Wealth Dynamics in Communities

Daniel Barron, Yingni Guo, and Bryony Reich*

July 8, 2020

Abstract

This paper studies wealth accumulation in communities. We develop a model of favor exchange among households in a community and their investment decisions. Our result identifies a key obstacle to wealth accumulation: wealth crowds out favor exchange. Thus, households under-invest, since growing their wealth would entail losing the support of the community. The result is a persistent wealth gap between such communities and the rest of the economy. Using numerical simulations, we show that increased productivity outside the community exacerbates under-investment inside the community, while strengthening kinship, friendship, or religious ties within the community encourages wealth accumulation.

*Barron: d-barron@kellogg.northwestern.edu; Guo: yingni.guo@northwestern.edu; Reich: bryony.reich@kellogg.northwestern.edu. We thank George Georgiadis, Oliver Hart, Dilip Mookherjee, Andrew Newman, Michael Powell, Benjamin Roth, Kathryn Spier, and the audience at the Harvard Contracts Lunch for comments. We thank Edwin Muñoz-Rodriguez for excellent research assistance.

1 Introduction

In recent decades, countries have grappled with growing internal inequality: some areas flourish while others languish. The result is a phenomenon of "left-behind" communities, whose members suffer from poor economic prospects. Yet, despite those problems, left-behind communities are not dying out (Economist 2017). Instead, households choose to stay, because such communities provide "practical and social support" (Economist 2020). In the United States, for example, staying in such a community means continuing to access "a crucial source of childcare" and other assistance (Economist 2017).

Given that left-behind communities provide this kind of support, it is all the more puzzling that they don't catch up with wealthier areas. Communal support frees up resources for households to save and invest. In practice, however, this support does not translate to growing wealth; instead, members of these communities tend to merely "get by" (Warren et al. 2001), even when they have opportunities to grow their wealth by paying down high-interest debts or making other high-return investments (Ananth et al. 2007, Stegman 2007, Mel et al. 2008). For all of its benefits, why doesn't the support of the community translate to growing wealth?

This paper studies wealth accumulation in communities. We explore how the support of the community shapes its members' wealth and welfare. To do so, we develop a model that combines favor-exchange in communities with investment dynamics. Our main result uncovers a key obstacle to wealth accumulation in the community: wealth crowds out favor exchange. Poor households therefore face a stark choice between growing their wealth and accessing support from their communities. The result is a persistent wealth gap between left-behind communities and the rest of the economy.

At the center for this finding is what we call the "too big for their boots" effect: wealth undermines trust among community members. In particular, neighbors are willing to support a household only if they trust it to reciprocate in the future. Wealth improves the household's alternative to the community: an anonymous and more productive market that we call the "city." Money makes the city more attractive, so wealthy households are more tempted to leave the community rather than repay neighbors for past support. Thus, as households get wealthier, they can no longer access significant community support. To avoid this "too big for their boots" effect, households keep their wealth low, foregoing even profitable investments. Thus, while communal support encourages consumption and improves welfare, it also depresses investment and leads to persistently low wealth.

In her seminal study of an impoverished United States community, the anthropologist Carol Stack vividly illustrates how wealth can undermine trust (Stack [1975]). Favor exchange is ubiquitous among the households that she studies, with neighbors trading clothing, transportation, childcare, and food with one another. These relationships are typically dynamic—neighbors repay one another in kind and over time—and are backed by the threat of social sanctions. Strikingly, Stack [1975] suggests that it is the wealthiest (though still poor) members of the community who are most at risk of being excluded from favor exchange:

> "Members of second-generation welfare families have calculated the risk of giving. As people say, 'The poorer you are, the more likely you are to pay back.' This criterion often determines which kin and friends are actively recruited into exchange networks." - *p. 43*

This rule of thumb might seem strange, since wealthier households presumably have more resources to trade. However, Stack [1975] points out that wealthier households are also less reliant on the community, since they can leave and meet their needs in the nearby city of Chicago.

To explore the implications of the "too big for their boots" effect, we develop a model of wealth dynamics in communities and cities. A long-lived household resides in a tight-knit community and trades consumption goods with neighbors. To compensate those neighbors, the household can make an up-front monetary payment, and it can also promise—but not commit to—an ex post, in-kind reward. At the end of each period, the household invests its remaining wealth. After each interaction, the household can choose to leave this community for the city, which is more productive than the community but also anonymous.

Anonymity implies that favor exchange is impossible in the city. Thus, households in the city use money alone to purchase goods and services. They optimally do so according to a standard consumption-investment problem, where wealth and consumption grow over time. In contrast, households in the community can augment consumption by promising to repay their neighbors with in-kind rewards. Such promises are credible only if the community can punish the household by excluding it from favor exchange. Since a wealthy household can escape these punishments by moving to the city, it is less trusted to repay favors. This gives rise to the "too big for their boots" effect: households lose access to favor exchange as their wealth grows.

Our main result identifies two reasons why "left behind" communities remain left behind. First, there is selection: initially wealthy households leave the community, so that only poorer households remain. This initial disparity is then compounded by a treatment effect: households in the community under-invest, so they experience sharply limited long-term wealth. Indeed, for some households, under-investment is so severe that wealth actually decreases over time. Thus, while households in the city take full advantage of investment opportunities and get richer, households in the community stay poor and may even get poorer.

We use numerical simulations to study how changes in the economic context influence investment and inequality. As productivity in the city improves, inequality gets worse, since productivity gains make the city more attractive, encouraging exit from the community while discouraging investment among those who remain. Conversely, factors that bind households to their community—such as family, religious, or social ties—alleviate the "too big for their boots" effect and encourage wealth accumulation.

The contribution of this paper is to explore how communities influence wealth dynamics. Since those dynamics occur in the shadow of the "city," we relate to papers on the interaction between formal and informal sectors (Kranton [1996], Banerjee and Newman [1998], Gagnon and Goyal [2017], Banerjee et al. [2018], Jackson and Xing [2019]). Unlike that literature, we focus on how this interaction shapes wealth dynamics. In so doing, we focus on an *intertemporal* spillover from future wealth to current cooperation. This focus also separates us from papers that study how communities distort decision-making (Austen-Smith and Fryer [2005], Hoff and Sen [2006]).

Our model of exchange draws on the relational contracting literature ([Macaulay, 1963, Bull, 1987, Levin, 2003, Malcomson, 2013]), particularly those papers that consider the role of outside options (Baker et al. [1994], Kovrijnykh [2013]). We contribute to this literature by introducing a new state variable, wealth, and modeling an anonymous city as an alternative to long-term relationships in the community. We are also related to papers that study cooperation in communities [Wolitzky, 2013, Ali and Miller, 2016, 2018, Miller and Tan, 2018]; however, we abstract from questions of network structure to focus on wealth dynamics.

Our central result is that communal exchange leads to under-investment and persistent poverty. Unlike papers that study under-investment due to lumpy costs (e.g., Nelson [1956], Advani [2019]), we assume that investment entails no fixed costs. The literature has proposed two explanations for this type of under-investment. First, individuals might be subject to time-inconsistent preferences (Bernheim et al. [2015]) or temptations (Banerjee and Mullainathan [2010]). Second, capital markets might be imperfect; for instance, monopolistic lenders might expropriate the returns on investment (Mookherjee and Ray [2002]) or impose restrictive covenants that limit long-term investment (Liu and Roth [2019]). Our model offers a different explanation, which is that households voluntarily limit investment in order to maintain access to communal support. Thus, even though our model assumes neither behavioral preferences nor capital-market imperfections, poor households do not take full advantage of investment opportunities and have persistently low wealth.

The idea that money eases commitment problems dates to Jevons [1875]'s argument that money solves the "double coincidence of wants." Prendergast and Stole [1999, 2000] build on this idea to compare market and barter economies. We build on a related idea to explore wealth dynamics.

2 Model

A long-lived **household** ("it") has initial wealth $w_0 \geqslant 0$ and discount factor $\delta \in (0,1)$. The household initially lives in the community. At the start of each period $t \in \{0, 1, ...\}$, it can choose to stay in the community or irreversibly move to a city.

If the household is still in the community in period t, it plays the following **community game** with a short-lived **neighbor** t ("she"), who represents another member of the community:

1. The household offers to pay $p_t \in [0, w_t]$, which cannot exceed its period-t wealth w_t, in exchange for consumption $c_t \geqslant 0$.

2. Neighbor t accepts or rejects this exchange, $d_t \in \{0, 1\}$. If she accepts ($d_t = 1$), then she receives p_t and incurs the cost of providing c_t. If she rejects ($d_t = 0$), then no trade occurs.

3. The household chooses the size of favor to do for neighbor t, $f_t \geqslant 0$.

4. The household invests its remaining wealth, $w_t - p_t d_t$, to generate w_{t+1}. Let $R(\cdot)$ denote the return on investment, so

$$w_{t+1} = R(w_t - p_t d_t).$$

The household's period-t payoff, π_t, equals $U(c_t d_t) - f_t$. Neighbor t's payoff is $(p_t - c_t)\, d_t + f_t$. Interactions are observed by all neighbors, reflecting the idea that the community is tight-knit.

We assume that consumption utility $U(\cdot)$ and investment returns $R(\cdot)$ are strictly increasing and strictly concave, with $U''(\cdot)$ and $R'(\cdot)$ continuous and $U(0) = 0$, $\lim_{c \downarrow 0} U'(c) = \infty$, and $\lim_{c \to \infty} U'(c) = 0$. Investment generates positive returns, and strictly so below a threshold $\bar{w} > 0$: $R(0) = 0$, $R'(w) > \frac{1}{\delta}$ for $w < \bar{w}$, and $R'(w) = \frac{1}{\delta}$ for $w \geqslant \bar{w}$.

If the household has moved to the city by period t, then it plays the **city game** with a short-lived **vendor** t ("she"). The city game is identical to the community game in all but two ways. First, each vendor observes only her own interaction with the household, so that transactions are anonymous in the city. Second, the city is more productive at providing goods and services. Let $\hat{U}(\cdot)$ be the household's consumption utility in the city, so that $\pi_t = \hat{U}(c_t d_t) - f_t$. We assume that each extra unit of consumption generates more utility in the city: $\hat{U}'(c) > U'(c)$ for all $c > 0$.Moreover, $\hat{U}(\cdot)$ satisfies the same conditions as $U(\cdot)$ does. A household in the city remains there forever.[1] We use a hat mark, like $\hat{U}$, to denote city-specific terms.

The household's continuation payoff from period t onward is

$$\Pi_t \equiv (1-\delta)\sum_{s=t}^{\infty} \delta^{s-t}\pi_s.$$

We characterize household-optimal equilibria, which are the Perfect Bayesian Equilibria that maximize the household's continuation payoff in $t = 0$.

We assume that strictly positive-return investments are available for a wide range of wealth levels. As we will show, this assumption ensures that households in the community have access to strictly positive-return investments.

