The Interval Structure of Optimal Disclosure

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MSU
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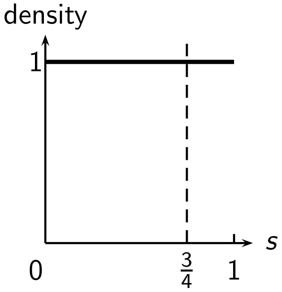
An example

An online platform promotes a product to a customer.

Customer's payoff from buying depends on an unknown s uniform on $[0, 1]$.

Customer's payoff from buying is $u(s) = s - 3/4$. Platform's payoff is 1.

Not buying gives both a payoff of 0.



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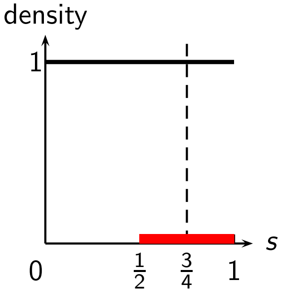
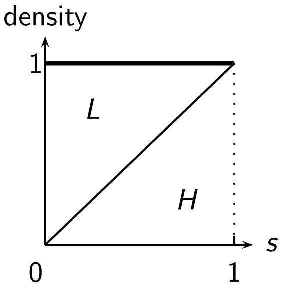


An example

Customer reads some product reviews/report and acquires private information about s.

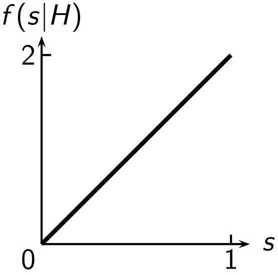
We refer to the private information as Customer's type.

Given s, Customer's type is H with prob. s and L with prob. $1 - s$.



An example

H type's belief (density) is $f(s|H) = 2s$. L type's is $f(s|L) = 2(1-s)$.



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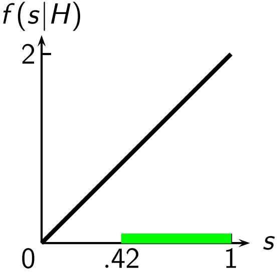

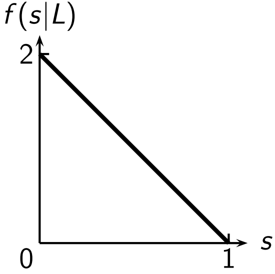

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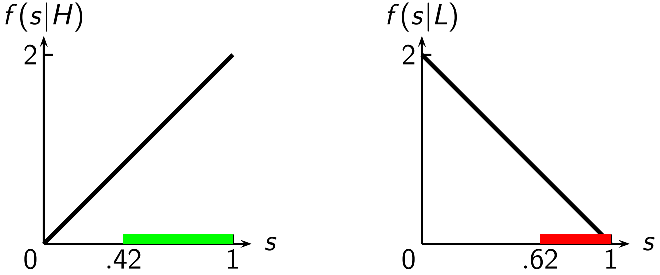
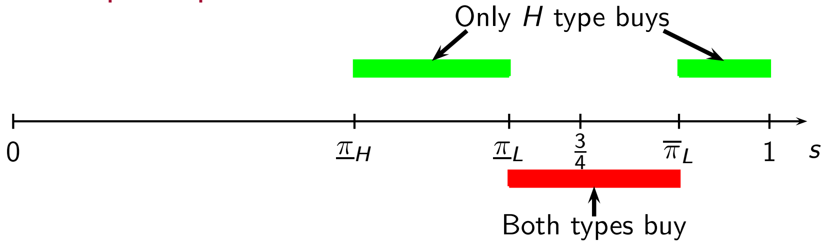


An example: optimal disclosure



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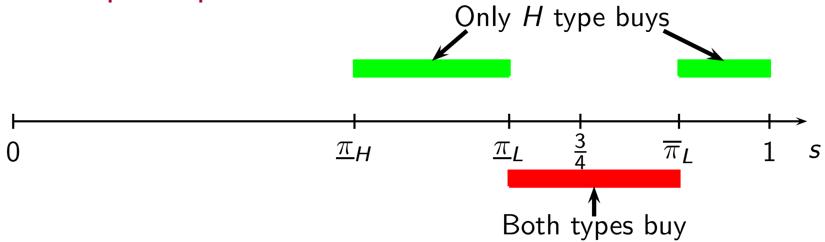

