Information Transmission and Voting *

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Abstract

I analyze an individual's incentive to disclose hard evidence in the context of committee voting. A committee consists of three members, one left-leaning, one right-leaning, and the third ex ante unbiased. They decide on whether to pursue a left or a right policy by majority rule. One of them has private information about the merits of the policies and can privately send verifiable messages to the others. If the informed member is unbiased, he withholds information to neutralize the other two's votes when preferences are sufficiently diverse. If the informed member is biased, then the others can better infer his information, knowing that any information favoring his agenda will be shared. In the latter case, because more information is effectively shared, higher social welfare results.

Keywords: committee decision-making, voting, verifiable disclosure.

JEL Codes: D71, D72, D83.

1 Introduction

Motivation. In many political and economic interactions, collective decisions are made by committees through voting. For instance, a legislative body decides about implementing a policy change. A board of directors decides to approve or reject a proposal. More often than not, committee members make their voting decisions under uncertainty. They are uncertain about the underlying state. They do

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not know others' preferences precisely. One of them may know the state better than the others. In such situations, if the better-informed member can send costless, verifiable messages to the others, will he share his information voluntarily? If yes, with whom will he share and to what extent?

As an example, consider a legislative body picking one of two policies. Legislators represent their constituents, whose preferences diverge in regard to which policy is more appropriate. We assume that legislators know their constituents' preference distributions, but learn their preferences precisely only before voting. This can be interpreted as the case in which legislators keep gathering information about their constituents' preferences, or the case in which the constituents' preferences are subject to changes before votes are cast.[1] Some legislator might know the policies' merits better than the others. Will he share the information? Which legislator, if informed, shares more information?

To address these questions, I study a three-member committee which selects a left-leaning policy or a right-leaning one by majority rule. Members are uncertain about a state variable that is payoff-relevant. One member, whose identity is commonly known, might have information (a signal) about the state and can share this information with the other two members in private. Communication takes the form of verifiable disclosure: the signal is costless to communicate and verifiable. Hence, the informed member's strategic choice is whether to share his information and, if so, with whom.

Each member's payoff depends on the state, the winning policy, and his preference type.[2] I assume that committee members' preference types are drawn from uniform distributions with different supports, and refer to the mean of a member's preference distribution as his ideology. Preference distributions are commonly known, yet each member privately learns his preference type right before he votes. This formulation is consistent with the observation that policy makers' ideologies are common knowledge, but their exact stances on any particular issue are private information and subject to idiosyncratic shocks. A stylized committee consists of three members: Left, Central, and Right. Ex ante, Left is biased toward the left-leaning policy and Right toward the right-leaning policy, while Central is unbiased. After the informed member shares his information, each member learns his preference type. Then, they vote simultaneously.

A preview of results. I explore which member, if informed, shares more information and what the welfare implications are. In particular, I compare the situation in which Central is informed with the situation in which a peripheral member is informed. The term "a peripheral member" refers to

[1]As argued by Millner and Ollivier (2016), "politicians are unlikely to be able to support policy choices that conflict with the worldview held by a majority of their constituents." Also, constituents' preferences about a policy issue may not be stable over time. As a result, politicians need to keep gathering information about their constituents' preferences.

[2]Austen-Smith (1990), Austen-Smith and Feddersen (2006), and Jackson and Tan (2013), for instance, also assume that a player' payoff depends on a common state and a private preference type.

a member who, ex ante, is leaning toward one of the policies, so both Left and Right are peripheral members. I show that more information is effectively shared when it is a peripheral member who has the private information. This is because Central's incentive to be pivotal deters him from sharing when the committee becomes sufficiently polarized. However, if a peripheral member is informed, then the others can better infer his information, knowing that any information favoring his ex ante preferred policy will be shared. My analysis leads to the somewhat surprising conclusion that social welfare is higher when an ex ante biased member is informed than when an ex ante neutral member is informed. Here, I use the sum of all members' expected payoffs as the social welfare measure, which is the same at the total utility in Jackson and Tan (2013).

To obtain some intuition, let me illustrate Central's incentives to share or to withhold information. Central has an incentive to share information with the uninformed players in order to coordinate their votes. With coordinated votes, when Central obtains information favoring one policy, that policy is more likely to be chosen by the majority rule. On the other hand, Central wants to neutralize the other players' votes in order to be pivotal. Being pivotal allows a member to adjust his vote according to his preference type, and is valuable in the setting in which members' preference types are subject to idiosyncratic shocks. When the committee is sufficiently polarized, Central chooses not to share any information at all. The other two always vote differently from each other, which makes Central a de facto dictator. This analysis prompts one intuitive conclusion: the more polarized the committee is, the less information Central shares.