Assumption 1 *Define $\bar{c} > 0$ as the solution to $U^{'}(\bar{c}) = 1$. Then, $R(\bar{w} - \bar{c}) > \bar{w}$.*

In the context of Stack [1975], the household and neighbors are members of a predominantly low-income community called "the Flats." Members of the Flats exchange a wide variety of goods and services (c_t), including food, clothing, childcare, and transportation. While households can pay one another with money (p_t), compensation typically also occurs via favor exchange: for example, the recipient of childcare ($c_t > 0$) agrees to reciprocate with future childcare ($f_t > 0$). Households invest ($R(\cdot)$) by repaying high-interest debt,

[1] In Online Appendix B, we show that the "too big for their boots" effect arises even if households in the city can return to the community.

taking fewer short-term loans, and saving. The Flats is tight-knit, and news quickly spreads among neighbors. As Stack [1975] emphasizes, households in the Flats can move to the city, Chicago, which harbors better opportunities for consumption ($\hat{U}' > U'$) but is far enough away that a household must leave the Flats to access them.

3 Life in the City

We first characterize wealth dynamics in the city. A household in the city faces a standard consumption-investment problem, and takes full advantage of investment opportunities.

Transactions are anonymous in the city, so $f_t = 0$ in equilibrium. Vendor t is therefore willing to accept any offer that covers his costs of providing consumption, $p_t \geq c_t$; consequently, every equilibrium entails $p_t = c_t$ in every $t \geqslant 0$, so that $w_{t+1} = R(w_t - c_t)$. For a household with wealth w, the resulting optimal consumption and payoff are given by:

$$\hat{C}(w) \in \arg\max_{c\in[0,w]} \Big((1-\delta)\hat{U}(c) + \delta\hat{\Pi}\left(R\left(w-c\right)\right)\Big) \text{ and}$$
$$\hat{\Pi}(w) = \max_{c\in[0,w]} \Big((1-\delta)\hat{U}(c) + \delta\hat{\Pi}\left(R\left(w-c\right)\right)\Big).$$

Our first result shows that $\hat{\Pi}(w)$ and $\hat{C}(w)$ are the unique equilibrium outcome in the city.

Proposition 1 *Both $\hat{\Pi}(\cdot)$ and $\hat{C}(\cdot)$ are strictly increasing, with $\hat{\Pi}(\cdot)$ continuous. In any equilibrium, $\Pi_t = \hat{\Pi}(w_t)$ and $c_t = \hat{C}(w_t)$ in any $t \geq 0$, with $(w_t)_{t=0}^{\infty}$ increasing and*

$$\lim_{t\to\infty} w_t \geq \bar{w}$$

on the equilibrium path.

The proof of Proposition 1 is routine and relegated to Online Appendix A. The standard Euler equation,

$$\hat{U}'(\hat{C}(w_t)) = \delta R'(w_t - \hat{C}(w_t))\hat{U}'(\hat{C}(w_{t+1})),\ \forall\, t, \tag{1}$$

together with the fact that $R'(w) > \frac{1}{\delta}$ for $w < \bar{w}$, imply that wealth increases until at least $\bar{w}$.

Figure 1 simulates equilibrium outcomes in the city. The left panel illustrates payoff and consumption as a function of w. The right panel illustrates equilibrium consumption and wealth dynamics.

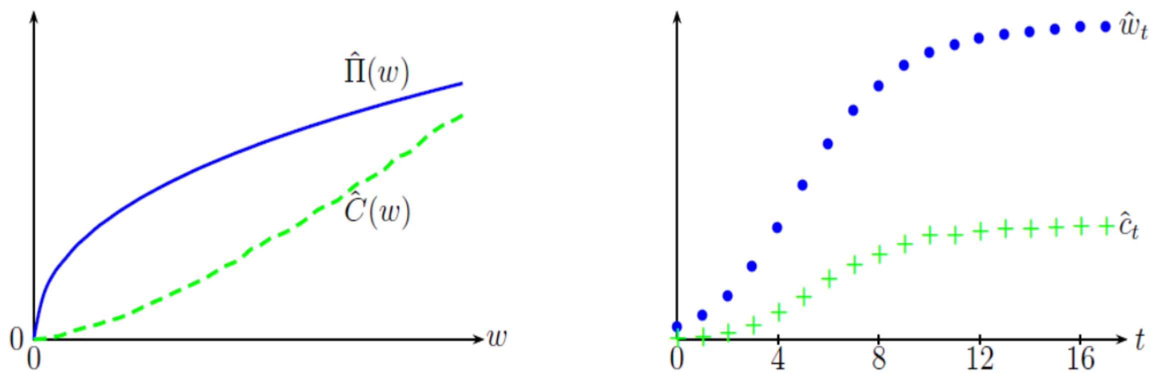

Figure 1: Left panel: the household's equilibrium payoff and consumption as a function of w; Right panel: consumption and wealth over time, starting at $w_0 = 0.108$.

4 Household-Optimal Wealth Dynamics

We now turn to our main result, which characterizes household-optimal equilibria in the community. Section 4.1 states this result and discusses its intuition. Section 4.2 gives the proof.

4.1 Life in the Community

Our main result identifies two reasons why wealth in the community remains substantially lower than wealth in the city. First, there is a *selection margin:* the wealthiest members of the community leave, so that only poorer households remain. Second, there is a *treatment effect:* households in the community do not take full advantage of investment opportunities. Consequently, long-run household wealth is sharply limited.

To build intuition for this result, note that the sole advantage of the community is that neighbors can observe and punish a household who reneges on $f_t > 0$. The threat of this punishment potentially induces the household to follow through on $f_t > 0$. Thus, unlike in the city, favor exchange is possible in the community. The city's advantage is that it is more productive.

Crucially, the relative advantages of the community versus the city depend on wealth. Households with $w_t \approx 0$ would not be able to consume much in the city. Thus, these households optimally remain in the community, since the possibility of favor exchange outweighs the minimal productivity gain that they would realize in the city. Conversely, wealthy households would consume a lot in the city and so derive a large benefit from its higher productivity. Moreover, such households have low marginal utility from consumption, so they would derive little benefit from using favor exchange to further augment consumption. Thus, the richest leave the community and the poorest stay.

Now, consider a household with wealth w_0 that stays in the community. To compensate its neighbor for providing c_0, the household pays p_0 upfront and promises favor f_0. The household optimally sets $p_0 + f_0 = c_0$, so its investment and future wealth equal $I_0 = w_0 + f_0 - c_0$ and $w_1 = R(I_0)$, respectively. Since the household can renege on f_0, leave, and earn $\hat{\Pi}(R(I_0))$, f_0 is bounded from above in equilibrium. In particular, denoting the household's on-path continuation payoff by $\Pi^*(R(I_0))$ and the maximum credible favor by $\bar{f}(I_0)$, we have

$$f_0 \leq \bar{f}(I_0) \equiv \frac{\delta}{1-\delta}(\Pi^*(R(I_0)) - \hat{\Pi}(R(I_0))). \tag{2}$$

Consider the following perturbation: the household increases I_0 and decreases p_0 by some small $\epsilon > 0$. Assuming that (2) binds, this perturbation changes the favor to $f_0 = \bar{f}(I_0 + \epsilon)$ and consumption to $c_0 = p_0 - \epsilon + \bar{f}(I_0 + \epsilon)$. In a household-optimal equilibrium, neither this perturbation nor its counterpart with $\epsilon < 0$ should improve the household's payoff.

Consequently, the following *modified* Euler equation holds:

$$U'(c_0) = \delta R'(I_0)U'(c_1) + \bar{f}'(I_0)(U'(c_0) - 1). \quad (3)$$

The first two terms of (3) are as in (1). The final term equals the marginal effect of investment on the maximum favor, $\bar{f}'(I_0)$, multiplied by the marginal effect of promising a larger favor on the household's current-period payoff, $U'(c_0) - 1$. The term $U'(c_0) - 1$ equals the marginal consumption utility, $U'(c_0)$, minus the marginal cost of a favor. We will show that for any household that optimally stays in the community, $U'(c_0) - 1 > 0$. Thus, the "too big for their boots" effect arises whenever $\bar{f}'(I_0) < 0$, so that the final term is negative. In that case, investment crowds out favor exchange. To avoid crowd out, the household "over"-consumes and "under"-invests.

This intuition elides a key complication: the maximum credible favor, $\bar{f}(I_0)$, depends on both $\Pi^*(\cdot)$ and $\hat{\Pi}(\cdot)$, which in turn depend on the household's future consumption and investment decisions. Wealth is a persistent state variable that affects all of these decisions, rendering a full characterization of household-optimal equilibria intractable.

Our main result, Proposition 2, instead characterizes how wealth determines selection and treatment. *Selection* is summarized by a wealth level, $w^{se} < \bar{w}$, such that the household leaves whenever $w_t \geq w^{se}$, and a set $\mathcal{W} \subseteq [0, w^{se}]$, such that the household stays forever whenever $w_t \in \mathcal{W}$. *Treatment* is summarized by a wealth level, $w^{tr} < w^{se}$, such that long-term wealth of a household in the community is below w^{tr}.

Proposition 2 *Impose Assumption 1. There exist wealth levels $w^{tr} < w^{se} \in (0, \bar{w})$ and a positive-measure set $\mathcal{W} \subseteq [0, w^{se}]$ such that in any household-optimal equilibrium:*

1. ***Selection.*** *The household stays in the community forever if $w_0 \in \mathcal{W}$, and otherwise leaves in $t = 0$.*

2. ***Treatment.*** *If the household stays in the community, then* $(w_t)_{t=0}^{\infty}$ *is monotone, with*

$$\lim_{t\to\infty} w_t \leqslant w^{tr}.$$

Moreover, $\mathcal{W} \cap [w^{tr}, w^{se}]$ *has positive measure.*

We defer the formal proof of this result to Section 4.2. We have already argued that wealthy households leave the community while at least some poorer households stay. Any household that stays, stays forever, since otherwise favor exchange would unravel from their last interaction within the community. This gives us selection: a wealth level above which households leave, w^{se}, and a positive-measure set, $\mathcal{W}$, of poorer wealth levels such that households stay forever.

To prove treatment, consider a household in the community whose initial wealth is *just below* w^{se}. Such a household is close to indifferent between leaving and staying. If this household's *future* wealth remains near w^{se}, then $f_t \approx 0$. Since staying is optimal only if $f_t \gg 0$ in some t, the household must under-invest so severely that its wealth decreases. The proof of Proposition 2 strengthens this result by showing that $(w_t)_{t=0}^{\infty}$ is monotone and that a positive measure of households, $\mathcal{W} \cap [w^{tr}, w^{se}]$, stay in the community despite having initial wealth near w^{se}. These households experience declining wealth.

Figure 2 summarizes Proposition 2. In this simulation, the household moves to the city if $w_0 \geqslant w^{se}$ and stays otherwise. Among those that stay, households with wealth below w^{tr} grow their wealth, but only to w^{tr}. Those with $w_0 \in (w^{tr}, w^{se})$ have declining wealth over time.