- Nested intervals
- U-shaped cutoff mechanism

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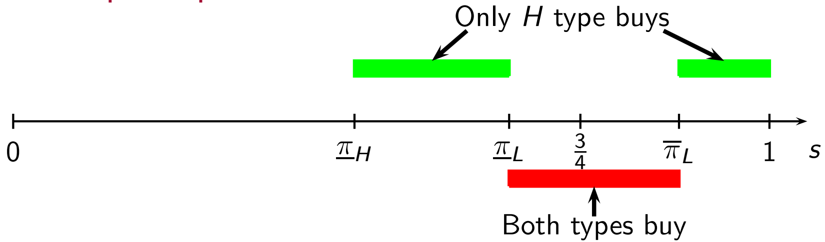

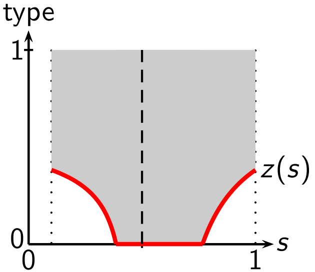

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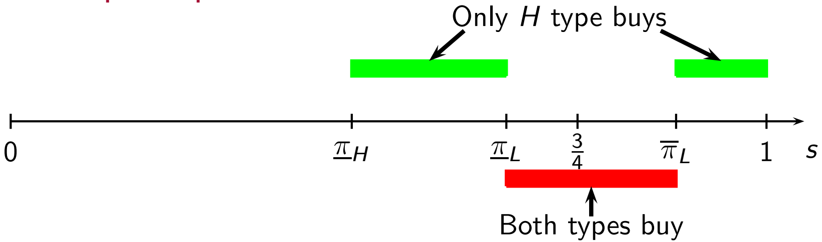

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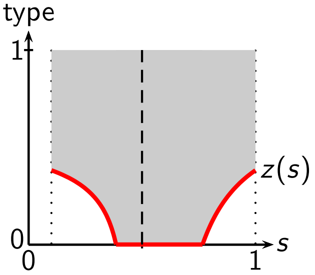

- A lobbyist sways a legislator's position on an issue.
- A media outlet promotes a candidate or an agenda.

Related literature

Information design (mostly single receiver):

Rayo and Segal (2010), Kamenica and Gentzkow (2011)

Aumann and Maschler (1995), Ely (2017)

Kolotilin (2016), Kolotilin, Li, Mylovanov, and Zapechelnyuk (2016)

Gentzkow and Kamenica (2016), Dworczak and Martini (2016)

Information design with multiple receivers:

Lehrer, Rosenberg and Shmaya (2010, 2013), Bergemann and Morris (2016a, 2016b), Mathevet, Perego and Taneva (2016), Taneva (2016)

Schnakenberg (2015), Alonso and Câmara (2016), Chan et al. (2016), Guo and Bardhi (2016), Arieli and Babichenko (2016)

Cheap talk with privately informed receiver:

Seidmann (1990), Watson (1996), Olszewski (2004), Chen (2009), Lai (2014)

Roadmap

1. Environment and main results
2. A simple algorithm
3. Sketch of proof

Environment

One Sender and one Receiver.

$s \in \mathcal{S} \subset \mathbf{R}$: set of states.

Receiver's utility from accepting is $u : \mathcal{S} \to \mathbf{R}$ (0 if Receiver rejects).

u is monotone increasing.

$t \in \mathcal{T} \subset \mathbf{R}$: set of types. Lowest type is $\underline{t}$.

f: distribution over $\mathcal{S} \times \mathcal{T}$.

Assumption:

$f(s, t)$ satisfies increasing monotone likelihood ratio, i.e., $\frac{f(s,t)}{f(s,t')}$ (weakly) increases in s for every $t' < t$.

