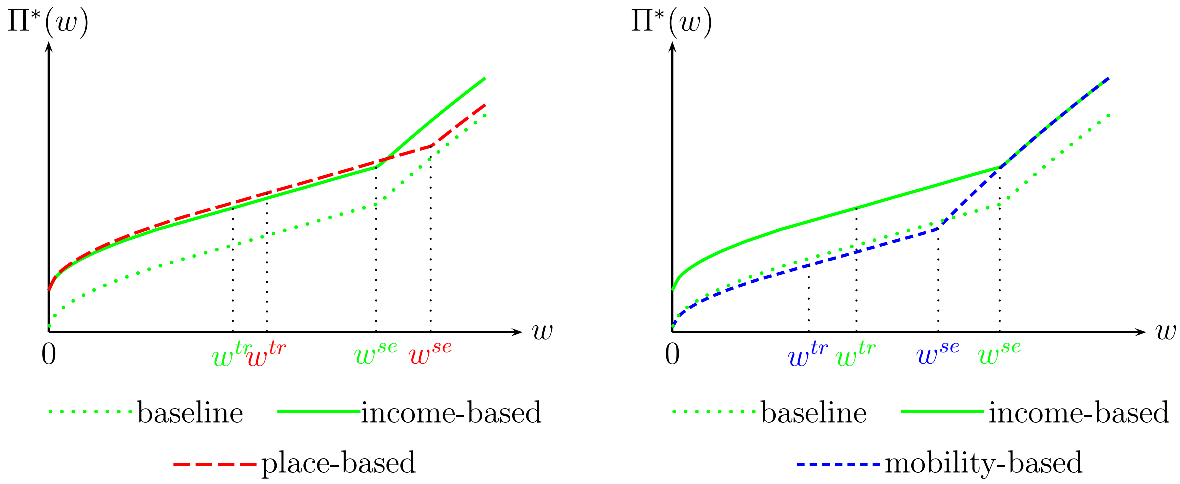

Figure 4: Simulated effects of income-, mobility-, and place-based policies.

and income-based policies respectively increase utility in the city, in the community, or in both locations. These simulations assume that following a deviation, players play a Markov Perfect Equilibrium.[5]

Income-based policies do not affect the household's relative payoff from the city versus from the community, resulting in a uniform increase in equilibrium payoffs. Thus, such policies affect neither selection nor treatment effects. In contrast, both mobility- and place-based policies affect the relative attractiveness of the city, which influences both selection and treatment effects.

Mobility-based policies increase equilibrium payoffs in the city, which encourages households to leave, leading to a lower w^{se}. Households that remain in the community post-policy are strictly worse off; they face a higher outside option, which tightens the dynamic enforcement constraint (DE), exacerbates the "too big for their boots" effect, and decreases the long-term wealth limit in the community, w^{tr}. Note that mobility-based policies have identical effects as increasing productivity in the city, as in right panel of Figure 3.

Place-based policies, in contrast, increase the relative value of staying in the community,

[5]For income- and mobility-based policies, the optimal penal code is indeed Markov. For place-based policies, the restriction to Markov Perfect Equilibrium off-path gives us an upper bound on punishment payoffs and hence a lower bound on the household's optimal equilibrium payoff in the community.

which relaxes (DE) and so mitigates the “too big for their boots” effect. Thus, place-based policies encourage both favor exchange and investment, leading to higher w^{se} and w^{tr}. Such policies are therefore especially effective at helping left-behind communities. By encouraging investment, these policies also help left-behind communities catch up, resulting in less long-term wealth inequality. Other benefits that are tied to the community, such as family, social, or religious ties, can similarly encourage both wealth accumulation and favor exchange.

6 Conclusion

Helping left behind communities requires understanding the social constraints faced by those experiencing poverty. This paper argues that wealth isolates households from community support. Thus, while communities are an essential source of support for their members, that support comes at the cost of lower investment and deeper long-term wealth inequality.

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A Routine Proofs

A.1 Proof of Proposition 1

Suppose that the household lives in the city. In any period t, since future vendors don't observe f_t, the household always chooses $f_t = 0$. Hence, vendor t accepts only if $p_t \geq c_t$. This means that $c_t \in [0, w_t]$ are the feasible consumptions, so that the household's equilibrium continuation payoff is at most $\hat{\Pi}(w_t)$ given wealth w_t.

The following equilibrium gives the household an equilibrium continuation payoff of $\hat{\Pi}(w_t)$. In period t, (i) the household proposes $(c_t, p_t) = (\hat{C}(w_t), \hat{C}(w_t))$; (ii) vendor t accepts if and only if $p_t \geq c_t$. Vendor t has no profitable deviation. This strategy attains $\hat{\Pi}$, so the household has no profitable deviation either.

Let $\{c_t^*\}_{t=0}^{\infty}$ be the consumption sequence in the equilibrium above, given initial wealth w. If $w = 0$, then $c_t^* = 0$ in all $t \geq 0$, so $\hat{\Pi}(0) = \hat{U}(0) = 0$ is the unique equilibrium payoff. If $w > 0$, then it must be true that $c_t^* > 0$ in every $t \geq 0$. Suppose otherwise. Let $\tau \geq 0$ be the first period in which $\min\{c_\tau^*, c_{\tau+1}^*\} = 0$ and $\max\{c_\tau^*, c_{\tau+1}^*\} > 0$. If $c_\tau^* > 0$ and $c_{\tau+1}^* = 0$, consider the perturbation $c_\tau = c_\tau^* - \epsilon_1$, $c_{\tau+1} = c_{\tau+1}^* + \epsilon_2$ for some small $\epsilon_1, \epsilon_2 > 0$ such that the wealth $w_{\tau+2}$ stays the same. If $c_\tau^* = 0$ and $c_{\tau+1}^* > 0$, consider the perturbation $c_\tau = c_\tau^* + \epsilon_1$, $c_{\tau+1} = c_{\tau+1}^* - \epsilon_2$ for some small $\epsilon_1, \epsilon_2 > 0$ such that the wealth $w_{\tau+2}$ stays the same. In either case, the perturbation gives a strictly higher payoff, since $\lim_{c \downarrow 0} \hat{U}'(c) = \infty$.