One implication of this figure is that one-time transfers do not necessarily spur further investment. In particular, so long as the household's post-transfer wealth satisfies $w_0 < w^{se}$, its long-term wealth remains below w^{tr}. Such transfers result in "over-consumption" and "under-investment." A transfer is large enough to lead to $w_0 \geq w^{se}$ *does* induce further investment, but only by spurring the household to leave the community. This result resonates

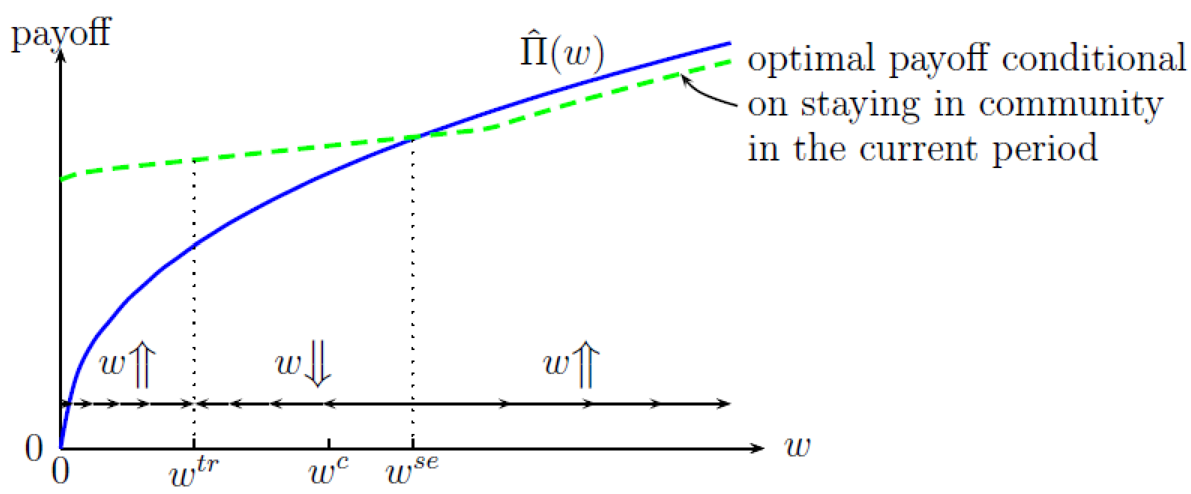

Figure 2: Household-optimal equilibrium payoffs and wealth dynamics

with Karlan et al. [2019], which finds that entrepreneurs receiving one-time debt relief tend to quickly fall back into debt.

4.2 The Proof of Proposition 2

Let $\Pi^*(w)$ be the maximum equilibrium payoff of a household with wealth w. Define

$$\begin{aligned} \Pi_c(w) \equiv \quad & \max_{c\geq 0, f\geq 0} \quad \{(1-\delta)(U(c)-f)+\delta\Pi^*(R(w+f-c))\} \\ & \text{s.t.} \qquad 0 \leq c-f \leq w \qquad (4) \\ & \qquad\qquad f \leq \frac{\delta}{1-\delta}\left(\Pi^*(R(w+f-c))-\hat{\Pi}(R(w+f-c))\right). \qquad (5) \end{aligned}$$

We show that $\Pi_c(w)$ is the household's maximum payoff *conditional* on staying in the community in the current period. Then, $\Pi^*(w)$ is the maximum of $\hat{\Pi}(w)$ and $\Pi_c(w)$.

Lemma 1 *The household's maximum equilibrium payoff is* $\Pi^*(w_0) = \max\left\{\hat{\Pi}(w_0), \Pi_c(w_0)\right\}$, *where* $\Pi_c(\cdot)$ *and* $\Pi^*(\cdot)$ *are strictly increasing.*

Proof of Lemma 1: We show that $\Pi_c(\cdot)$ is the household's maximum equilibrium payoff conditional on staying in the community in the current period. In any equilibrium, neighbor 0 accepts only if $c_0 \le p_0 + f_0$. The household's continuation payoff is at most $\Pi^*(R(w_0 - p_0))$ and at least $\hat{\Pi}(R(w_0 - p_0))$. Hence, it is willing to choose f_0 only if

$$f_0 \le \frac{\delta}{1-\delta}\left(\Pi^*(R(w_0 - p_0)) - \hat{\Pi}(R(w_0 - p_0))\right).$$

Setting $c_0 = p_0 + f_0$ yields $\Pi_c(w_0)$ as an upper bound on the household's payoff from staying.

This bound is tight. For any (c_0, f_0) that satisfies (4) and (5), it is an equilibrium to set $p_0 = c_0 - f_0 \ge 0$, play a household-optimal continuation equilibrium on-path, and respond to any deviation with the household leaving and $f_t = 0$ in all future periods.[2] Thus, $\Pi_c(\cdot)$ is the household's maximum equilibrium payoff conditional on staying. It follows that $\Pi^*(w) = \max\{\hat{\Pi}(w), \Pi_c(w)\}$. Since $\Pi_c(\cdot)$ is strictly increasing by inspection and $\hat{\Pi}(\cdot)$ is strictly increasing by Proposition 1, $\Pi^*(\cdot)$ is strictly increasing. □

The next four lemmas characterize household-optimal equilibria in the community. First, we show that households that stay in the community, stay forever.

Lemma 2 *If $w_0 \ge 0$ is such that $\Pi^*(w_0) > \hat{\Pi}(w_0)$, then in any $t \ge 0$ of any household-optimal equilibrium, $\Pi^*(w_t) > \hat{\Pi}(w_t)$ on the equilibrium path.*

Proof of Lemma 2: Suppose $t > 0$ is the first period in which $\Pi^*(w_t) = \hat{\Pi}(w_t)$. Let $\{c_{t-1}, f_{t-1}\}$ achieve $\Pi_c(w_{t-1})$. Since $\Pi^*(w_t) = \hat{\Pi}(w_t)$, (5) implies $f_{t-1} = 0$. The household therefore earns a higher payoff by exiting in $t-1$ and choosing the same c_{t-1}. This contradicts $\Pi^*(w_{t-1}) > \hat{\Pi}(w_{t-1})$. □

Second, we bound consumption from above in the community.

Lemma 3 *Fix $w_0 \in \{w : \Pi^*(w) > \hat{\Pi}(w)\}$, and let $\{c_t\}_{t=0}^{\infty}$ be consumption in a household-optimal equilibrium. Then $U'(c_t) \ge 1$ in all $t \ge 0$.*

[2] If $f_t = 0$ in all $t \ge 0$, then the household is willing to leave because $U \le \hat{U}$.

Proof of Lemma 3: Consider a household in the community with wealth w_t, and suppose $U'(c_t) < 1$ in period t. If $f_t > 0$, consider decreasing f_t and c_t by $\epsilon > 0$. Doing so is feasible and increases the household's payoff at rate $1 - U'(c_t) > 0$ as $\epsilon \to 0$.

Suppose $f_t = 0$, and let $\tau > t$ be the first period after t such that $f_\tau > 0$. Consider decreasing p_t and c_t by $\epsilon > 0$, increasing p_τ by $\chi(\epsilon)$, and decreasing f_τ by $\chi(\epsilon)$, where $\chi(\epsilon)$ is chosen so that $w_{\tau+1}$ remains constant. Then, $\chi(\epsilon) \geq \frac{\epsilon}{\delta^{\tau-t}}$ because $R'(\cdot) \geq \frac{1}{\delta}$. As $\epsilon \to 0$, this perturbation increases the household's payoff by at least $\delta^{\tau-t}\frac{1}{\delta^{\tau-t}} - U'(c_t) > 0$. It is an equilibrium because $f_s = 0$ for all $s \in [t, \tau - 1]$, so (5) still holds in these periods.

We conclude that if $U'(c_t) < 1$, then $f_s = 0$ in all $s \geq t$. But then $\Pi_c(w_t) < \hat{\Pi}(w_t)$, so $\Pi_c(w_0) \leq \hat{\Pi}(w_0)$ by Lemma 2. Contradiction of $w_0 \in \{w : \Pi^*(w) > \hat{\Pi}(w)\}$. $\square$

Third, we show that wealthy households leave the community, while poorer households stay.

Lemma 4 *The set* $\left\{w : \Pi^*(w) > \hat{\Pi}(w)\right\}$ *has positive measure. Moreover,* $w^{se} \equiv \sup\left\{w : \Pi^*(w) > \hat{\Pi}(w)\right\}$ *satisfies* $0 < w^{se} < \infty$.

Proof of Lemma 4: First, we show that $\Pi^*(0) > \hat{\Pi}(0) = 0$. Because $\lim_{c\downarrow 0} U'(c) = \infty$, there exists a $c > 0$ such that $c \leq \delta U(c)$. Suppose that in all $t \geq 0$, $f_t = c_t = c$ and $p_t = 0$ on the equilibrium path. Any deviation is punished by $f_t = 0$ in all future periods and the household immediately exiting. This strategy delivers a strictly positive payoff. It is an equilibrium because $c \leq \delta U(c)$ implies (5). Thus, $\Pi^*(0) > 0$. Since $\Pi^*(w)$ is increasing and $\hat{\Pi}(w)$ is continuous, there exists an open interval around 0 such that $\Pi^*(w) > \hat{\Pi}(w)$. So $\left\{w : \Pi^*(w) > \hat{\Pi}(w)\right\}$ has positive measure.

Next, we show that $w^{se} < \infty$. Let $\bar{c}$ satisfy $U'(\bar{c}) = 1$. From Lemma 3, we know that $\Pi^*(w) \leq U(\bar{c})$ for every $w \in \{w : \Pi^*(w) > \hat{\Pi}(w)\}$. For any w_0 such that $R(w_0 - \bar{c}) \geq w_0$, the household can set $p_t = c_t = \bar{c}$ in every $t \geq 0$ in the city. For such w_0, $\hat{\Pi}(w_0) > \hat{U}(c) > U(c)$, so w^{se} satisfies $R(w^{se} - \bar{c}) < w_0$ and hence $w^{se} < \infty$. $\square$

Finally, household-optimal equilibria exhibit monotone wealth dynamics.

Lemma 5 *In any household-optimal equilibrium, $(w_t)_{t=0}^{\infty}$ is monotone.*

The (tedious) proof of Lemma 5 is relegated to Appendix A. The key step of this proof shows that household-optimal investment, $w_0 - p_0$, increases in w_0. Wealth dynamics are therefore monotone: if $w_1 = R(w_0 - p_0) \geq w_0$, then $w_2 = R(w_1 - p_1) \geq R(w_0 - p_0) = w_1$ and so on, and similarly if $w_1 \leq w_0$.

We can now prove Proposition 2.