A (disclosure) mechanism is a triple:

$$(\mathcal{X},\ \kappa,\ r)$$

set of signals	Markov kernel from $\mathcal{S}$ to $\mathcal{X}$	recommendation function $r : \mathcal{X} \times \mathcal{T} \to \{1,0\}$

When the state is s, the mechanism

- randomizes a signal x according to $\kappa(s,\cdot)$;
- recommends that type t accept if and only if $r(x,t) = 1$.

- Fully reveal the state: $\mathcal{X} = \mathcal{S}$ and $\kappa(s, \cdot) = \delta_s$.
- Reveal whether s is above or below π:

  $\mathcal{X} = \{above, below\}$;
  for $s \geqslant \pi$, $\kappa(s, \cdot) = \delta_{above}$;
  for $s < \pi$, $\kappa(s, \cdot) = \delta_{below}$.
- For $B \subset \mathcal{S}$, reveal whether the state is in B or not.
- Randomize:

  $\mathcal{X} = \{above, below, null\}$;
  for $s \geqslant \pi$, $\kappa(s, \cdot) = 1/2\delta_{above} + 1/2\delta_{null}$;
  for $s < \pi$, $\kappa(s, \cdot) = 1/2\delta_{below} + 1/2\delta_{null}$.

A strategy for type t is

$$\sigma : \mathcal{X} \to \{0, 1\}.$$

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A mechanism is publicly incentive-compatible if for every t

$$\sigma_t^* \in \arg\max \int f(s,t)u(s)\left(\int \sigma(x)\ \kappa(s,\,\mathrm{d}x)\right)\,\mathrm{d}s.$$

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If type t obeys the recommendation, his acceptance probability at s, is

$$\rho(s, t) = \int r(x, t)\ \kappa(s, \mathrm{d}x).$$

Sender's problem is

$$\text{Maximize} \iint f(s,t) \underbrace{\left(\int r(x,t)\ \kappa(s,\ \mathrm{d}x) \right)}_{=\rho(s,t)}\ \mathrm{d}s\,\mathrm{d}t$$

among all publicly IC mechanisms.

Structural theorem

A mechanism is a cutoff mechanism if

- $\mathcal{X} = \mathcal{T} \cup \{\infty\}$,
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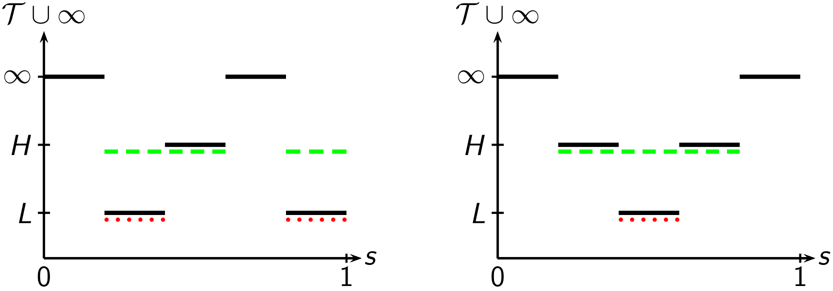
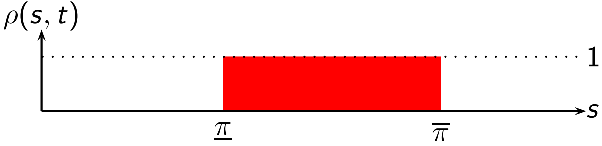


Structural theorem

A mechanism recommends t to accept on an interval if



Structural theorem

Theorem 1:

The optimal publicly IC mechanism is a cutoff mechanism that recommends that each type accept on an interval.

Private incentive compatibility

Receiver doesn't observe x. He reports t' and observes $r(x, t')$.