Next, we show that $\hat{\Pi}(w)$ is the household's *unique* equilibrium payoff. At $w = 0$, $\hat{\Pi}(0) = 0$, so the household's unique equilibrium payoff is indeed $\hat{\Pi}(0)$. For $w > 0$, the household can choose $(c_t, p_t) = ((1-\epsilon)c_t^*, c_t^*)$ in every $t \geq 0$ for $\epsilon > 0$ small. Vendor t strictly prefers to accept. As $\epsilon \downarrow 0$, the consumption sequence $\{(1-\epsilon)c_t^*\}_{t=0}^{\infty}$ gives the household a payoff that converges to $\hat{\Pi}(w)$. So the household must earn at least $\hat{\Pi}(w)$ in any equilibrium.

Turning to properties of $\hat{\Pi}(\cdot)$, we claim that $\hat{\Pi}(\cdot)$ is strictly increasing. Pick $0 \leq w < \tilde{w}$. Let $\{c_t^*\}_{t=0}^{\infty}$ be the sequence associated with w. If the initial wealth is $\tilde{w}$, it is feasible to choose $c_0 = c_0^* + \tilde{w} - w$ and $c_t = c_t^*$ for $t \geq 1$. Since $\hat{U}(\cdot)$ is strictly increasing, so too is $\hat{\Pi}(\cdot)$.

It remains to show that $\hat{\Pi}(\cdot)$ is continuous for all $w > 0$. If $w > 0$, then $\hat{C}(w) > 0$. For $\tilde{w}$ sufficiently close to w, setting $c_0 = \hat{C}(w) + (\tilde{w} - w)$ and $c_t = \hat{C}(w_t)$ for $t \geq 1$ is feasible. The household's payoffs converge to $\hat{\Pi}(w)$ as $\tilde{w} \to w$ under this perturbation, which means that $\lim_{\tilde{w}\uparrow w} \hat{\Pi}(\tilde{w}) \geq \hat{\Pi}(w)$ and $\lim_{\tilde{w}\downarrow w} \hat{\Pi}(\tilde{w}) \geq \hat{\Pi}(w)$. Since $\hat{\Pi}(\cdot)$ is increasing, we conclude that $\hat{\Pi}(\cdot)$ is continuous at every $w > 0$.

We now show that $\hat{\Pi}(\cdot)$ is continuous at $w = 0$. Consider $\lim_{w\downarrow 0} \hat{\Pi}(w)$. Since $R'(\bar{w}) = \frac{1}{\delta}$, the line tangent to $R(\cdot)$ at $\bar{w}$ is $\hat{R}(w) = R(\bar{w}) + \frac{w-\bar{w}}{\delta}$. Since $R(\cdot)$ is concave, $R(w) \leq \hat{R}(w)$ for all $w \geq 0$. Therefore, $\hat{\Pi}(w)$ is bounded from above by the household's maximum payoff if we replace $R(\cdot)$ with $\hat{R}(\cdot)$. For consumption path $\{c_t\}_{t=0}^{\infty}$ to be feasible under $\hat{R}(\cdot)$, it must satisfy

$$(1-\delta)\sum_{t=0}^{\infty}\delta^t c_t \leq (1-\delta)w_0 + \delta R(\bar{w}) - \bar{w}.$$

This means that the payoff of a household with initial wealth w_0 is at most

$$\hat{U}((1-\delta)w_0 + \delta R(\bar{w}) - \bar{w}).$$

Pick any small $\epsilon > 0$. There exists $T < \infty$ and sufficiently small $w_0 > 0$ such that

$$\delta^T \hat{U}\left((1-\delta)R^T(w_0) + \delta R(\bar{w}) - \bar{w}\right) < \frac{\epsilon}{2},$$

where $R^T(w_0)$ denotes the function that applies $R(\cdot)$ T-times to w_0.

Consider a hypothetical setting that is more favorable to the household: we allow the household to both consume *and* save her wealth until period T, after which she must play the original city game. The household's payoff from this hypothetical is strictly larger than $\hat{\Pi}(w_0)$ and is bounded from above by

$$(1-\delta)\sum_{t=0}^{T-1}\delta^t(\hat{U}(R^t(w_0)) + \delta^T\hat{U}((1-\delta)R^T(w_0) + \delta R(\bar{w}) - \bar{w}).$$

As $w_0 \downarrow 0$, $R^T(w_0) \downarrow 0$, so $R^t(w_0) \downarrow 0$ for any $t < T$. Thus,

$$\hat{\Pi}(w_0) \leq (1-\delta) \sum_{t=0}^{T-1} \delta^t \hat{U}(R^t(w_0)) + \delta^T \hat{U}((1-\delta)R^T(w_0) + \delta R(\bar{w}) - \bar{w}) < \epsilon.$$

This is true for any $\epsilon > 0$, so $\lim_{w \downarrow 0} \hat{\Pi}(w) = 0$.

Finally, consider any equilibrium in the city. If $w_0 = 0$, then $w_t = 0$ in any $t \geq 0$. If $w_0 > 0$, then we have shown that $c_t > 0$ in every $t \geq 0$, so $w_t > c_t > 0$. A standard argument (see below) implies the following Euler equation:

$$\hat{U}'(c_t) = \delta R'(w_t - c_t) \hat{U}'(c_{t+1}). \tag{3}$$

Together with $R'(\cdot) \geq \frac{1}{\delta}$ and $\hat{U}(\cdot)$ strictly concave, (3) implies $c_t \leq c_{t+1}$, and strictly so if $w_t < \bar{w}$.

Next, we argue that $\hat{C}(\cdot)$ is strictly increasing in w. Let $\{c_t\}_{t=0}^{\infty}$ and $\{\tilde{c}_t\}_{t=0}^{\infty}$ be the equilibrium consumption sequences for $w > 0$ and $\tilde{w} > w$, respectively. Suppose $c_0 \geq \tilde{c}_0$, and let $\tau \geq 1$ be the first period such that $c_t < \tilde{c}_t$, which must exist because $\hat{\Pi}(\cdot)$ is strictly increasing. Then, $c_{\tau-1} \geq \tilde{c}_{\tau-1}$, $w_{\tau-1} - c_{\tau-1} < \tilde{w}_{\tau-1} - \tilde{c}_{\tau-1}$, and $c_\tau < \tilde{c}_\tau$, so at least one of $(c_{\tau-1}, w_{\tau-1}, c_\tau)$ and $(\tilde{c}_{\tau-1}, \tilde{w}_{\tau-1}, \tilde{c}_\tau)$ violates (3). Hence, $\hat{C}(w)$ is strictly increasing in w. Therefore, $c_{t+1} \geq c_t$ implies $w_{t+1} \geq w_t$, with strict inequalities if $w_t \leq \bar{w}$.