Importantly, Central's withholding information polarizes the committee even further. Left and Right vote like a partisan, even if their ex ante bias toward the left-leaning and right-leaning policies, respectively, are just moderate. The reason is that, conditioning on being pivotal, a peripheral member puts too much weight on the event that Central's information favors his ex ante preferred policy, even though Central's information is equally likely to favor each policy. Take the perspective of Left as an example. He conditions on his vote being pivotal, in which case Right (most likely) votes for the right-leaning policy while Central votes for the left-leaning policy, making Left's vote decisive. Being pivotal makes Left infer from Central's vote that Central's information favors the left-leaning policy. Therefore, Left is even more likely to vote for the left-leaning policy.

Central's withholding information and its polarizing effect have two implications. First, peripheral members' votes fail to respond to information. Second, peripheral members behave more polarized than they truly are, leading to less preference aggregation. Both implications contribute to a lower welfare level than in the case in which a peripheral member is informed.

A peripheral member is less likely to be pivotal. In addition, a peripheral member, if informed, is inclined to share information that promotes his ex ante preferred policy, enabling the uninformed members to better infer the signal that a peripheral member obtains. With more information being shared and without the polarizing effect, the voting outcome better reflects the information and members' preferences. Thus, a higher welfare level results.

Related literature. This paper is closely related to the literature on verifiable disclosure, which studies scenarios where verifiable information is available and disclosure is discretionary (e.g., Grossman (1981), Milgrom (1981), Verrecchia (1983), Dye (1985), Milgrom and Roberts (1986), Shin (1994*b*)). A common theme is the "unravelling argument" which says that, if the sender is known to have information for sure, then the "skeptical posture" by the receiver "forces" the sender to reveal all information. Shin (1994*a*) and Bhattacharya and Mukherjee (2013) study the disclosure game when multiple senders try to influence a single receiver's action. Jackson and Tan (2013) studies the equilibria when multiple informed senders try to influence uninformed voters' actions, and analyze how different voting rules affect the disclosure and voting behavior. In all these papers, the informed parties do not participate in the decision process. In contrast, I examine an informed voter's incentives to disclose hard evidence to other uninformed voters. As a result, other voters can learn hidden information about the state from being pivotal.[3] Moreover, an informed party's incentives differ when he is part of the collective decision-making process than when he is simply advising the receivers.

From this aspect, the paper is closely related to Schulte (2010) which assumes that each voter has a verifiable signal and characterizes the conditions under which full information aggregation is possible. This occurs when voters have information for sure or when their interests are completely aligned. In a different setting than that in Schulte (2010), I show that the central voter, if informed, discloses less information than a peripheral voter due to the incentives to be pivotal, and that the central voter's withholding behavior polarizes the committees even further.

This paper is closely related to the literature on deliberative voting, which focuses on the role of communication prior to voting in aggregating private information and/or preferences (e.g., Coughlan (2000), Austen-Smith and Feddersen (2005, 2006), Gerardi and Yariv (2007)). In this literature, communication is modeled as a cheap-talk stage (e.g., Crawford and Sobel (1982)). Coughlan (2000) examines the situation in which members have common values and shows that cheap-talk communication can restore *full-information equivalence*—the voting outcome is the same as if all private information were publicly known—under a unanimous voting rule.[4] Coughlan (2000) and Austen-Smith and Feddersen

[3] Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1998) also illustrate that learning hidden information from being pivotal affects voters' behavior in a subtle way.

[4] If committee members' preferences over two alternatives are identical whenever they have the same information, they share common values.

(2006) show that this result relies on the assumption of common values. If conflicts of interests prevail, however, communication cannot guarantee full-information equivalence regardless of voting rules. Gerardi and Yariv (2007) shows that all voting rules except the two unanimity rules generate the same set of equilibrium outcomes when cheap-talk communication is allowed. Although my paper also focuses on the relationship between communication and voting, it focuses on settings in which the informed voter can conceal information but cannot lie. I show that full-information equivalence is the case only when voters' preferences are sufficiently congruent.