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$$\sigma = \bar{\sigma}(r(x, t')) \text{ for some type } t' \in \mathcal{T} \text{ and some } \bar{\sigma} : \{0, 1\} \to \{0, 1\}.$$

Private incentive compatibility

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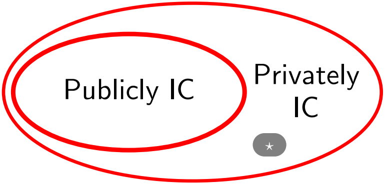
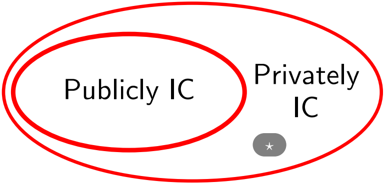

Publicly IC
Privately IC
⋆

Equivalence theorem



Theorem 2:

No privately IC mechanism gives Sender a higher payoff than the optimal publicly IC mechanism.

Roadmap

1. Environment and main results
2. A simple algorithm
3. Sketch of proof

Binary-type case: a simple algorithm

$\mathcal{T} = \{H, L\}$, $\mathcal{S} = [0, 1]$.

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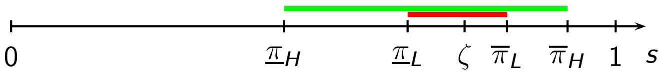

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Sender has one IC constraint for each type:

$$\begin{aligned}
\underset{\underline{\pi}_H, \underline{\pi}_L, \overline{\pi}_L, \overline{\pi}_H}{\text{Maximize}} \quad & \int_{\underline{\pi}_L}^{\overline{\pi}_L} f(s, L)\,\mathrm{d}s + \int_{\underline{\pi}_H}^{\overline{\pi}_H} f(s, H)\,\mathrm{d}s \\
\text{subject to} \quad & \int_{\underline{\pi}_L}^{\overline{\pi}_L} f(s, L) u(s)\,\mathrm{d}s \geqslant 0, \\
& \int_{\underline{\pi}_H}^{\underline{\pi}_L} f(s, H) u(s)\,\mathrm{d}s + \int_{\overline{\pi}_L}^{\overline{\pi}_H} f(s, H) u(s)\,\mathrm{d}s \geqslant 0.
\end{aligned}$$

Binary-type case: pooling vs separating

Proposition:

Pooling is optimal if and only if

$$\frac{f(1,H)}{f(1,L)} - \frac{f(\underline{\pi}_L^*,H)}{f(\underline{\pi}_L^*,L)} < 1 - \frac{u(\underline{\pi}_L^*)}{u(1)},$$

where $\underline{\pi}_L^*$ is such that $\int_{\underline{\pi}_L^*}^1 f(s,L)u(s)\,\mathrm{d}s = 0$. In this case the mechanism recommends to both types to accept on $[\pi_L^*, 1]$.

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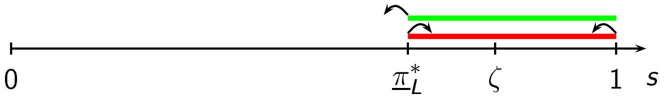

Binary-type case: a comparison with cavU approach

Revisit the Platform-Customer example:

$$f(s, H) = s, \quad f(s, L) = 1 - s, \quad u(s) = s - \frac{3}{4}.$$

The problem amounts to an infinite-dimensional LP problem:

$$\begin{aligned}
\underset{g_L(s)\geq 0, g_H(s)\geq 0}{\text{Maximize}} \quad & \int_0^1 (g_L(s) + s g_H(s))\, \mathrm{d}s \\
\text{subject to} \quad & g_L(s) + g_H(s) \leqslant 1, \ \forall s, \\
& \int_0^1 (1 - s)\left(s - \frac{3}{4}\right) g_L(s)\, \mathrm{d}s \geqslant 0, \\
& \int_0^1 s\left(s - \frac{3}{4}\right) g_H(s)\, \mathrm{d}s \geqslant 0.
\end{aligned}$$

Continuous-type example: a screening perspective

$\mathcal{T} = [0,1]$, $\mathcal{S} = [-1,1]$.

$u(s) = -\eta < 0$ for every $s < 0$ and $u(s) \geqslant 0$ for $s \geqslant 0$.

$\underline{f} : \mathcal{T} \to \mathbf{R}_+$ such that $f(s,t) = \underline{f}(t)$ for every $s < 0$.