Since $(w_t)_{t=0}^{\infty}$ is monotone, it converges on $\mathbb{R}_+ \cup \{\infty\}$. Suppose $\lim_{t\to\infty} w_t \leq \bar{w}$. Since $c_t = w_t - R^{-1}(w_{t+1})$, $(c_t)_{t=0}^{\infty}$ converges as well. Then, $R'(w_t - c_t)$ converges to a number strictly above $\frac{1}{\delta}$. Hence, (3) is violated as $t \to \infty$. We conclude that $\lim_{t\to\infty} w_t > \bar{w}$. ■

A.2 Deriving the Euler Equation

Consider a household in the city, and let its optimal consumption and wealth sequence be $\{c_t^*, w_t^*\}_{t=0}^{\infty}$. We prove that if $w_0 > 0$, then

$$\hat{U}'(c_t^*) = \delta R'(w_t^* - c_t^*) \hat{U}'(c_{t+1}^*)$$

in every $t \geq 0$.

The proof of Proposition 1 says that $c_t^* > 0$, $c_{t+1}^* > 0$, and $w_t^* - c_t^* > 0$. Suppose that $\hat{U}'(c_t^*) > \delta R'(w_t^* - c_t^*)\hat{U}'(c_{t+1}^*)$. Then, we can perturb (c_t^*, c_{t+1}^*) to $(c_t^* + \epsilon, c_{t+1}^* - \chi(\epsilon))$, where $\chi(\epsilon)$ is chosen such that w_{t+2}^* remains the same as before the perturbation. In particular,

$$R(w_t^* - (c_t^* + \epsilon)) - (c_{t+1}^* - \chi(\epsilon)) = R(w_t^* - c_t^*) - c_{t+1}^*.$$

Hence, $\chi'(\epsilon) = R'(w_t^* - (c_t^* + \epsilon))$.

As $\epsilon \downarrow 0$, this perturbation strictly increases the household's payoff:

$$\begin{aligned} \lim_{\epsilon \downarrow 0} \left\{ \hat{U}'(c_t^* + \epsilon) - \delta \hat{U}'(c_{t+1}^* - \chi(\epsilon))\chi'(\epsilon) \right\} &= \lim_{\epsilon \downarrow 0} \left\{ \hat{U}'(c_t^* + \epsilon) - \delta \hat{U}'(c_{t+1}^* - \chi(\epsilon)) R'(w_t^* - c_t^* - \epsilon) \right\} \\ &= \hat{U}'(c_t^*) + \delta R'(w_t^* - c_t^*)\hat{U}'(c_{t+1}^*) > 0. \end{aligned}$$

This contradicts the fact that (c_t^*, c_{t+1}^*) is optimal. Using a similar argument, we can show that $\hat{U}'(c_t^*) < \delta R'(w_t^* - c_t^*)\hat{U}'(c_{t+1}^*)$ is not possible either. ■

A.3 Proof of Lemma 4

We break the proof of this lemma into four steps.

A.3.1 Step 1: Locally Bounding the Slope of $\Pi^*(\cdot)$ From Below

We claim that for any $w \in [0, w^{se})$, there exists $\epsilon_w > 0$ such that for any $\epsilon \in (0, \epsilon_w)$,

$$\Pi^*(w + \epsilon) - \Pi^*(w) > (1 - \delta)\epsilon.$$

First, suppose $\hat{\Pi}(w) \geq \Pi_c(w)$, and let $\{w_t, c_t\}_{t=0}^{\infty}$ be the wealth and consumption sequences if the household enters the city. The proof of Lemma 3 implies that for any $w_0 < w^{se}$, $R(w_0 - \bar{c}) < w_0$. Proposition 1 says that $\{w_t\}_{t=0}^{\infty}$ is increasing, so $c_0 < \bar{c}$. Hence, there exists $\epsilon_w > 0$ such that $U'(c_0 + \epsilon_w) > 1$. Since $\hat{U}'(c) \geq U'(c)$ for all $c > 0$, $\hat{U}'(c_0 + \epsilon_w) > 1$.

For any $\epsilon < \epsilon_w$, if $w_0 = w+\epsilon$, then the household can enter the city and choose $\hat{c}_0 = c_0+\epsilon$, with $\hat{c}_t = c_t$ in all $t > 0$. We can bound $\Pi^*(w+\epsilon)$ from below by the payoff from this strategy,

$$\Pi^*(w+\epsilon) \geq (1-\delta)\left(\hat{U}(c_0+\epsilon) - \hat{U}(c_0)\right) + \hat{\Pi}(w) > (1-\delta)\epsilon + \hat{\Pi}(w) = (1-\delta)\epsilon + \Pi^*(w).$$

We conclude that $\Pi^*(w+\epsilon) - \Pi^*(w) > (1-\delta)\epsilon$, as desired.

Now, suppose $\hat{\Pi}(w) < \Pi_c(w)$. Let $\{w_t, c_t, f_t\}_{t=0}^{\infty}$ be the wealth, consumption, and reward sequence in a household-optimal equilibrium. There exists $\tau \geq 0$ such that $f_\tau > 0$ for the first time in period τ; otherwise, the household could implement the same consumption sequence in the city. Choose $\epsilon_w > 0$ to satisfy $\epsilon_w < \delta^\tau f_\tau$.

For $\epsilon \in (0, \epsilon_w)$ and initial wealth $w_0 = w + \epsilon$, consider the perturbed strategy such that $\hat{p}_t = p_t$, $\hat{c}_t = c_t$, and $\hat{f}_t = f_t$ in every period *except* τ. In period τ, $\hat{f}_\tau = f_\tau - \frac{\epsilon}{\delta^\tau}$ and $\hat{p}_\tau = p_\tau + \chi$, where χ is chosen so that $\hat{w}_{t+1} = w_{t+1}$. Then, $\hat{c}_\tau = c_\tau + \chi - \frac{\epsilon}{\delta^\tau}$. Based on the proof of Proposition 2, $w_t < w^{se}$ for all $t \leq \tau$. This observation together with $w^{se} \leq \bar{w}$ and Assumption 1, implies that we can choose a sufficiently small ϵ_w such that the marginal return from capital in every $t < \tau$ is strictly higher than $\frac{1}{\delta}$ even if the inital wealth is $w+\epsilon$ rather than w. Hence, $\chi > \frac{\epsilon}{\delta^\tau}$.