Statement 1 of Proposition 2: Follows immediately from Lemma 2. The set $\mathcal{W}$ has positive measure by Lemma 4. □

Statement 2 of Proposition 2: By the proof of Lemma 4, $R(w^{se} - \bar{c}) < \bar{c}$, so Assumption 1 implies that $w^{se} < \bar{w}$. Statement 2 therefore follows from Proposition 1. □

Statement 3 of Proposition 2: We argue that there exists $w^c < w^{se}$ such that for any $w_t \in (w^c, w^{se})$, if the household stays in the community, then $w_{t+1} \leq w^c$. Consider $w_t < w^{se}$, and suppose $w_{t+1} > w^c$. Since $\Pi^*(\cdot)$ is increasing, (5) holds only if

$$f_t \leq \Delta(w_t) \equiv \frac{\delta}{1-\delta}(\Pi^*(w^{se}) - \hat{\Pi}(w_t)).$$

Proposition 1 implies that $\Delta(w_t)$ is strictly decreasing and continuous.

Continuity implies that $\lim_{w \uparrow w^{se}} \hat{\Pi}(w) = \hat{\Pi}(w^{se})$. If $\Pi^*(w^{se}) > \hat{\Pi}(w^{se})$, then $\Pi^*(\cdot)$ increasing and $\hat{\Pi}(\cdot)$ continuous imply $\Pi^*(w) > \hat{\Pi}(w)$ just above w^{se}, contradicting the definition of w^{se}. Therefore, $\Pi^*(w^{se}) = \hat{\Pi}(w^{se})$ and $\lim_{w \uparrow w^{se}} \Delta(w) = 0$.

Next, we construct w^c. For any $w \in [R^{-1}(w^{se}), w^{se}]$, define

$$G(w) \equiv \hat{U}(w - R^{-1}(w^{se})) - (U(w^{se} - R^{-1}(w) + \Delta(w)) + \Delta(w)).$$

Then, $G(\cdot)$ is strictly increasing, continuous, and strictly crosses 0 from below. Define $w^c \in (R^{-1}(w^{se}), w^{se})$ as the unique wealth such that $G(w^c) = 0$.

Consider a household with $w_t > w^c$ such that $\Pi_c(w_t) > \hat{\Pi}(w_t)$, with corresponding household-optimal choices $(c_t, p_t, _t)$. Towards contradiction, suppose $w_{t+1} > w^c$. Then, this household can exit, choose $\hat{c}_t = \hat{p}_t = p_t = c_t - f_t$ and $\hat{f}_t = 0$, where $p_t \geq 0$ because $w^c \geq R^{-1}(w^{se})$. This deviation leaves w_{t+1} unchanged and results in continuation payoff $\hat{\Pi}(w_{t+1})$.

We argue that this deviation is profitable:

$$
\begin{aligned}
(1-\delta)\hat{U}(\hat{c}_t) + \delta\hat{\Pi}(w_{t+1}) &\geq (1-\delta)\hat{U}(c_t - f_t) + \delta\hat{\Pi}(w^c) \\
&> (1-\delta)(U(c_t) + \Delta(w^c)) + \delta\hat{\Pi}(w^c) \\
&= (1-\delta)(U(c_t) + \Delta(w^c)) + \delta\Pi^*(w^{se}) - (1-\delta)\Delta(w^c) \\
&= (1-\delta)U(c_t) + \delta\Pi^*(w^{se}) \\
&\geq (1-\delta)U(c_t) + \delta\Pi^*(w_{t+1}) \\
&= \Pi^*(w_t).
\end{aligned}
$$

Here, the first line follows from $\hat{c}_t = c_t - f_t$ and $w_{t+1} \geq w^c$; the second line is proven below; the third line from the definition of $\Delta(w^c)$; and the fifth line because, by Lemma 2, $w_{t+1} \leq w^{se}$. The fourth and sixth lines are algebra.

The second line follows because for any $w_t > w^c$, $\hat{U}(c_t - f_t) > U(c_t) + \Delta(w^c)$. To see this, note

$$w^{se} \geq w_{t+1} = R(w_t - p_t) > R(w^c - p_t).$$

Hence, $p_t > w^c - R^{-1}(w^{se})$. Similarly,

$$w^c < w_{t+1} = R(w_t - p_t) \leq R(w^{se} - p_t),$$

so $p_t < w^{se} - R^{-1}(w^{se})$. Therefore,

$$\begin{aligned}
\hat{U}(c_t - f_t) = \hat{U}(p_t) &> \hat{U}(w^c - R^{-1}(w^{se})) \\
&\geq U(w^{se} - R^{-1}(w^c) + \Delta(w^c)) + \Delta(w^c) \\
&> U(p_t + \Delta(w^c)) + \Delta(w^c) \\
&\geq U(p_t + \Delta(w_t)) + \Delta(w^c) \\
&\geq U(c_t) + \Delta(w^c).
\end{aligned}$$

Here, the first and third lines follow from $w^{se} - R^{-1}(w^c) > p_t > w^c - R^{-1}(w^{se})$; the second line from $G(w^c) = 0$; the fourth line from the fact that $w^c < w_t$ and $\Delta(\cdot)$ strictly decreasing; and the last line from $p_t + \Delta(w_t) \geq p_t + f_t = c_t$.

The household therefore has a profitable deviation if $w_{t+1} > w^c$, so $w_{t+1} \leq w^c$ whenever $w_t \in (w^c, w^{se})$. Since $(w_t)_{t=0}^{\infty}$ is monotone by Lemma 5, it converges. Hence, $\lim_{t\to\infty} w_t \leq w^c \equiv w^{tr}$.

The final step is to show $\mathcal{W} \cap (w^c, w^{se})$ has positive measure. By definition of w^{se}, we can find $w \in (w^c, w^{se})$ such that $\Pi^*(w) > \hat{\Pi}(w)$. Since $\Pi^*(\cdot)$ is increasing and $\hat{\Pi}(\cdot)$ is continuous, $\Pi^*(\cdot) > \hat{\Pi}(\cdot)$ on an open set around w. ■

5 Policy Simulations

This section considers two policy-relevant numerical simulations. In the United States, the productivity gap between left-behind communities and the rest of the economy has widened over the past two decades (Economist 2017). We first analyze how an improvement in the city's productivity affects wealth dynamics in the community. We show that an increase in city productivity encourages households to leave the community, and it worsens the "too big for their boots" effect for those who remain. Second, we examine whether a sense of "belonging" in the community—modeled as a per-period benefit for households that stay—can mitigate the "too big for their boots" effect.

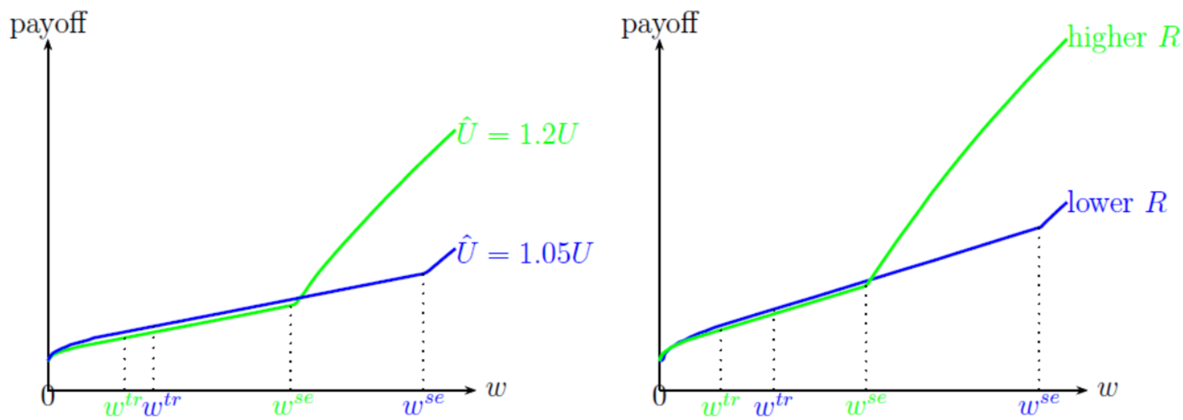

Figure 3: Left panel: improving productivity in the city; right panel: improving investment opportunities

Figure 3 illustrates how an increase in $\hat{U}(\cdot)$, an improvement in the productivity of the city, exacerbates long-term inequality. The left-hand panel simulates an increase in $\hat{U}(\cdot)$. The better the city, the lower the wealth threshold, w^{se}, at which households leave the community. Those that remain in the community face a higher outside option, engage in less favor exchange, and are worse off. To mitigate this effect, households invest even less, exacerbating the "too big for their boots" effect and decreasing w^{tr}.

As illustrated in the right-hand panel of Figure 3, increasing $R(\cdot)$ has similar effects on w^{se} and w^{tr}, since the household takes full advantage of investment opportunities in the city but not in the community. However, unlike an increase in $\hat{U}(\cdot)$, increasing $R(\cdot)$ can benefit the poorest members of the community, since better investment opportunities let them grow their wealth to w^{tr} more quickly.

In our model, the only reason to stay in the community is to engage in favor exchange. In practice, communities provide other benefits, including a sense of belonging, a platform for family, social, and religious interactions, and so on. To capture these benefits, we enrich the model so that the household earns a per-period benefit B as long as it remains in the community. As Figure 4 illustrates, increasing this per-period benefit discourages exit (w^{se} increases) and encourages investment from those who remain (w^{tr} increases). By mitigating the "too big for their boots" phenomenon, communal benefits can encourage investment and

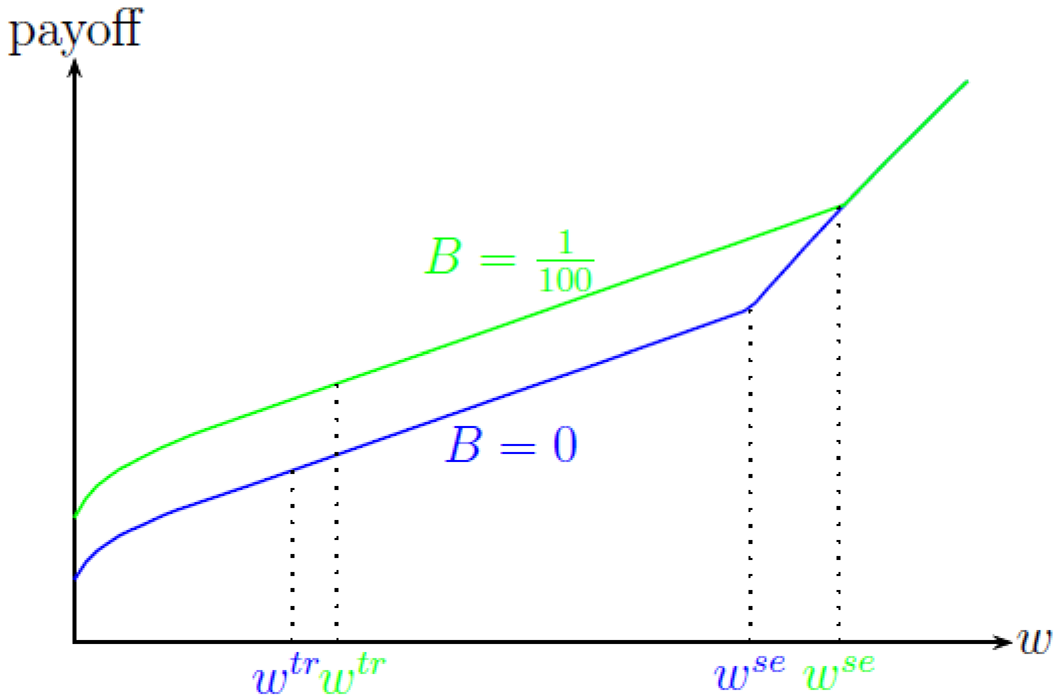

Figure 4: Increasing the per-period benefit of remaining in the community

help left-behind communities catch up.