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Type t's payoff from accepting on $[\underline{\pi}, \overline{\pi}]$ is

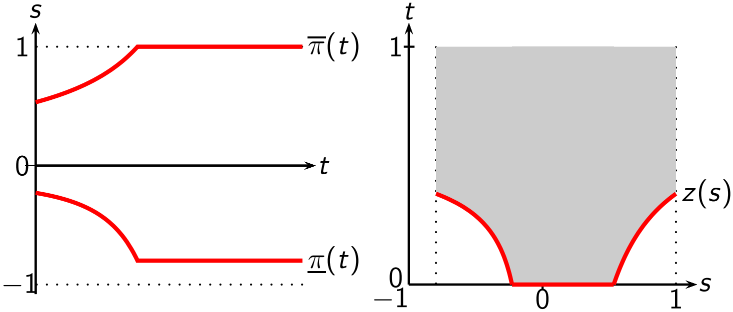
$$\begin{aligned}\int_{\underline{\pi}}^{\overline{\pi}} f(s,t)u(s)\,\mathrm{d}s &= \underline{f}(t)\left(\frac{\int_0^{\overline{\pi}} f(s,t)u(s)\,\mathrm{d}s}{\underline{f}(t)} + \eta\underline{\pi}\right) \\ &= \underline{f}(t)\left(\overline{U}(\overline{\pi},t) + \eta\underline{\pi}\right).\end{aligned}$$

Continuous-type example: a screening perspective

Type t is a "buyer" with quasilinear utility:

- utility $\overline{U}(\overline{\pi}, t)$ from receiving quality $\overline{\pi}$,
- transfer payment $-\eta\underline{\pi}$.

The IC constraints translate to the standard IC constraints for selling the good when the buyer's type is private.



Roadmap

1. Environment and main results
2. A simple algorithm
3. Sketch of proof

Sketch of proof:

A mechanism is downward incentive-compatible if for every t

$$\sigma_t^* \in \arg\max \int f(s,t)u(s)\left(\int \sigma(x)\ \kappa(s,\,\mathrm{d}x)\right)\,\mathrm{d}s,$$

over $\sigma = \sigma_{t'}^*$ for every $t' \leqslant t$, and

$$\int f(s,t)u(s)\left(\int \sigma_{\underline{t}}^*(x)\ \kappa(s,\,\mathrm{d}x)\right)\,\mathrm{d}s \geqslant 0.$$

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Downward IC depends only on acceptance probabilities $\{\rho(s,t)\}$.

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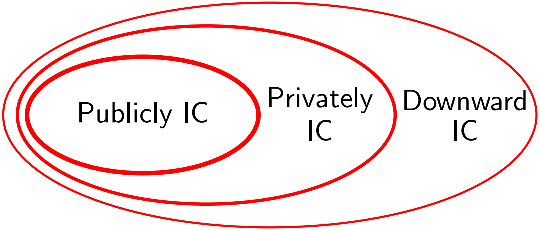
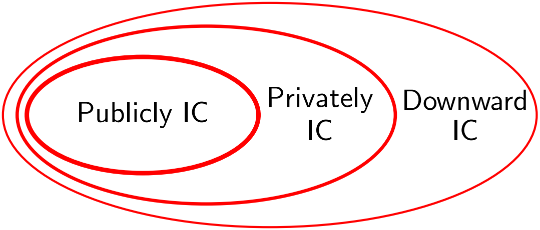


Sketch of proof: create intervals



For any downward IC mechanism, we can create a U-shaped downward IC mechanism that is better for Sender.

Sketch of proof: create intervals

Without loss, we let $\mathcal{S} = \mathbf{R}$ and $u(0) = 0$.

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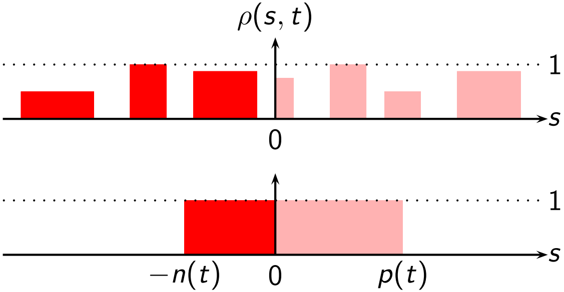
Without loss, we let $\mathcal{S} = \mathbf{R}$ and $u(0) = 0$.