Under this perturbed strategy, (2) is satisfied in all $t < \tau$ because $f_t = 0$ in these periods; in $t = \tau$ because $\hat{f}_\tau < f_\tau$ and $\hat{w}_{\tau+1} = w_{\tau+1}$; and in $t > \tau$ because play is unchanged after τ. Moreover, $f_\tau - \frac{\epsilon}{\delta^\tau} > 0$ because $\epsilon < \epsilon_w$, and $\hat{f}_\tau + \hat{p}_\tau = \hat{c}_\tau$, so this strategy is feasible. Thus, it is an equilibrium. Consequently, $\Pi^*(w+\epsilon)$ is bounded from below by the household's payoff from this strategy,

$$\Pi^*(w+\epsilon) > (1-\delta)\delta^\tau \frac{\epsilon}{\delta^\tau} + \Pi_c(w) = (1-\delta)\epsilon + \Pi^*(w),$$

as desired.

A.3.2 Step 2: Moving from Local to Global Bound on Slope

Next, we show that for any $0 \leq w < w' < w^{se}$, $\Pi^*(w') - \Pi^*(w) > (1-\delta)(w'-w)$.

Let

$$z(w) = \sup\{w'' | w < w'' \leq w^{se}, \text{ and } \forall w' \in (w, w''], \Pi^*(w') - \Pi^*(w) > (1-\delta)(w'-w)\}.$$

By Step 1, $z(w) \geq w$ exists. Moreover,

$$\Pi^*(z(w)) - \Pi^*(w) \geq \lim_{\tilde{w} \uparrow z(w)} \Pi^*(\tilde{w}) - \Pi^*(w) \geq (1-\delta)(z(w)-w),$$

where the first inequality follows because $\Pi^*(\cdot)$ is increasing, and the second inequality follows by definition of $z(w)$.

Suppose that $z(w) < w^{se}$. By Step 1, there exists $\epsilon_{z(w)}$ such that for any $\epsilon < \epsilon_{z(w)}$,

$$\Pi^*(z(w)+\epsilon) - \Pi^*(z(w)) > (1-\delta)\epsilon.$$

Hence,

$$\Pi^*(z(w)+\epsilon) - \Pi^*(w) = \Pi^*(z(w)+\epsilon) - \Pi^*(z(w)) + \Pi^*(z(w)) - \Pi^*(w) > (1-\delta)\epsilon + (1-\delta)(z(w)-w).$$

This contradicts the definition of $z(w)$, so $z(w) \geq w^{se}$.

For any $w' < w^{se}$, $w' < z(w)$ and so $\Pi^*(w') - \Pi^*(w) > (1-\delta)(w'-w)$, as desired.

A.3.3 Step 3: Investment is increasing in wealth.

Consider two wealth levels, $0 \leq w_L < w_H < w^{se}$, and suppose that $\Pi_c(w_L) > \hat{\Pi}(w_L)$ and $\Pi_c(w_H) > \hat{\Pi}(w_H)$. Given any household-optimal equilibria, let p_H, p_L be the respective period-0 payments under w_H, w_L. We prove that $w_H - p_H \geq w_L - p_L$. Define $I_k \equiv w_k - p_k$, $k \in \{L, H\}$. Towards contradiction, suppose that $I_H < I_L$.

We first show that $c_H > c_L + (w_H - w_L)$. Suppose instead that $c_H \le c_L + (w_H - w_L)$. Since $I_H < I_L$, we have $p_H > p_L + (w_H - w_L)$. But then $f_H < f_L$, since

$$f_H = c_H - p_H < c_H - (p_L + (w_H - w_L)) \le c_L + (w_H - w_L) - (p_L + (w_H - w_L)) = f_L.$$

Consider the following perturbation: $\hat{p}_H = p_L + (w_H - w_L) \in (p_L, p_H)$, $\hat{f}_H = f_H + p_H - \hat{p}_H \ge f_H$, and $\hat{c}_H = c_H$. Under this perturbation, $\hat{I}_H = w_H - \hat{p}_H = I_L$. Thus, to show that the perturbation satisfies (2), we need only show that $\hat{f}_H \le f_L$. Indeed:

$$\hat{f}_H = f_H + p_H - (p_L + (w_H - w_L)) = c_H - (c_L - f_L) - (w_H - w_L) = f_L + c_H - (c_L + w_H - w_L) \le f_L,$$

where the final inequality holds because $c_H \le c_L + (w_H - w_L)$ by assumption. Thus, this perturbation is also an equilibrium.

We claim that a household with initial wealth w_H strictly prefers this equilibrium to the original equilibrium, which is true so long as

$$\begin{aligned} & (1-\delta)(U(c_H) - \hat{f}_H) + \delta\Pi^*(R(I_L)) > (1-\delta)(U(c_H) - f_H) + \delta\Pi^*(R(I_H)) \\ \iff \quad & (1-\delta)(\hat{f}_H - f_H) < \delta(\Pi^*(R(I_L)) - \Pi^*(R(I_H))) \\ \iff \quad & (1-\delta)(p_H - \hat{p}_H) < \delta(\Pi^*(R(I_L)) - \Pi^*(R(I_H))). \end{aligned}$$

We know that $I_L = I_H + p_H - \hat{p}_H$. Since the household stays in the community, $I_H < I_L < w^{se}$, so $R'(I_H), R'(I_L) > \frac{1}{\delta}$. Thus,

$$R(I_L) - R(I_H) > \frac{1}{\delta}(I_L - I_H) = \frac{1}{\delta}(p_H - \hat{p}_H).$$

By Step 2, $\Pi^*(\cdot)$ increases at rate strictly greater than $(1-\delta)$, so we conclude

$$\delta(\Pi^*(R(I_L)) - \Pi^*(R(I_H))) > \delta(1-\delta)\frac{1}{\delta}(p_H - \hat{p}_H),$$

as desired. Thus, if $I_H < I_L$, then $c_H > c_L + (w_H - w_L)$.