6 Conclusion

To understand the phenomenon of "left behind" communities, we must understand the social constraints faced by those in poverty. This paper studies the sacrifices that households make to access support from their communities. Those who grow their wealth risk being excluded from exchange. Thus, although the community is an important source of support for its members, it can also discourage investment, exacerbate inequality, and perpetuate poverty.

References

Arun Advani. Insurance networks and poverty traps. Working Paper, 2019.

S. Nageeb Ali and David Miller. Ostracism and forgiveness. *American Economic Review*, 106(8):2329–2348, 2016.

S. Nageeb Ali and David Miller. Enforcing cooperation in networked societies. Working Paper, 2018.

Bindu Ananth, Dean Karlan, and Sendhil Mullainathan. Microentrepreneurs and their money: three anomalies. Working Paper, 2007.

David Austen-Smith and Roland G. Fryer. An economic analysis of 'acting white'. *The Quarterly Journal of Economics*, 120(2):551–583, 2005.

George Baker, Robert Gibbons, and Kevin Murphy. Subjective performance measures in optimal incentive contracts. *The Quarterly Journal of Economics*, 109(4):1125–1156, 1994.

Abhijit Banerjee, Arun G. Chandrasekhar, Esther Duflo, and Matthew O. Jackson. Changes in social network structure in response to exposure to formal credit markets. Working Paper, 2018.

Abhijit V. Banerjee and Sendhil Mullainathan. The shape of temptation: Implications for the economic lives of the poor. Working Paper, 2010.

Abhijit V. Banerjee and Andrew F. Newman. Information, the dual economy, and development. *The Review of Economic Studies*, 65:631–653, 1998.

B. Douglas Bernheim, Debraj Ray, and Sevin Yeltekin. Poverty and self-control. *Econometrica*, 83(5):1877–1911, 2015.

Clive Bull. The existence of self-enforcing implicit contracts. *The Quarterly Journal of Economics*, 102(1):147–159, 1987.

Economist. Globalisation has marginalized many regions in the rich world. *The Economist*, October 21st 2017.

Economist. Economists grapple with rising american mortality. *The Economist*, January 9th 2020.

Julien Gagnon and Sanjeev Goyal. Networks, markets, and inequality. *American Economic Review*, 107(1):1–30, 2017.

Karla Hoff and Arijit Sen. The kin system as a poverty trap? In Samuel Bowles, Steven N. Durlauf, and Karla Hoff, editors, *Poverty Traps*, pages 95–115. 2006.

Matthew O. Jackson and Yiqing Xing. The complementarity between community and government in enforcing norms and contracts, and their interaction with religion and corruption. Working Paper, 2019.

William Stanley Jevons. *Money and the Mechanism of Exchange*. London: Appleton, 1875.

Dean Karlan, Sendhil Mullainathan, and Benjamin Roth. Debt traps? market vendors and moneylender debt in india and the philippines. *forthcoming in American Economic Review: Insights*, 2019.

Natalia Kovrijnykh. Debt contracts with partial commitment. *the American Economic Review*, 103(7):2848–2874, 2013.

Rachel E. Kranton. Reciprocal exchange: A self-sustaining system. *The American Economic Review*, 86(4):830–851, 1996.

Jonathan Levin. Relational incentive contracts. *The American Economic Review*, 93(3): 835–857, 2003.

Ernest Liu and Benjamin N. Roth. Keeping the little guy down: A debt trap for lending with limited pledgability. Working Paper, 2019.

Stewart Macaulay. Non-contractual relations in business: A preliminary study. *Sociological Review*, 28(1):55–67, 1963.

James Malcomson. Relational incentive contracts. In Robert Gibbons and John Roberts, editors, *Handbook of Organizational Economics*, pages 1014–1065. 2013.

Suresh De Mel, David McKenzie, and Christopher Woodruff. Returns to capital in microenterprises: evidence from a field experiment. *Quarterly Journal of Economics*, 123(4): 1329–1372, 2008.

David Miller and Xu Tan. Seeking relationship support: Strategic network formation and robust cooperation. Working Paper, 2018.

Dilip Mookherjee and Debraj Ray. Contractual structure and wealth accumulation. *The American Economic Review*, 92(4):818–849, 2002.

Richard R. Nelson. A theory of the low-level equilibrium trap in underdeveloped economies. *American Economic Review*, 46(5):894–908, 1956.

Canice Prendergast and Lars Stole. Restricting the means of exchange within organizations. *European Economic Review*, 43:1007–1019, 1999.

Canice Prendergast and Lars Stole. Barter relationships. In Paul Seabright, editor, *Barter in Post-Socialist Economics*. Cambridge University Press, 2000.

Carol B. Stack. *All Our Kin: Strategies for Survival in a Black Community*. Harper, 1975.

Michael A. Stegman. Payday lending. *Journal of Economic Perspectives*, 21(1):169–190, 2007.

M.R. Warren, P.J. Thompson, and S. Saegert. *Social Capital and Poor Communities*, chapter The Role of Social Capital in Combating Poverty. Russell Sage Foundation Press, 2001.

Alexander Wolitzky. Cooperation with network monitoring. *The Review of Economic Studies*, 80(1):395–427, 2013.

A Routine Proofs

A.1 Proof of Proposition 1

Suppose that the household lives in the city. In any period t, since future vendors don't observe f_{t}, the household always chooses $f_t = 0$. Hence, vendor t accepts only if $p_t \geq c_t$. This means that $c_t \in [0, w_t]$ are the feasible consumptions, so that the household's equilibrium continuation payoff is at most $\hat{\Pi}(w_t)$ given wealth w_t.

The following equilibrium gives the household an equilibrium continuation payoff of $\hat{\Pi}(w_t)$. In period t, (i) the household proposes $(c_t, p_t) = (\hat{C}(w_t), \hat{C}(w_t))$; (ii) vendor t accepts if and only if $p_t \geq c_t$. Vendor t has no profitable deviation. This strategy attains $\hat{\Pi}$, so the household has no profitable deviation either.

Let $\{c_t^*\}_{t=0}^{\infty}$ be the consumption sequence in the equilibrium above, given initial wealth w. If $w = 0$, then $c_t^* = 0$ in all $t \geq 0$, so $\hat{\Pi}(0) = \hat{U}(0) = 0$ is the unique equilibrium payoff. If $w > 0$, then it must be true that $c_t^* > 0$ in every $t \geq 0$. Suppose otherwise. If $c_0^* > 0$, then let $\tau > 0$ be the first period in which $c_t^* = 0$. Consider the perturbation $c_{\tau-1} = c_{\tau-1}^* - \epsilon$, $c_\tau = c_\tau^* + \epsilon$ for some small $\epsilon > 0$. Since $\lim_{c \downarrow 0} \hat{U}'(c) = \infty$, this perturbation gives a strictly higher payoff, and it is feasible because $R'(\cdot) \geq 1$. If $c_0^* = 0$ instead, then let $\tau > 0$ be the first period in which $c_\tau^* > 0$. Then the perturbation $c_{\tau-1} = c_{\tau-1}^* + \epsilon$, $c_\tau = c_\tau^* - \epsilon$ again gives a strictly higher payoff for $\epsilon > 0$ small. Hence, $c_t^* > 0$ in every $t \geq 0$.

Next, we show that $\hat{\Pi}(w)$ is the household's *unique* equilibrium payoff. At $w = 0$, $\hat{\Pi}(0) = 0$, so the household's unique equilibrium payoff is indeed $\hat{\Pi}(0)$. For $w > 0$, the household can choose $(c_t, p_t) = ((1 - \epsilon_t)c_t^*, c_t^*)$ in every $t \geq 0$ for $\epsilon_t > 0$ small. Vendor t strictly prefers to accept. As $\epsilon_t \downarrow 0$ for all $t \geq 0$, the consumption sequence $\{(1 - \epsilon_t)c_t^*\}_{t=0}^{\infty}$ gives the household a payoff that converges to $\hat{\Pi}(w)$. So the household must earn at least $\hat{\Pi}(w)$ in any equilibrium.

Turning to properties of $\hat{\Pi}(\cdot)$, we claim that $\hat{\Pi}(\cdot)$ is strictly increasing. Pick $0 \leq w < \tilde{w}$. Let $\{c_t^*\}_{t=0}^{\infty}$ be the sequence associated with w. If the initial wealth is $\tilde{w}$, it is feasible to choose $c_0 = c_0^* + \tilde{w} - w$ and $c_t = c_t^*$ for $t \geq 1$. Since $\hat{U}(\cdot)$ is strictly increasing, so too is $\hat{\Pi}(\cdot)$.

It remains to show that $\hat{\Pi}(\cdot)$ is continuous for all $w > 0$. If $w > 0$, then $\hat{C}(w) > 0$. For $\tilde{w}$ sufficiently close to w, setting $c_0 = \hat{C}(w) + (\tilde{w} - w)$ and $c_t = \hat{C}(w_t)$ for $t \geq 1$ is feasible. The household's payoffs converge to $\hat{\Pi}(w)$ as $\tilde{w} \to w$ under this perturbation, which means that $\lim_{\tilde{w} \uparrow w} \hat{\Pi}(\tilde{w}) \geq \hat{\Pi}(w)$ and $\lim_{\tilde{w} \uparrow w} \hat{\Pi}(\tilde{w}) \geq \hat{\Pi}(w)$. Since $\hat{\Pi}(\cdot)$ is increasing, we conclude that $\hat{\Pi}(\cdot)$ is continuous at every $w > 0$.