For every t, let $p(t)$ and $n(t)$ be such that

$$\int_0^\infty f(s,t)u(s)\rho(s,t)\,\mathrm{d}s = \int_0^{p(t)} f(s,t)u(s)\,\mathrm{d}s,$$
$$\int_{-\infty}^0 f(s,t)u(s)\rho(s,t)\,\mathrm{d}s = \int_{-n(t)}^0 f(s,t)u(s)\,\mathrm{d}s.$$



Sketch of proof: create intervals

For every t, Sender is better off

$$\int_{-\infty}^{\infty} f(s,t)\rho(s,t)\,\mathrm{d}s \leqslant \int_{-n(t)}^{p(t)} f(s,t)\,\mathrm{d}s.$$

Sketch of proof: create intervals

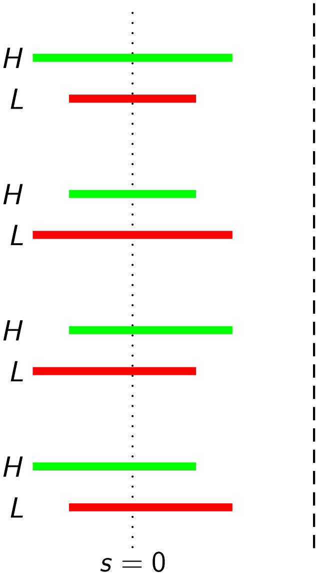
For every t, Sender is better off

$$\int_{-\infty}^{\infty} f(s,t)\rho(s,t)\,\mathrm{d}s \leqslant \int_{-n(t)}^{p(t)} f(s,t)\,\mathrm{d}s.$$

High type's IC is maintained. From i.m.l.r., for $t' < t$

$$\int_{-\infty}^{\infty} f(s,t)u(s)\rho(s,t')\,\mathrm{d}s \geqslant \int_{-n(t')}^{p(t')} f(s,t)u(s)\,\mathrm{d}s.$$

Sketch of proof: create nested intervals



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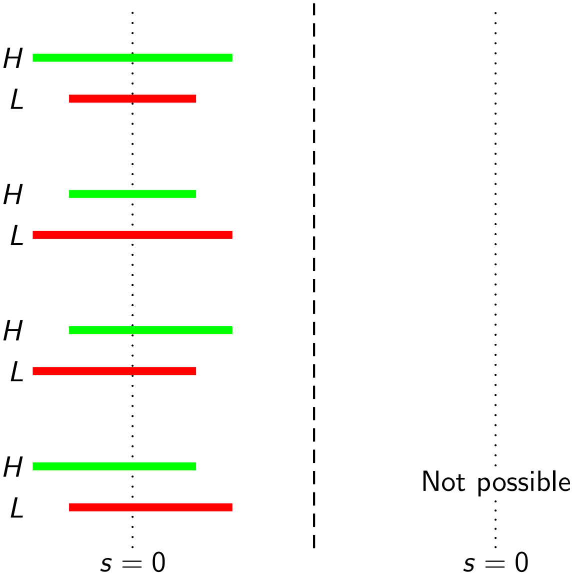

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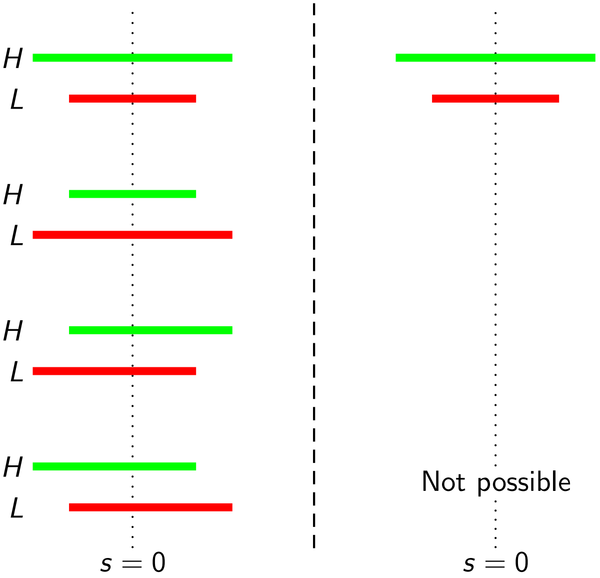