We are now ready to prove that $I_H < I_L$ contradicts household optimality. To do so, we consider two perturbations: one at w_L and one at w_H. At w_H, consider setting

$$\begin{aligned} \hat{c}_H &= c_L + (w_H - w_L) > c_L \geq 0, \\ \hat{p}_H &= p_L + w_H - w_L \in (p_L, w_H], \\ \hat{f}_H &= \hat{c}_H - \hat{p}_H = f_L. \end{aligned}$$

By construction, $w_H - \hat{p}_H = I_L$. Thus, $\hat{f}_H$ satisfies (2) because f_L does. Moreover, $\hat{p}_H + \hat{f}_H = \hat{c}_H$, so the neighbor is willing to accept. This perturbed strategy is therefore an equilibrium. For the original equilibrium to be household-optimal, we must therefore have

$$(1-\delta)(U(c_H) - f_H) + \delta\Pi^*(R(I_H)) \geq (1-\delta)(U(\hat{c}_H) - \hat{f}_H) + \delta\Pi^*(R(\hat{I}_H)). \tag{4}$$

At w_L, consider setting

$$\begin{aligned} \hat{c}_L &= c_H - (w_H - w_L) > c_L \geq 0, \\ \hat{p}_L &= p_H - (w_H - w_L) \in (p_L, w_L]\ , \\ \hat{f}_L &= \hat{c}_L - \hat{p}_L = f_H. \end{aligned}$$

By construction, $w_L - \hat{p}_L = I_H$. Thus, $\hat{f}_L$ satisfies (2) because f_H does. This perturbed strategy is again an equilibrium, so the original equilibrium is household-optimal only if

$$(1-\delta)(U(c_L) - f_L) + \delta\Pi^*(R(I_L)) \geq (1-\delta)(U(\hat{c}_L) - \hat{f}_L) + \delta\Pi^*(R(\hat{I}_L)). \tag{5}$$

Combining (4) and (5) and plugging in definitions, we have

$$U(c_H) - U(c_H - (w_H - w_L)) \geq U(c_L + (w_H - w_L)) - U(c_L).$$

However, $c_H > c_L + w_H - w_L$ and $U(\cdot)$ is strictly concave, so this inequality cannot hold. Thus, if $I_H < I_L$, then at least one of the equilibria at w_H and w_L cannot be household-optimal.

A.3.4 Step 4: Establishing Monotonicity

We have shown that investment, $I(w)$, is increasing in w. Consider a household-optimal equilibrium with $w_1 \geq w_0$. Then, $I(w_1) \geq I(w_0)$, so $w_2 = R(I(w_1)) \geq R(I(w_0)) = w_1$. Thus, $w_2 \geq w_1$, and $w_{t+1} \geq w_t$ for all $t > 1$ by the same argument. Similarly, if $w_1 \leq w_0$, then $I(w_1) \leq I(w_0)$, $w_2 \leq w_1$, and $w_{t+1} \leq w_t$ in all $t \geq 0$. We conclude that $(w_t)_{t=0}^{\infty}$ is monotone in any household-optimal equilibrium. ■

B Extensions to the Model

B.1 Reversible Exit

This appendix shows that mis-investment occurs even if the household can return to the community after leaving for the city. Formally, we modify the game in Section 2 so that at the start of every period while the household is in the city, it can return to the community. If it does, then it plays the community game until it again chooses to leave for the city. Payoffs and information structures are the same as in Section 2, and so neighbors observe all of the household's interactions with neighbors, while vendors observe only their own interactions.

We impose a slightly stronger version of Assumption 1.

Assumption 2 *Define $\bar{c}_m$ as the solution to $\hat{U}'(\bar{c}_m) = 1$, and let $\hat{w}_m$ satisfy $R(\hat{w}_m - \bar{c}_m) = \hat{w}_m$. Then, $R'(\hat{w}_m) > \frac{1}{\delta}$.*

Under this assumption, we can prove that mis-investment occurs even if exit is reversible.

Proposition 3 *Impose Assumption 2. There exists a w^{**}, a $w^{cc} < w^{**}$, and a positive-measure interval $\mathcal{W} \subseteq [0, w^{**})$ such that the household permanently exits the community if $w_0 \notin \mathcal{W}$. If $w_0 \in \mathcal{W}$, then in any household-optimal equilibrium, the household is in the community for an infinite number of periods.*

Moreover, if $w_0 \notin \mathcal{W}$, then in any equilibrium,

$$\lim_{t\to\infty} w_t > w^{**},$$

while if $w_0 \in \mathcal{W}$, then for every $t \geq 0$, $w_t < w^{**}$. Moreover, $w_{t+1} < w_t$ whenever $w_t \in (w^{cc}, w^{**})$.

B.1.1 Proof of Proposition 3

Much like the proof of Proposition 2, we break this proof into a sequence of lemmas. We begin by showing that the household's worst equilibrium payoff equals $\hat{\Pi}(\cdot)$, its worst equilibrium payoff from the game with reversible exit.

Lemma 5 *For any initial wealth $w \geq 0$, the household's worst equilibrium payoff is $\hat{\Pi}(\cdot)$.*

Proof of Lemma 5: This proof is similar to the proof of Proposition 1. It is an equilibrium for $f_t = 0$ in every $t \geq 0$, in which case it is optimal for the household to permanently leave the community. In the city, vendor t accepts only if $p_t \geq c_t$. Therefore, $\hat{\Pi}(\cdot)$ gives the maximum equilibrium payoff if the household permanently leaves the community. But as in the proof of Proposition 1, the household cannot earn less than $\hat{\Pi}(\cdot)$, because vendor t must accept whenever $p_t > c_t$. $\square$

Now, we turn to the household's maximum equilibrium payoff. Define $\Pi^{**}(w)$ as the maximum equilibrium payoff with initial wealth w. Define $\Pi_c^*(\cdot)$ identically to $\Pi_c(\cdot)$, except that $\Pi^*(\cdot)$ is replaced by $\Pi^{**}(\cdot)$. Define

$$\Pi_m^*(w) \equiv \max_{0\leq c\leq w} \left\{(1-\delta)\hat{U}(c) + \delta\Pi^{**}(R(w-c))\right\}$$

as the household's maximum equilibrium payoff if it chooses the city in the current period. The key difference between this model and the baseline model is that $\Pi_m^*(\cdot)$ might entail the household returning to the community to take advantage of relational contracts in the future. Therefore, $\Pi_m^*(\cdot) \geq \hat{\Pi}(\cdot)$, since the latter entails staying in the city forever.

Lemma 6 *Both* $\Pi_c^*(\cdot)$ *and* $\Pi_m^*(\cdot)$ *are strictly increasing. For all* $w \geq 0$*,*

$$\Pi^{**}(w) \equiv \max \{\Pi_c^*(w), \Pi_m^*(w)\}.$$

Proof of Lemma 6: By Lemma 5, the household earns no less than $\hat{\Pi}(\cdot)$ following a deviation. As in Lemma 1, conditional on choosing the community in period 0, the household's maximum equilibrium payoff equals $\Pi_c^*(w_0)$. If the household instead chooses the city in period t, then $f_t = 0$ in any equilibrium., since the continuation equilibrium is independent of f_t. Thus, the household optimally sets $p_t = c_t$, so its maximum equilibrium continuation payoff equals $\Pi^{**}(R(w-c))$. We conclude that $\Pi_m^*(w)$ is the household's maximum equilibrium payoff conditional on choosing the city. It then immediately follows that $\Pi^{**}(w) \equiv \max \{\Pi_c^*(w), \Pi_m^*(w)\}$. Both $\Pi_c^*(\cdot)$ and $\Pi_m^*(\cdot)$ are strictly increasing by inspection. □

Apart from some details of the proof, the next result is similar to Lemma 2.