I assume that the informed voter can choose to share the information with a subgroup of the uninformed voters. This is referred to as the private communication instead of the public one. There are many ways in which communication might be organized. In particular, subgroups may conduct private meetings in which meaningful communication takes place. Private communication allows the informed member to share with one fellow member while keeping the other in the dark. Hence, I am able to examine with whom the informed member shares his private information. From this aspect, the paper is related to Farrell and Gibbons (1989) and Goltsman and Pavlov (2011). Both papers study a cheap-talk communication game with one sender and two receivers, and examine how the presence of one receiver affects the sender's communication with the other receiver. In the context of voting, the informed member's incentives to share with one uninformed member are naturally affected by the presence of the other uninformed member. I show that the informed member tends to share information that confirms an uninformed member's ex ante bias. In section 7, I argue that the main messages of the paper still hold under public communication.

The paper is organized as follows. The model and the solution concept are introduced in section 2. Section 3 illustrates the preliminary results. In section 4 and 5, I present the equilibria for when an ex ante neutral member is informed and those for when an ex ante biased member is informed. I compare the two scenarios and perform a welfare analysis in section 6. Section 7 concludes.

2 The setup

2.1 The model

Environment. A committee consists of three players, $\ell \in N = \{1, 2, 3\}$. They select a policy, $p \in \{L, R\}$, by majority rule. Let L and R be $-w < 0$ and $w > 0$, respectively. No restriction is imposed on the size of w since the distance between two policies does not affect the analysis, as I show later. If policy p is selected, then the outcome is $p - s$, where s is an unknown state drawn uniformly from $[-1/2, 1/2]$. This unknown state represents players' uncertainty regarding the effect of the candidate policies.

For a fixed policy p, its outcome $p - s$ is also uniformly distributed. The supports of $L - s$ and $R - s$ are $[-w - 1/2, -w + 1/2]$ and $[w - 1/2, w + 1/2]$, which are symmetric with respect to the origin. Figure 1 shows the density functions of outcome $L - s$ and $R - s$, when w equals $1/5$.

Preferences. Player ℓ prefers policies that minimize the distance between his bias b_ℓ and outcome $p - s$. His realized payoff is given by the quadratic loss function:

$$u_\ell = -(b_\ell - (p - s))^2 = -(p - (b_\ell + s))^2.$$

The higher the state is or the higher the bias is, the more likely it is that player ℓ prefers R. Player ℓ's bias b_ℓ is drawn uniformly from support B_ℓ, independently across players and of the state. The support B_ℓ is parameterized by a positive constant β, as follows:

$$(B_1, B_2, B_3) = \left(\left[-\frac{1}{2} - \beta, \frac{1}{2} - \beta \right], \left[-\frac{1}{2}, \frac{1}{2} \right], \left[-\frac{1}{2} + \beta, \frac{1}{2} + \beta \right] \right).$$

The support of player 2's bias distribution is symmetric around the origin. The supports of player 1's and player 3's bias distributions are symmetric around $-\beta$ and β, respectively. Here, β captures the ex ante misalignment of players' preferences (or how polarized the committee is). Let G_ℓ and g_ℓ designate the cdf and pdf of b_ℓ.

The bias distributions are common knowledge, but each player learns his bias privately before voting. Figure 2 shows the pdf of the bias distributions when β equals $3/10$. I focus on the parameter region $\beta \in (0, 2/3)$. It becomes clear later that the analysis is trivial for $\beta \geqslant 2/3$ since player 1 and 3 become partisans.[5]

For a fixed policy p, player ℓ's expected payoff, before s and b_ℓ are realized, is:

$$\mathrm{E}[u_\ell \mid p] = \mathrm{E}[-(b_\ell - (p - s))^2 \mid p] = -\left(p - (\mathrm{E}[b_\ell] + \mathrm{E}[s])\right)^2 - \mathrm{Var}[s] - \mathrm{Var}[b_\ell],$$

where $\mathrm{E}[s]$ and $\mathrm{Var}[s]$ are the expected value and variance of s; and $\mathrm{E}[b_\ell]$ and $\mathrm{Var}[b_\ell]$ are the expected value and variance of b_ℓ. Ex ante, player ℓ's induced preference over policies is single-peaked at $\mathrm{E}[b_l] + \mathrm{E}[s]$. In particular, player 2's induced preference over policies is single-peaked at zero, implying that ex ante he is indifferent between policy L and R. Since $\mathrm{E}[b_1] + \mathrm{E}[s] < 0$ and $\mathrm{E}[b_3] + \mathrm{E}[s] > 0$, ex ante player 1 strictly prefers L and player 3 strictly prefers R. For this reason, players 1, 2, and 3 are also called the left, central, and right players, respectively.