We now show that $\hat{\Pi}(\cdot)$ is continuous at $w = 0$. Consider $\lim_{w \downarrow 0} \hat{\Pi}(w)$. Since $R'(\bar{w}) = \frac{1}{\delta}$, the line tangent to $R(\cdot)$ at $\bar{w}$ is $\hat{R}(w) = R(\bar{w}) + \frac{w - \bar{w}}{\delta}$. Since $R(\cdot)$ is concave, $R(w) \leq \hat{R}(w)$ for all $w \geq 0$. Therefore, $\hat{\Pi}(w)$ is bounded from above by the household's maximum payoff if we replace $R(\cdot)$ with $\hat{R}(\cdot)$. For consumption path $\{c_t\}_{t=0}^{\infty}$ to be feasible under $\hat{R}(\cdot)$, it must satisfy

$$(1-\delta) \sum_{t=0}^{\infty} \delta^t c_t \leq (1-\delta) w_0 + \delta R(\bar{w}) - \bar{w}.$$

This means that the payoff of a household with initial wealth w_0 is at most

$$\hat{U}((1-\delta) w_0 + \delta R(\bar{w}) - \bar{w}).$$

Pick any small $\epsilon > 0$. There exists $T < \infty$ and sufficiently small $w_0 > 0$ such that

$$\delta^T \hat{U} \left((1-\delta) R^T(w_0) + \delta R(\bar{w}) - \bar{w} \right) < \frac{\epsilon}{2},$$

where $R^T(w_0)$ denotes the function that applies $R(\cdot)$ T-times to w_0.

Consider a hypothetical setting that is more favorable to the household: we allow the household to both consume *and* save her wealth until period T, after which she must play the original city game. The household's payoff from this hypothetical is strictly larger than $\hat{\Pi}(w_0)$ and is bounded from above by

$$(1-\delta) \sum_{t=0}^{T-1} \delta^t (\hat{U}(R^t(w_0)) + \delta^T \hat{U}((1-\delta) R^T(w_0) + \delta R(\bar{w}) - \bar{w}).$$

As $w_0 \downarrow 0$, $R^T(w_0) \downarrow 0$, so $R^t(w_0) \downarrow 0$ for any $t < T$. Thus,

$$\hat{\Pi}(w_0) \leq (1-\delta)\sum_{t=0}^{T-1} \delta^t \hat{U}(R^t(w_0)) + \delta^T \hat{U}((1-\delta)R^T(w_0) + \delta R(\bar{w}) - \bar{w}) < \epsilon.$$

This is true for any $\epsilon > 0$, so $\lim_{w \downarrow 0} \hat{\Pi}(w) = 0$.

Finally, consider any equilibrium in the city. If $w_0 = 0$, then $w_t = 0$ in any $t \geq 0$. If $w_0 > 0$, then we have shown that $c_t > 0$ in every $t \geq 0$, so $w_t > c_t > 0$. A standard argument (see below) implies the following Euler equation:

$$\hat{U}'(c_t) = \delta R'(w_t - c_t)\hat{U}'(c_{t+1}). \tag{6}$$

Together with $R'(\cdot) \geq \frac{1}{\delta}$ and $U(\cdot)$ strictly concave, (6) implies $c_t \leq c_{t+1}$, and strictly so if $w_t < \bar{w}$.

Next, we argue that $\hat{C}(\cdot)$ is strictly increasing in w. Let $\{c_t\}_{t=0}^{\infty}$ and $\{\tilde{c}_t\}_{t=0}^{\infty}$ be the equilibrium consumption sequences for $w > 0$ and $\tilde{w} > w$, respectively. Suppose $c_0 \geq \tilde{c}_0$, and let $\tau \geq 1$ be the first period such that $c_t < \tilde{c}_t$, which must exist because $\hat{\Pi}(\cdot)$ is strictly increasing. Then, $c_{\tau-1} \geq \tilde{c}_{\tau-1}$, $w_{\tau-1} - c_{\tau-1} < \tilde{w}_{\tau-1} - \tilde{c}_{\tau-1}$, and $c_\tau < \tilde{c}_\tau$, so at least one of $(c_{\tau-1}, w_{\tau-1}, c_\tau)$ and $(\tilde{c}_{\tau-1}, \tilde{w}_{\tau-1}, \tilde{c}_\tau)$ violates (6). Hence, $\hat{C}(w)$ is strictly increasing in w. Therefore, $c_{t+1} \geq c_t$ implies $w_{t+1} \geq w_t$, with strict inequalities if $w_t \leq \bar{w}$.

Since $(w_t)_{t=0}^{\infty}$ is monotone, it converges on $\mathbb{R}_+ \cup \{\infty\}$. Suppose $\lim_{t\to\infty} w_t < \bar{w}$. Then, $R'(w_t)$ is uniformly bounded away from $\frac{1}{\delta}$. But $c_t = w_t - R^{-1}(w_{t+1})$, so $(c_t)_{t=0}^{\infty}$ converges. Hence, (6) is violated as $t \to \infty$. We conclude that $\lim_{t\to\infty} w_t \geq \bar{w}$. ■

A.2 Deriving the Euler Equation

Consider a household in the city, and let its optimal consumption and wealth sequence be $\{c_t^*, w_t^*\}_{t=0}^{\infty}$. We prove that if $w_0 > 0$, then

$$\hat{U}'(c_t^*) = \delta R'(w_t^* - c_t^*)\hat{U}'(c_{t+1}^*)$$

in every $t \geq 0$.

The proof of Proposition 1 says that $c_t^* > 0$, $c_{t+1}^* > 0$, and $w_t^* - c_t^* > 0$. Suppose that $\hat{U}'(c_t^*) > \delta R'(w_t^* - c_t^*)\hat{U}'(c_{t+1}^*)$. Then, we can perturb (c_t^*, c_{t+1}^*) to $(c_t^* + \epsilon, c_{t+1}^* - \chi(\epsilon))$, where $\chi(\epsilon)$ is chosen such that w_{t+2}^* remains the same as before the perturbation. In particular,

$$R(w_t^* - (c_t^* + \epsilon)) - (c_{t+1}^* - \chi(\epsilon)) = R(w_t^* - c_t^*) - c_{t+1}^*.$$

Hence, $\chi'(\epsilon) = R'(w_t^* - (c_t^* + \epsilon))$.

As $\epsilon \downarrow 0$, this perturbation strictly increases the household's payoff:

$$\begin{aligned} \lim_{\epsilon \downarrow 0} \left\{ \hat{U}'(c_t^* + \epsilon) - \delta \hat{U}'(c_{t+1}^* - \chi(\epsilon))\chi'(\epsilon) \right\} &= \lim_{\epsilon \downarrow 0} \left\{ \hat{U}'(c_t^* + \epsilon) - \delta \hat{U}'(c_{t+1}^* - \chi(\epsilon)) R'(w_t^* - c_t^* - \epsilon) \right\} \\ &= \hat{U}'(c_t^*) + \delta R'(w_t^* - c_t^*)\hat{U}'(c_{t+1}^*) > 0. \end{aligned}$$

This contradicts the fact that (c_t^*, c_{t+1}^*) is optimal. Using a similar argument, we can show that $\hat{U}'(c_t^*) < \delta R'(w_t^* - c_t^*)\hat{U}'(c_{t+1}^*)$ is not possible either. ■

A.3 Proof of Lemma 5

We break the proof of this lemma into four steps.

A.3.1 Step 1: Locally Bounding the Slope of $\Pi^*(\cdot)$ From Below

We claim that for any $w \in [0, w^{se})$, there exists $\epsilon_w > 0$ such that for any $\epsilon \in (0, \epsilon_w)$,

$$\Pi^*(w + \epsilon) - \Pi^*(w) > (1 - \delta)\epsilon.$$

First, suppose $\hat{\Pi}(w) \geq \Pi_c(w)$, and let $\{w_t, c_t\}_{t=0}^{\infty}$ be the wealth and consumption sequences if the household enters the city. Assumption 1 and the proof of Lemma 4 imply $R(w^{se} - \bar{c}) < w^{se}$, so $R(w_0 - \bar{c}) < w_0$. Proposition 1 says that $\{w_t\}_{t=0}^{\infty}$ is increasing, so $c_0 < \bar{c}$. Hence, there exists $\epsilon_w > 0$ such that $U'(c_0 + \epsilon_w) > 1$.

For any $\epsilon < \epsilon_w$, if $w_0 = w+\epsilon$, then the household can enter the city and choose $\hat{c}_0 = c_0+\epsilon$, with $\hat{c}_t = c_t$ in all $t > 0$. We can bound $\Pi^*(w+\epsilon)$ from below by the payoff from this strategy,

$$\Pi^*(w+\epsilon) \geq (1-\delta)\left(U(c_0+\epsilon) - U(c_0)\right) + \hat{\Pi}(w) > (1-\delta)\epsilon + \hat{\Pi}(w) = (1-\delta)\epsilon + \Pi^*(w).$$

We conclude that $\Pi^*(w+\epsilon) - \Pi^*(w) > (1-\delta)\epsilon$, as desired.

Now, suppose $\hat{\Pi}(w) < \Pi_c(w)$. Let $\{w_t, c_t, f_t\}_{t=0}^{\infty}$ be the wealth, consumption, and reward sequence in a household-optimal equilibrium. There exists $\tau \geq 0$ such that $f_\tau > 0$ for the first time in period τ; otherwise, the household could implement the same consumption sequence in the city. Choose $\epsilon_w > 0$ to satisfy $\epsilon_w < \delta^\tau f_\tau$.

For $\epsilon \in (0, \epsilon_w)$ and initial wealth $w_0 = w + \epsilon$, consider the perturbed strategy such that $\hat{w}_t = w_t$, $\hat{c}_t = c_t$, and $\hat{f}_t = f_t$ in every period *except* τ. In period τ, $\hat{f}_\tau = f_\tau - \frac{\epsilon}{\delta^\tau}$ and $\hat{p}_\tau = p_\tau + \chi$, where χ is chosen so that $\hat{w}_{t+1} = w_{t+1}$. Then, $\hat{c}_\tau = c_\tau + \chi - \frac{\epsilon}{\delta^\tau}$. Since the household stays in the community forever by Lemma 2, $w_t \leq w^{se}$ for all $t \leq \tau$, which together with $R(w^{se} - \bar{c}) < w^{se}$ and Assumption 1, implies $R'(w_t - p_t) > \frac{1}{\delta}$. Hence, $\chi > \frac{\epsilon}{\delta^\tau}$.

Under this perturbed strategy, (5) is satisfied in all $t < \tau$ because $f_t = 0$ in these periods; in $t = \tau$ because $\hat{f}_\tau < f_\tau$ and $\hat{w}_{\tau+1} = w_{\tau+1}$; and in $t > \tau$ because play is unchanged after τ. Moreover, $f_\tau - \frac{\epsilon}{\delta^\tau} > 0$ because $\epsilon < \epsilon_w$, and $\hat{f}_\tau + \hat{p}_\tau = \hat{c}_\tau$, so this strategy is feasible. Thus, it is an equilibrium. Consequently, $\Pi^*(w+\epsilon)$ is bounded from below by the household's payoff from this strategy,

$$\Pi^*(w+\epsilon) > (1-\delta)\delta^\tau \frac{\epsilon}{\delta^\tau} + \Pi_c(w) = (1-\delta)\epsilon + \Pi^*(w),$$

as desired.

A.3.2 Step 2: Moving from Local to Global Bound on Slope

Next, we show that for any $0 \leq w < w' < w^{se}$, $\Pi^*(w') - \Pi^*(w) > (1-\delta)(w' - w)$.