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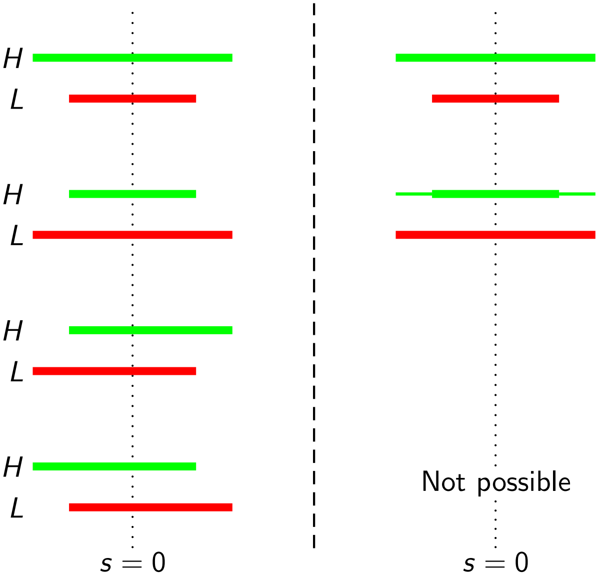

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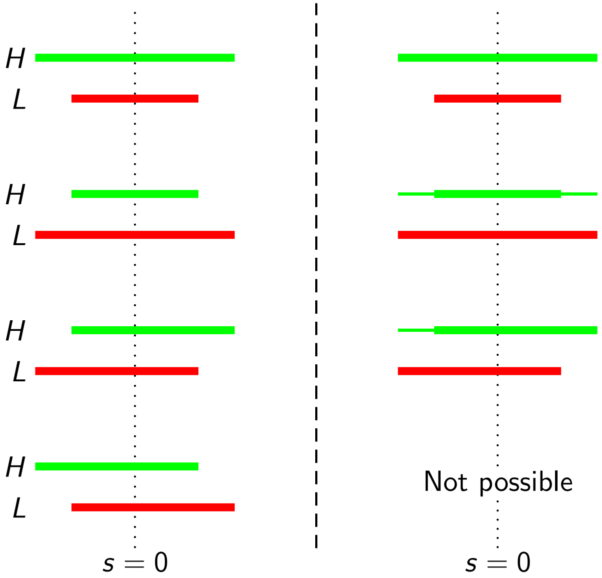
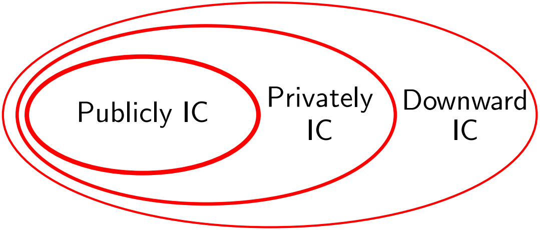


Sketch of proof: formally

Lemma:

For every downward IC mechanism, there exists a U-shaped function $z : \mathcal{S} \to \mathcal{T} \cup \{\infty\}$, such that the cutoff mechanism given by z is downward IC and weakly dominates the original mechanism.