Lemma 7 *If* $\Pi^{**}(w_0) > \hat{\Pi}(w_0)$*, then* $\Pi^{**}(w_t) > \hat{\Pi}(w_t)$ *in all* $t \geq 0$ *of any household-optimal equilibrium.*

Proof of Lemma 7: Suppose not, and let $\tau > 0$ be the first period such that $\Pi^{**}(w_\tau) = \hat{\Pi}(w_\tau)$. In period $\tau - 1$, (2) implies that $f_t = 0$ if the household stays in the community. Therefore, it is optimal for the household to leave the community in $\tau - 1$. But then it is optimal for the household to *permanently* leave the community in $\tau - 1$, since $\Pi^{**}(w_\tau) = \hat{\Pi}(w_\tau)$. So $\Pi^{**}(w_{\tau-1}) = \hat{\Pi}(w_{\tau-1})$, contradicting the definition of τ. □

Lemma 8 *Suppose that* $\Pi^{**}(w_0) > \hat{\Pi}(w_0)$*. Then in every* $t \geq 0$*,* $\hat{U}'(c_t) \geq 1$*, and there exists* $\tau > t$ *such that the household stays in the community in period* τ*.*

Proof of Lemma 8: Towards contradiction, suppose that there exists $t \geq 0$ such that $\hat{U}'(c_t) < 1$, so that *a fortiori,* $U'(c_t) < 1$. If $f_t > 0$, then we can decrease f_t and c_t by the same $\epsilon > 0$. This perturbation is also an equilibrium, and increases the household's period-t payoff at rate $1 - \hat{U}'(c_t) > 0$ as $\epsilon \to 0$. Thus, $f_t = 0$, which implies that $p_t = c_t > 0$.

By Lemma 7, $\Pi^*(w_{t+1}) > \hat{\Pi}(w_{t+1})$. Therefore, there exists a $\tau > t$ such that $f_\tau > 0$, since otherwise the household could do no better than exiting the city permanently. Let τ be the *first* period after t such that $f_\tau > 0$. Note that the household must be in the community in period τ.

Consider the following perturbation: decrease p_t and c_t by $\epsilon > 0$, and increase p_τ and decrease f_τ by $\chi(\epsilon)$, where $\chi(\epsilon)$ is chosen so that $w_{\tau+1}$ remains constant. Then, $\chi(\epsilon) \geq \frac{\epsilon}{\delta^{\tau-t}}$ because $R'(\cdot) \geq \frac{1}{\delta}$. As $\epsilon \to 0$, $\chi(\epsilon) \to 0$. Hence, this perturbation is feasible for small enough $\epsilon > 0$. It is an equilibrium, since (2) is trivially satisfied in all $t' \in [t, \tau - 1]$ because $f_{t'} = 0$ in those periods. This perturbation changes the household's period-t continuation payoff at rate no less than

$$-(1-\delta)\hat{U}'(c_t) + \delta^{\tau-t}(1-\delta)\frac{1}{\delta^{\tau-t}} > 0$$

as $\epsilon \to 0$. Thus, the original equilibrium could not have been household-optimal. $\square$

Next, we show that the household stays in the community for sufficiently low initial wealth levels.

Lemma 9 *The set*

$$\left\{w | \Pi^{**}(w) > \hat{\Pi}(w)\right\}$$

has positive measure, with

$$w^{**} \equiv \sup \left\{w | \Pi^{**}(w) > \hat{\Pi}(w)\right\} < \infty.$$

Proof of Lemma 9: The proof that $\left\{w | \Pi^{**}(w) > \hat{\Pi}(w)\right\}$ is identical to the proof in Lemma 3, since the same constructions work at $w = 0$, $\Pi^{**}(\cdot)$ is increasing, and $\hat{\Pi}(\cdot)$ is continuous. To prove that $w^{**} < \infty$, suppose w_0 is such that

$$R(w_0 - \bar{c}_m) > w_0.$$

For any such w_0,

$$\hat{\Pi}(w_0) > \hat{U}(\bar{c}_m) \geq \Pi^{**}(w_0),$$

where the second inequality follows by Lemma 8 and the assumption that $\Pi^{**}(w) > \hat{\Pi}(w)$. Contradiction. So $\hat{\Pi}(w_0) = \Pi^{**}(w_0)$ for any $w_0 > \hat{w}_m$. We conclude $w^{**} < \infty$. $\square$

We are now in a position to prove Proposition 3. So far, the argument has hewn closely to the proof of Proposition 2. The rest of the proof marks a more substantial departure.

Lemma 9 shows that a positive-measure set $\mathcal{W}$ exists such that $\Pi^{**}(w_0) > \hat{\Pi}(w_0)$ for all $w_0 \in \mathcal{W}$. Lemma 7 implies that for any $w_0 \in \mathcal{W}$, we can construct an infinite sequence of periods such that the household remains in the community for each period in that sequence. For any $w_0 \notin \mathcal{W}$, $\Pi^{**}(w_0) = \hat{\Pi}(w_0)$ and so the household permanently exits the community. This proves the first part of Proposition 3.

From the proof of Lemma 8, we know that $w^{**} \leq \hat{w}_m$. Assumption 2 then implies that $R'(w^{**}) > \frac{1}{\delta}$. As in the proof of Proposition 2, if the household permanently exits the community, then w_t is increasing, with $\lim_{t\to\infty} w_t > w^{**}$. This proves the second part of Proposition 3.

Suppose $w_0 \in \mathcal{W}$. Then, Lemma 7 and the definition of w^{**} immediately imply that $w_t < w^{**}$ in every $t \geq 0$. It remains to identify a $w^{cc} < w^{**}$ such that if $w_0 \in \mathcal{W}$, then whenever $w_t \in (w^{cc}, w^{**})$ in a household-optimal equilibrium, $w_{t+1} < w_t$. By an argument similar to Proposition 2,

$$\lim_{w \uparrow w^{**}} \Pi^{**}(w) = \Pi^{**}(w^{**}) = \hat{\Pi}(w^{**}),$$

because $\Pi^{**}(\cdot)$ is increasing, $\hat{\Pi}(\cdot)$ is continuous, $\Pi^{**}(\cdot) \geq \hat{\Pi}(\cdot)$, and $\Pi^{**}(w) = \hat{\Pi}(w)$ for all

$w > w^{**}$. Therefore, we can define a wealth level $w^{cc1} < w^{**}$ similarly to w^c in the proof of Proposition 2.