Let

$$z(w) = \sup\{w'' | w < w'' \leq w^{se}, \text{ and } \forall w' \in (w, w''], \Pi^*(w') - \Pi^*(w) > (1-\delta)(w'-w)\}.$$

By Step 1, $z(w) \geq w$ exists. Moreover,

$$\Pi^*(z(w)) - \Pi^*(w) \geq \lim_{\tilde{w} \uparrow z(w)} \Pi^*(\tilde{w}) - \Pi^*(w) \geq (1-\delta)(z(w) - w),$$

where the first inequality follows because $\Pi^*(\cdot)$ is increasing, and the second inequality follows by definition of $z(w)$.

Suppose that $z(w) < w^{se}$. By Step 1, there exists $\epsilon_{z(w)}$ such that for any $\epsilon < \epsilon_{z(w)}$,

$$\Pi^*(z(w) + \epsilon) - \Pi^*(z(w)) > (1-\delta)\epsilon.$$

Hence,

$$\Pi^*(z(w)+\epsilon) - \Pi^*(w) = \Pi^*(z(w)+\epsilon) - \Pi^*(z(w)) + \Pi^*(z(w)) - \Pi^*(w) > (1-\delta)\epsilon + (1-\delta)(z(w) - w).$$

This contradicts the definition of $z(w)$, so $z(w) \geq w^{se}$.

For any $w' < w^{se}$, $w' < z(w)$ and so $\Pi^*(w') - \Pi^*(w) > (1-\delta)(w'-w)$, as desired.

A.3.3 Step 3: Investment is increasing in wealth.

Consider two wealth levels, $0 \leq w_L < w_H < w^{se}$, and suppose that $\Pi_c(w_L) > \hat{\Pi}(w_L)$ and $\Pi_c(w_H) > \hat{\Pi}(w_H)$. Given any household-optimal equilibria, let p_H, p_L be the respective period-0 payments under w_H, w_L. We prove that $w_H - p_H \geq w_L - p_L$. Define $I_k \equiv w_k - p_k$, $k \in \{L, H\}$. Towards contradiction, suppose that $I_H < I_L$.

We first show that $c_H > c_L + (w_H - w_L)$. Suppose instead that $c_H \leq c_L + (w_H - w_L)$.

Since $I_H < I_L$, we have $p_H > p_L + (w_H - w_L)$. But then $f_H < f_L$, since

$$f_H = c_H - p_H < c_H - (p_L + (w_H - w_L)) \leq c_L + (w_H - w_L) - (p_L + (w_H - w_L)) = f_L.$$

Consider the following perturbation: $\hat{p}_H = p_L + (w_H - w_L) \in (p_L, p_H)$, $\hat{f}_H = f_H + p_H - \hat{p}_H \geq f_H$, and $\hat{c}_H = c_H$. Under this perturbation, $\hat{I}_H = w_H - \hat{p}_H = I_L$. Thus, to show that the perturbation satisfies (5), we need only show that $\hat{f}_H \leq f_L$. Indeed:

$$\hat{f}_H = f_H + p_H - (p_L + (w_H - w_L)) = c_H - (c_L - f_L) - (w_H - w_L) = f_L + c_H - (c_L + w_H - w_L) \leq f_L,$$

where the final inequality holds because $c_H \leq c_L + (w_H - w_L)$ by assumption. Thus, this perturbation is also an equilibrium.

We claim that a household with initial wealth w_H strictly prefers this equilibrium to the original equilibrium, which is true so long as

$$\begin{aligned} & (1-\delta)(U(c_H) - \hat{f}_H) + \delta\Pi^*(R(I_L)) > (1-\delta)(U(c_H) - f_H) + \delta\Pi^*(R(I_H)) \\ \iff \quad & (1-\delta)(\hat{f}_H - f_H) < \delta(\Pi^*(R(I_L)) - \Pi^*(R(I_H))) \\ \iff \quad & (1-\delta)(p_H - \hat{p}_H) < \delta(\Pi^*(R(I_L)) - \Pi^*(R(I_H))). \end{aligned}$$

We know that $I_L = I_H + p_H - \hat{p}_H$. Since the household stays in the community, $I_H < I_L < w^{se}$, so $R'(I_H), R'(I_L) > \frac{1}{\delta}$. Thus,

$$R(I_L) - R(I_H) > \frac{1}{\delta}(I_L - I_H) = \frac{1}{\delta}(p_H - \hat{p}_H).$$

By Step 2, $\Pi^*(\cdot)$ increases at rate strictly greater than $(1-\delta)$, so we conclude

$$\delta(\Pi^*(R(I_L)) - \Pi^*(R(I_H))) > \delta(1-\delta)\frac{1}{\delta}(p_H - \hat{p}_H),$$

as desired. Thus, if $I_H < I_L$, then $c_H > c_L + (w_H - w_L)$.

We are now ready to prove that $I_H < I_L$ contradicts household optimality. To do so, we consider two perturbations: one at w_L and one at w_H. At w_H, consider setting

$$\hat{c}_H = c_L + (w_H - w_L) > c_L \geq 0,$$
$$\hat{p}_H = p_L + w_H - w_L \in (p_L, w_H],$$
$$\hat{f}_H = \hat{c}_H - \hat{p}_H = f_L.$$

By construction, $w_H - \hat{p}_H = I_L$. Thus, $\hat{f}_H$ satisfies (5) because f_L does. Moreover, $\hat{p}_H + \hat{f}_H = \hat{c}_H$, so the neighbor is willing to accept. This perturbed strategy is therefore an equilibrium. For the original equilibrium to be household-optimal, we must therefore have

$$(1-\delta)(U(c_H) - f_H) + \delta\Pi^*(R(I_H)) \geq (1-\delta)(U(\hat{c}_H) - \hat{f}_H) + \delta\Pi^*(R(\hat{I}_H)). \tag{7}$$

At w_L, consider setting

$$\hat{c}_L = c_H - (w_H - w_L) > c_L \geq 0,$$
$$\hat{p}_L = p_H - (w_H - w_L) \in (p_L, w_L] \ ,$$
$$\hat{f}_L = \hat{c}_L - \hat{p}_L = f_H.$$

By construction, $w_L - \hat{p}_L = I_H$. Thus, $\hat{f}_L$ satisfies (5) because f_H does. This perturbed strategy is again an equilibrium, so the original equilibrium is household-optimal only if

$$(1-\delta)(U(c_L) - f_L) + \delta\Pi^*(R(I_L)) \geq (1-\delta)(U(\hat{c}_L) - \hat{f}_L) + \delta\Pi^*(R(\hat{I}_L)). \tag{8}$$

Combining (7) and (8) and plugging in definitions, we have

$$U(c_H) - U(c_H - (w_H - w_L)) \geq U(c_L + (w_H - w_L)) - U(c_L).$$

However, $c_H > c_L + w_H - w_L$ and $U(\cdot)$ is strictly concave, so this inequality cannot hold. Thus, if $I_H < I_L$, then at least one of the equilibria at w_H and w_L cannot be household-optimal.

A.3.4 Step 4: Establishing Monotonicity

We have shown that investment, $I(w)$, is increasing in w. Consider a household-optimal equilibrium with $w_1 \geq w_0$. Then, $I(w_1) \geq I(w_0)$, so $w_2 = R(I(w_1)) \geq R(I(w_0)) = w_1$. Thus, $w_2 \geq w_1$, and $w_{t+1} \geq w_t$ for all $t > 1$ by the same argument. Similarly, if $w_1 \leq w_0$, then $I(w_1) \leq I(w_0)$, $w_2 \leq w_1$, and $w_{t+1} \leq w_t$ in all $t \geq 0$. We conclude that $(w_t)_{t=0}^{\infty}$ is monotone in any household-optimal equilibrium. ■

B Reversible Exit

B.1 Household can return to the community

This appendix shows that mis-investment occurs even if the household can return to the community after leaving for the city. Formally, we modify the game in Section 2 so that at the start of every period while the household is in the city, it can return to the community. If it does, then it plays the community game until it again chooses to leave for the city. Payoffs and information structures are the same as in Section 2, and so neighbors observe all of the household's interactions with neighbors, while vendors observe only their own interactions.

We impose a slightly stronger version of Assumption 1.

Assumption 2 *Define $\bar{c}_m$ as the solution to $\hat{U}'(\bar{c}_m) = 1$, and let $\hat{w}_m$ satisfy $R(\hat{w}_m - \bar{c}_m) = \hat{w}_m$. Then, $R'(\hat{w}_m) > \frac{1}{\delta}$.*

Under this assumption, we can prove that mis-investment occurs even if exit is reversible.

Proposition 3 *Impose Assumption 2. There exists a w^{**}, a $w^{cc} < w^{**}$, and a positive-measure interval $\mathcal{W} \subseteq [0, w^{**})$ such that the household permanently exits the community if $w_0 \notin \mathcal{W}$. If $w_0 \in \mathcal{W}$, then in any household-optimal equilibrium, the household is in the community for an infinite number of periods.*

Moreover, if $w_0 \notin \mathcal{W}$, then in any equilibrium,

$$\lim_{t\to\infty} w_t > w^{**},$$

while if $w_0 \in \mathcal{W}$, then for every $t \geq 0$, $w_t < w^{**}$. Moreover, $w_{t+1} < w_t$ whenever $w_t \in (w^{cc}, w^{**})$.

B.1.1 Proof of Proposition 3

Much like the proof of Proposition 2, we break this proof into a sequence of lemmas. We begin by showing that the household's worst equilibrium payoff equals $\hat{\Pi}(\cdot)$, its worst equilibrium payoff from the game with reversible exit.

Lemma 6 *For any initial wealth $w \geq 0$, the household's worst equilibrium payoff is $\hat{\Pi}(\cdot)$.*

Proof of Lemma 6: This proof is similar to the proof of Proposition 1. It is an equilibrium for $f_t = 0$ in every $t \geq 0$, in which case it is optimal for the household to permanently leave the community. In the city, vendor t accepts only if $p_t \geq c_t$. Therefore, $\hat{\Pi}(\cdot)$ gives the maximum equilibrium payoff if the household permanently leaves the community. But as in the proof of Proposition 1, the household cannot earn less than $\hat{\Pi}(\cdot)$, because vendor t must accept whenever $p_t > c_t$. $\square$

Now, we turn to the household's maximum equilibrium payoff. Define $\Pi^{**}(w)$ as the maximum equilibrium payoff with initial wealth w. Define $\Pi^*_c(\cdot)$ identically to $\Pi_c(\cdot)$, except that $\Pi^*(\cdot)$ is replaced by $\Pi^{**}(\cdot)$. Define

$$\Pi^*_m(w) \equiv \max_{0\leq c\leq w} \left\{(1-\delta)\hat{U}(c) + \delta\Pi^{**}(R(w-c))\right\}$$

as the household's maximum equilibrium payoff if it chooses the city in the current period. The key difference between this model and the baseline model is that $\Pi^*_m(\cdot)$ might entail the household returning to the community to take advantage of relational contracts in the future. Therefore, $\Pi_m^*(\cdot) \geq \hat{\Pi}(\cdot)$, since the latter entails staying in the city forever.