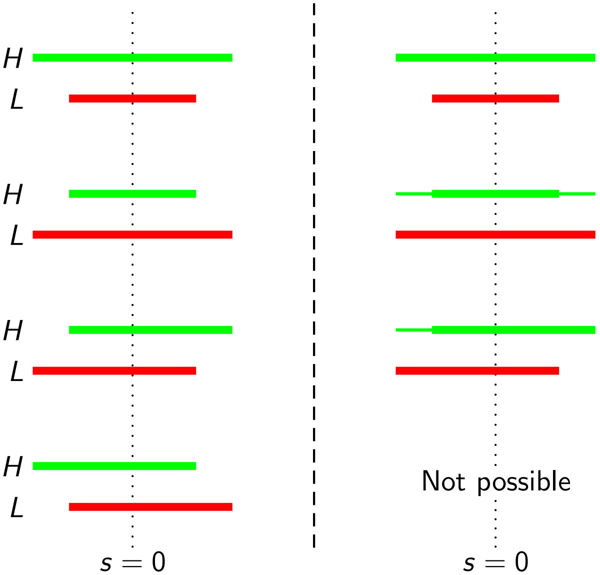
Sketch of proof: create intervals



For any downward IC mechanism, we can create a U-shaped downward IC mechanism that is better for Sender.

The induced publicly IC mechanism does better for Sender.

Sketch of proof: publicly IC mechanism does better



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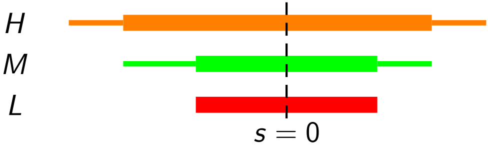


Lemma:

For every cutoff mechanism $(\mathcal{X}, \kappa, r)$ that is downward IC, the publicly IC mechanism induced by this mechanism has the property that type t accepts when $t \geqslant x$.

More general environments

- Receiver's payoff from accepting is $u(s,t)$;
- Sender's payoff from accepting is $v(s,t) > 0$ for every s, t;
- Receiver's belief is $f(s,t)$; Sender's belief is $g(s,t) > 0$ for every s, t.

Our results hold if

- $\frac{f(s,t)u(s,t)}{g(s,t)v(s,t)}$ is monotone in s for every t;
- $\frac{f(s,t)u(s,t)}{f(s,t')u(s,t')}$ is monotone in s for every $t' < t$.

End

The i.m.l.r. assumption

Example:

$\mathcal{S} = \{-4, -3, 3\}$ with $u(s) = s$, and $\mathcal{T} = \{T, B\}$. The prior is given by

	-4	-3	3
T	$1/6,$	$1/6,$	$1/6$
B	$3/54,$	$20/54,$	$4/54$

The optimal publicly IC pools the two types:

$$\rho(-4) = 12/17, \quad \rho(-3) = 1/17, \quad \rho(3) = 1.$$

The optimal privately IC recommends T to accept if $s = 3$ or -3.

It recommends B to accept according to the probabilities above.

The common prior assumption

Example:

$\mathcal{S} = \{-2, -1, 1\}$ with $u(s) = s$, and $\mathcal{T} = \{H, L\}$. Receiver's and Sender's beliefs are given by

Receiver's belief

	-2	-1	1
H	$1/10,$	$2/10,$	$2/10$
L	$4/12,$	$1/12,$	$1/12$

Sender's belief

	-2	-1	1
H	$8/20,$	$4/20,$	$4/20$
L	$2/20,$	$1/20,$	$1/20$

The optimal publicly IC mechanism gives up on L and recommends H to accept if $s = -2$ or 1.

The optimal privately IC mechanism recommends L to accept if $s = -1$ or 1 and H to accept if $s = -2$ or 1.

Nonequivalence between public and private IC

Example:

$\mathcal{S} = \{-1000, 1, 10\}$ with $u(s) = s$, and $\mathcal{T} = \{H, L\}$. The prior is given by

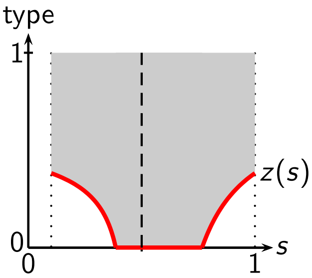
	-1000	1	10
H	$5/22$,	$5/22$,	$1/22$
L	$20/82$,	$20/82$,	$1/82$

A mechanism recommends H to accept if $s = 10$ and L to accept if $s = 1$.

This mechanism is privately IC (and of course not optimal).

H accepts with the interim prob. $1/11$ and L accepts with the interim prob. $20/41$.

Back



Thank you!