Define the function

$$F(w) \equiv (1-\delta)\hat{U}(w^{**} - R^{-1}(w)) + \delta\hat{\Pi}(w^{**}) - \hat{\Pi}(w)$$

on $w \in [0, w^{**}]$. Then $F(\cdot)$ is strictly decreasing and continuous, with

$$F(0) = (1-\delta)\hat{U}(w^{**}) + \delta\hat{\Pi}(w^{**}) > 0.$$

At $w_0 = w^{**}$, it is feasible for the household to permanently leave the community and consume $c_t = w^{**} - R^{-1}(w^{**})$ in each $t \geq 0$. However, doing so violates (Euler), since $R'(w^{**}) > \frac{1}{\delta}$. Therefore, this consumption path must be dominated by some other feasible consumption path once the household permanently leaves the community, which implies

$$\hat{U}(w^{**} - R^{-1}(w^{**}) < \hat{\Pi}(w^{**}).$$

Consequently,

$$F(w^{**}) = (1-\delta)\hat{U}(w^{**} - R^{-1}(w^{**})) - (1-\delta)\hat{\Pi}(w^{**}) < 0.$$

We conclude that there exists a unique $w^{cc2} \in (0, w^{**})$ such that $F(w^{cc2}) = 0$.

Set $w^{cc} = \max\{w^{cc1}, w^{cc2}\}$. For any $w_0 \in (w^{cc}, w^{**})$ such that $\Pi^{**}(w_0) > \hat{\Pi}(w_0)$, the household must either stay in the community or leave in $t = 0$. If it stays in the community, then we can follow the steps of the proof of Proposition 2, Statement 3, to conclude that $w_{t+1} \leq w^{cc1} < w_t$.

Suppose the household is in the city in $t = 0$. Towards contradiction, suppose that

$w_{t+1} \geq w_t$. Therefore, the household's payoff satisfies

$$\begin{aligned}
\Pi^{**}(w_0) &\leq (1-\delta)\hat{U}(w^{**} - R^{-1}(w_0)) + \delta\Pi^{**}(w^{**}) \\
&= (1-\delta)\hat{U}(w^{**} - R^{-1}(w_0)) + \delta\hat{\Pi}(w^{**}) \\
&< \hat{\Pi}(w_0),
\end{aligned}$$

where the first inequality follows because $f_0 = 0$, so that $p_0 = c_0 = w_0 - R^{-1}(w_1) \leq w^{**} - R^{-1}(w_0)$; the equality follows because $\Pi^{**}(w^{**}) = \hat{\Pi}(w^{**})$, and the final, strict inequality follows because $w_0 > w^{cc2}$ and so $F(w_0) < 0$. But $\Pi^{**}(\cdot) \geq \hat{\Pi}(\cdot)$, proving a contradiction. So $w_{t+1} < w_t$ if the household is in the city in $t = 0$.

We conclude that if $w_0 \in \mathcal{W}$, then in any household-optimal equilibrium, $w_t < w^{**}$ for all $t \geq 0$, and $w_{t+1} < w_t$ whenever $w_t > w^{cc}$. This completes the proof. ■

B.2 Non-Linear Favors

B.2.1 Model and Statement of Result

In this appendix, we show that a (slightly weaker version of) Proposition 2 holds if the principal's payoff is convex in f_t.

Consider the **game with non-linear favors,** which is identical to the game in Section 2 except that the household's stage-game payoff is

$$\pi_t = \begin{cases} U(c_t d_t) - k(f_t) & \text{in the community} \\ \hat{U}(c_t, d_t) - k(f_t) & \text{in the city.} \end{cases}$$

Assume that $k(\cdot)$ is strictly increasing, weakly convex, twice continuously differentiable, and satisfies $k(0) = 0$ and $\lim_{f\to\infty} k'(f) = \infty$. Note that a special case of this game has $k = U^{-1}$, which corresponds to a setting in which the neighbors value f_t according to the same utility function as the household's consumption.

Define $f^*(c)$ as the unique solution to $U'(c) = k'(f^*(c))$ and c^* as the solution to $U(c^*) =$

$\hat{U}(c^* - f^*(c^*))$. Note that c^* exists and is unique because $\hat{U}'(c) > U'(c)$, $f^*(\cdot)$ is decreasing, and $\lim_{c\to\infty} f^*(c) = 0$. We impose a modified version of Assumption 1.

Assumption 3 *Define $\tilde{c}(w)$ as the unique solution to $R(w - \tilde{c}(w)) = w$. Assume that*

$$\hat{U}(\tilde{c}(\bar{w}) - f^*(\tilde{c}(\bar{w}))) - U(\tilde{c}(\bar{w})) > \frac{\delta}{1-\delta}U(c^*).$$

Suppose that the household has wealth $\bar{w}$, which is the wealth level at which strictly positive-return investments are exhausted. Assumption 3 ensures that this household's value from moving to the market and consuming $\tilde{c}(\bar{w}) - f^*(\tilde{c}(\bar{w}))$ forever after is substantially greater than its utility from staying in the community and consuming $\tilde{c}(\bar{w})$ forever. Note that $\tilde{c}(w)$ is increasing because $R'(\cdot) \geq \frac{1}{\delta} > 1$.

We now prove our main result for the game with non-linear favors.

Proposition 4 *Impose Assumption 3 in the game with non-linear favors. Then, there exists a $w^{se} < \bar{w}$, a $w^{tr} \in [0, w^{se})$, and a positive-measure set $\mathcal{W} \subseteq [0, w^{se})$ with $\mathcal{W} \cap [w^{tr}, w^{se})$, such that in any household-optimal equilibrium,*

1. ***Selection:*** *The household stays in the community forever if $w_0 \in \mathcal{W}$ and otherwise leaves immediately.*

2. ***Treatment:*** *If $w_0 \in \mathcal{W}$ and $w_t \geq w^{tr}$ on the equilibrium path, then $w_{t+1} < w_t$.*

B.2.2 Proof of Proposition 4

We focus only on those parts of the proof that differ substantially from the proof of Proposition 2.