Lemma 7 *Both* $\Pi_c^*(\cdot)$ *and* $\Pi_m^*(\cdot)$ *are strictly increasing. For all* $w \geq 0$, $\Pi^{**}(w) \equiv \max\{\Pi_c^*(w), \Pi_m^*(w)\}$.

Proof of Lemma 7: By Lemma 6, the household earns no less than $\hat{\Pi}(\cdot)$ following a deviation. As in Lemma 1, conditional on choosing the community in period 0, the household's maximum equilibrium payoff equals $\Pi_c^*(w_0)$. If the household instead chooses the city in period t, then $f_t = 0$ in any equilibrium., since the continuation equilibrium is independent of f_t. Thus, the household optimally sets $p_t = c_t$, so its maximum equilibrium continuation payoff equals $\Pi^{**}(R(w-c))$. We conclude that $\Pi_m^*(w)$ is the household's maximum equilibrium payoff conditional on choosing the city. It then immediately follows that $\Pi^{**}(w) \equiv \max\{\Pi_c^*(w), \Pi_m^*(w)\}$. Both $\Pi_c^*(\cdot)$ and $\Pi_m^*(\cdot)$ are strictly increasing by inspection. $\square$

Apart from some details of the proof, the next result is similar to Lemma 2.

Lemma 8 *If* $\Pi^{**}(w_0) > \hat{\Pi}(w_0)$, *then* $\Pi^{**}(w_t) > \hat{\Pi}(w_t)$ *in all* $t \geq 0$ *of any household-optimal equilibrium.*

Proof of Lemma 8: Suppose not, and let $\tau > 0$ be the first period such that $\Pi^{**}(w_\tau) = \hat{\Pi}(w_\tau)$. In period $\tau - 1$, (5) implies that $f_t = 0$ if the household stays in the community. Therefore, it is optimal for the household to leave the community in $\tau - 1$. But then it is optimal for the household to *permanently* leave the community in $\tau - 1$, since $\Pi^{**}(w_\tau) = \hat{\Pi}(w_\tau)$. So $\Pi^{**}(w_{\tau-1}) = \hat{\Pi}(w_{\tau-1})$, contradicting the definition of τ. $\square$

Our next result is the analogue to Lemma 3 in this setting.

Lemma 9 *Suppose that* $\Pi^{**}(w_0) > \hat{\Pi}(w_0)$. *Then in every* $t \geq 0$, $\hat{U}'(c_t) \geq 1$, *and there exists* $\tau > t$ *such that the household stays in the community in period* τ.

Proof of Lemma 9: Towards contradiction, suppose that there exists $t \geq 0$ such that $\hat{U}'(c_t) < 1$, so that *a fortiori,* $U'(c_t) < 1$. If $f_t > 0$, then we can decrease f_t and c_t by the same $\epsilon > 0$. This perturbation is also an equilibrium, and increases the household's period-t payoff at rate $1 - \hat{U}'(c_t) > 0$ as $\epsilon \to 0$. Thus, $f_t = 0$, which implies that $p_t = c_t > 0$.

By Lemma 8, $\Pi^*(w_{t+1}) > \hat{\Pi}(w_{t+1})$. Therefore, there exists a $\tau > t$ such that $f_\tau > 0$, since otherwise the household could do no better than exiting the city permanently. Let τ be the *first* period after t such that $f_\tau > 0$. Note that the household must be in the community in period τ.

Consider the following perturbation: decrease p_t and c_t by $\epsilon > 0$, and increase p_τ and decrease f_τ by $\chi(\epsilon)$, where $\chi(\epsilon)$ is chosen so that $w_{\tau+1}$ remains constant. Then, $\chi(\epsilon) \geq \frac{\epsilon}{\delta^{\tau-t}}$ because $R'(\cdot) \geq \frac{1}{\delta}$. As $\epsilon \to 0$, $\chi(\epsilon) \to 0$. Hence, this perturbation is feasible for small enough $\epsilon > 0$. It is an equilibrium, since (5) is trivially satisfied in all $t' \in [t, \tau - 1]$ because $f_{t'} = 0$ in those periods. This perturbation changes the household's period-t continuation payoff at rate no less than

$$-(1-\delta)\hat{U}'(c_t) + \delta^{\tau-t}(1-\delta)\frac{1}{\delta^{\tau-t}} > 0$$

as $\epsilon \to 0$. Thus, the original equilibrium could not have been household-optimal. □

Next, we show that the household stays in the community for sufficiently low initial wealth levels.

Lemma 10 *The set*

$$\left\{w | \Pi^{**}(w) > \hat{\Pi}(w)\right\}$$

has positive measure, with

$$w^{**} \equiv \sup \left\{w | \Pi^{**}(w) > \hat{\Pi}(w)\right\} < \infty.$$

Proof of Lemma 10: The proof that $\left\{w | \Pi^{**}(w) > \hat{\Pi}(w)\right\}$ is identical to the proof in Lemma 4, since the same constructions work at $w = 0$, $\Pi^{**}(\cdot)$ is increasing, and $\hat{\Pi}(\cdot)$ is continuous. To prove that $w^{**} < \infty$, suppose w_0 is such that

$$R(w_0 - \bar{c}_m) > w_0.$$

For any such w_0,

$$\hat{\Pi}(w_0) > \hat{U}(\bar{c}_m) \geq \Pi^{**}(w_0),$$

where the second inequality follows by Lemma 9 and the assumption that $\Pi^{**}(w) > \hat{\Pi}(w)$. Contradiction. So $\hat{\Pi}(w_0) = \Pi^{**}(w_0)$ for any $w_0 > \hat{w}_m$. We conclude $w^{**} < \infty$. □

We are now in a position to prove Proposition 3. So far, the argument has hewn closely to the proof of Proposition 2. The rest of the proof marks a more substantial departure.

Lemma 10 shows that a positive-measure set $\mathcal{W}$ exists such that $\Pi^{**}(w_0) > \hat{\Pi}(w_0)$ for all $w_0 \in \mathcal{W}$. Lemma 8 implies that for any $w_0 \in \mathcal{W}$, we can construct an infinite sequence of periods such that the household remains in the community for each period in that sequence. For any $w_0 \notin \mathcal{W}$, $\Pi^{**}(w_0) = \hat{\Pi}(w_0)$ and so the household permanently exits the community. This proves the first part of Proposition 3.

From the proof of Lemma 9, we know that $w^{**} \leq \hat{w}_m$. Assumption 2 then implies that $R'(w^{**}) > \frac{1}{\delta}$. As in the proof of Proposition 2, if the household permanently exits the community, then w_t is increasing, with $\lim_{t\to\infty} w_t > w^{**}$. This proves the second part of Proposition 3.

Suppose $w_0 \in \mathcal{W}$. Then, Lemma 8 and the definition of w^{**} immediately imply that $w_t < w^{**}$ in every $t \geq 0$. It remains to identify a $w^{cc} < w^{**}$ such that if $w_0 \in \mathcal{W}$, then whenever $w_t \in (w^{cc}, w^{**})$ in a household-optimal equilibrium, $w_{t+1} < w_t$. By an argument similar to Proposition 2,

$$\lim_{w \uparrow w^{**}} \Pi^{**}(w) = \Pi^{**}(w^{**}) = \hat{\Pi}(w^{**}),$$

because $\Pi^{**}(\cdot)$ is increasing, $\hat{\Pi}(\cdot)$ is continuous, $\Pi^{**}(\cdot) \geq \hat{\Pi}(\cdot)$, and $\Pi^{**}(w) = \hat{\Pi}(w)$ for all

$w > w^{**}$. Therefore, we can define a wealth level $w^{cc1} < w^{**}$ similarly to w^c in the proof of Proposition 2.

Define the function

$$F(w) \equiv (1-\delta)\hat{U}(w^{**} - R^{-1}(w)) + \delta\hat{\Pi}(w^{**}) - \hat{\Pi}(w)$$

on $w \in [0, w^{**}]$. Then $F(\cdot)$ is strictly decreasing and continuous, with

$$F(0) = (1-\delta)\hat{U}(w^{**}) + \delta\hat{\Pi}(w^{**}) > 0.$$

At $w_0 = w^{**}$, it is feasible for the household to permanently leave the community and consume $c_t = w^{**} - R^{-1}(w^{**})$ in each $t \geq 0$. However, doing so violates (6), since $R'(w^{**}) > \frac{1}{\delta}$. Therefore, this consumption path must be dominated by some other feasible consumption path once the household permanently leaves the community, which implies

$$\hat{U}(w^{**} - R^{-1}(w^{**}) < \hat{\Pi}(w^{**}).$$

Consequently,

$$F(w^{**}) = (1-\delta)\hat{U}(w^{**} - R^{-1}(w^{**})) - (1-\delta)\hat{\Pi}(w^{**}) < 0.$$

We conclude that there exists a unique $w^{cc2} \in (0, w^{**})$ such that $F(w^{cc2}) = 0$.

Set $w^{cc} = \max\{w^{cc1}, w^{cc2}\}$. For any $w_0 \in (w^{cc}, w^{**})$ such that $\Pi^{**}(w_0) > \hat{\Pi}(w_0)$, the household must either stay in the community or leave in $t = 0$. If it stays in the community, then we can follow the steps of the proof of Proposition 2, Statement 3, to conclude that $w_{t+1} \leq w^{cc1} < w_t$.

Suppose the household is in the city in $t = 0$. Towards contradiction, suppose that

$w_{t+1} \geq w_t$. Therefore, the household's payoff satisfies

$$\begin{aligned} \Pi^{**}(w_0) &\leq (1-\delta)\hat{U}(w^{**} - R^{-1}(w_0)) + \delta \Pi^{**}(w^{**}) \\ &= (1-\delta)\hat{U}(w^{**} - R^{-1}(w_0)) + \delta \hat{\Pi}(w^{**}) \\ &< \hat{\Pi}(w_0), \end{aligned}$$

where the first inequality follows because $f_0 = 0$, so that $p_0 = c_0 = w_0 - R^{-1}(w_1) \leq w^{**} - R^{-1}(w_0)$; the equality follows because $\Pi^{**}(w^{**}) = \hat{\Pi}(w^{**})$, and the final, strict inequality follows because $w_0 > w^{cc2}$ and so $F(w_0) < 0$. But $\Pi^{**}(\cdot) \geq \hat{\Pi}(\cdot)$, proving a contradiction. So $w_{t+1} < w_t$ if the household is in the city in $t = 0$.

We conclude that if $w_0 \in \mathcal{W}$, then in any household-optimal equilibrium, $w_t < w^{**}$ for all $t \geq 0$, and $w_{t+1} < w_t$ whenever $w_t > w^{cc}$. This completes the proof. ■