The proof of Proposition 1 goes through without change, since $f_t = 0$ in every period of any equilibrium in the city. Similarly, once we substitute the cost function $k(f)$ into the household's payoff, the proofs of Lemmas 1 and 2 go through without change.

The next step of the proof, which shows that wealthy households leave the community and poorer households stay, requires a new argument.

Lemma 10 *The set* $\left\{w : \Pi^*(w) > \hat{\Pi}(w)\right\}$ *has positive measure. Moreover,* $w^{se} \equiv \sup\left\{w : \Pi^*(w) > \hat{\Pi}(w)\right\}$ *satisfies* $0 < w^{se} < \infty$.

Proof of Lemma 10: The argument that $\Pi^*(0) > \hat{\Pi}(0) = 0$ goes through essentially without change. Thus, we need to show only that $w^{se} < \infty$.

Step 1: In any $t \geq 0$ of any household-optimal equilibrium, $f_t \leq f^*(c_t)$. If the household is in the city, $f_t = 0 \leq f^*(c_t)$. If it is in the community, then suppose that $f_t > f^*(c_t)$. Consider perturbing the equilibrium by decreasing c_t and f_t by $\epsilon > 0$. This perturbation is feasible as $\epsilon \to 0$, and moreover, increases the household's stage-game payoff at rate $k'(f_t) - U'(c_t) > k'(f^*(c_t)) - U'(c_t) = 0$. Thus, the original equilibrium was not household-optimal. This proves Step 1.

Step 2: For any $w \geq 0$, $\Pi^*(w) - \hat{\Pi}(w) \leq U(c^*)$. If $\Pi^*(w) = \hat{\Pi}(w)$, then the result is immediate. Suppose that $\Pi^*(w) > \hat{\Pi}(w)$. By Lemma 2, a household with $w_0 = w$ stays in the community in any $t \geq 0$ of any household-optimal equilibrium.

Fixing such an equilibrium, we can derive a lower bound on $\hat{\Pi}(w)$ using the following perturbation: the household leaves the community, chooses $\tilde{p}_t = p_t$ in each $t \geq 0$, and request consumption $\tilde{c}_t = c_t - f_t \geq 0$. This strategy is feasible, and the resulting payoff satisfies

$$\begin{aligned}
\textstyle\sum_{t=0}^{\infty} \delta^t (1-\delta)\hat{U}(c_t - f_t) &= \textstyle\sum_{t|c_t \geq c^*} \delta^t (1-\delta)\hat{U}(c_t - f_t) + \sum_{t|c_t < c^*} \delta^t (1-\delta)\hat{U}(c_t - f_t) \\
&\geq \textstyle\sum_{t|c_t \geq c^*} \delta^t (1-\delta)\hat{U}(c_t - f^*(c_t)),
\end{aligned}$$

where the inequality follows because $f_t \leq f^*(c_t)$ and $c_t - f_t \geq 0$. Therefore,

$$\begin{aligned} \Pi^*(w) - \hat{\Pi}(w) &\leq \sum_{t=0}^{\infty} \delta^t (1-\delta) \left(U(c_t) - \hat{U}(c_t - f_t) \right) \\ &\leq \sum_{t|c_t \geq c^*} \delta^t (1-\delta) \left(U(c_t) - \hat{U}(c_t - f^*(c_t)) \right) + \sum_{t|c_t < c^*} \delta^t (1-\delta) U(c_t) \\ &\leq U(c^*), \end{aligned}$$

where the first two inequalities follow from our lower bound on $\hat{\Pi}(w)$, and the final inequality holds because $U(c_t) \leq \hat{U}(c_t - f^*(c_t))$ for all $c_t \geq c^*$, and $U(c_t) \leq U(c^*)$ for all $c_t < c^*$. This proves Step 2.

Step 3: There exists a w^{se} such that for all $w \geq w^{se}$, $\Pi^*(w) = \hat{\Pi}(w)$. Fix a wealth level such that $\Pi^*(w) > \hat{\Pi}(w)$ and $\hat{\Pi}(w) > U(c^*)$, and consider a household-optimal equilibrium with $w_0 = w$. By Lemma 2, the household must stay in the community forever. Therefore, there exists $t \geq 0$ such that $U(c_t) \geq \hat{\Pi}(w)$, so $c_t > c^*$.

In this period t, we must have $\Pi^*(w_{t+1}) > \hat{\Pi}(w_{t+1})$. Since the household can always leave the community, consume $c_t - f^*(c_t)$, and earn $\hat{\Pi}(w_{t+1})$, $\Pi^*(w_t) > \hat{\Pi}(w_t)$ holds only if

$$(1-\delta)U(c_t) + \delta \Pi^*(w_{t+1}) > (1-\delta)\hat{U}(c_t - f^*(c_t)) + \delta \hat{\Pi}(w_{t+1})$$

or

$$\frac{\delta}{1-\delta} \left(\Pi^*(w_{t+1}) - \hat{\Pi}(w_{t+1}) \right) > \hat{U}(c_t - f^*(c_t)) - U(c_t).$$

By Step 2, this inequality holds only if

$$\frac{\delta}{1-\delta} U(c^*) > \hat{U}(c_t - f^*(c_t)) - U(c_t). \tag{6}$$

Now, choose w^{se} such that $\hat{U}(\tilde{c}(w^{se}) - f^*(\tilde{c}(w^{se}))) - U(\tilde{c}(w^{se})) = \frac{\delta}{1-\delta} U(c^*)$; note that w^{se} exists and satisfies $w^{se} < \bar{w}$ by Assumption 3. For any $w \geq w^{se}$, (6) cannot hold, which means that in any household-optimal equilibrium, there exists a $t \geq 0$ such that

$\Pi^*(w_t) = \hat{\Pi}(w_t)$. Lemma 2 then implies that for all $w \geq w^{se}$, $\Pi^*(w_0) = \hat{\Pi}(w_0)$. We conclude that $w^{se} < \bar{w} < \infty$, as desired. $\square$

We can now complete the proof of Proposition 4. As in the proof of Proposition 2, **selection** follows from Lemmas 2 and 10. The argument for **treatment** is similar to the argument in Proposition 2, with two notable exceptions. First, we must substitute $k(f_t)$ into the household's payoff. Second, we have not proven that wealth is monotone, so we must replace w_0 with w_t throughout the argument. With this change, the proof shows that $w_{t+1} < w_t$ whenever $w_t > w^c$. We can therefore take $w^c = w^{tr}$ to prove Proposition 4. $\blacksquare$