[5] A player is partisan if he always votes for one policy regardless of his signal.

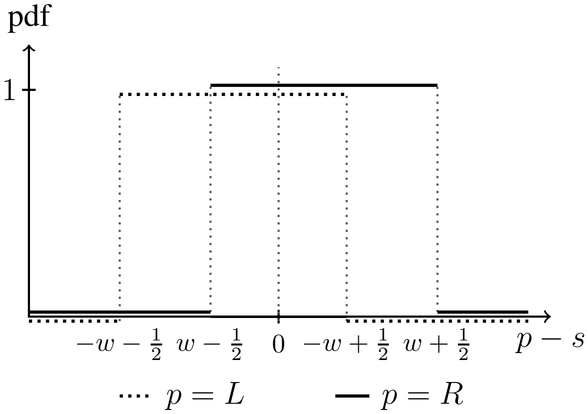

Figure 1: Distributions of outcome, $p-s$

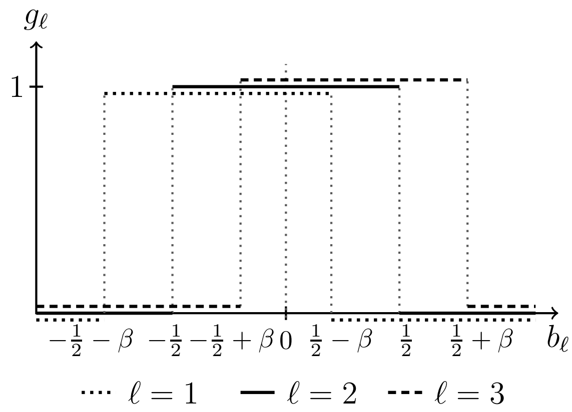

Figure 2: Distributions of bias, b_ℓ

Communication. Players do not know the state s. But one player, labeled i, has superior information and is called the *informed player*. Player i's identity is commonly known. With probability $1-h \in (0,1)$, player i obtains a binary signal about the state, $\theta \in \Theta = \{\theta_r, \theta_l\}$. The signal takes value θ_r with probability $1/2+s$ and θ_l with probability $1/2-s$. With complementary probability h, he receives no signal at all, denoted by $\varnothing$. I characterize the equilibria for $h \in (0,1)$ with a focus on the limiting behavior and payoffs as the probability of no signal becomes negligible (i.e., $h \to 0$).

The other two are the *uninformed players*, denoted by j_1 and j_2. After observing the signal, the informed player can disclose the signal to the uninformed players. The signal is hard evidence and the communication is private, so player i chooses with whom to share the signal if he obtains one. An uninformed player observes the signal if player i shares with him, and observes nothing if player i does not, so an uninformed player does not observe whether player i has shared with the other uninformed player. If the informed player does not receive a signal, then he has no strategic choice to make. Therefore, if an uninformed player receives no signal from i, either player i chooses not to share with him or player i himself does not receive a signal. The uninformed players are not allowed to talk with each other.[6]

Timeline. The game consists of a communication stage and a voting stage. In the communication stage, nature draws the state s and the signal θ, and reveals signal θ to player i with probability $1-h$. If player i obtains a signal, then he decides whether to share it with j_1, j_2. Otherwise, he has no strategic choice to make in this stage. In the voting stage, each player learns his bias privately and votes under

[6]I discuss in section 7 the impact of the assumption that the informed player can carry out private disclosure and the assumption that uninformed players cannot talk to each other. The main results for the $h \to 0$ case still hold if these two assumptions are relaxed.

majority rule. Then payoff is realized. I use ℓ to represent a generic player when discussing biases, payoffs, and the voting stage, while I use i to represent the informed player and j_1, j_2 to represent the uninformed players when discussing the communication stage.

Before proceeding, I want to emphasize three key features of the model. The first is the assumption that players are asymmetrically informed. This assumption lays the foundation for analyzing the better-informed player's incentives to disclose information. The second feature is that players have heterogeneous ideologies, measured by the constant β. Introducing heterogeneous ideologies enriches the model in two ways. I can analyze with whom an informed player shares the information. In addition, I can compare the equilibria in which the central player is informed with the equilibria in which a peripheral player is informed. The third feature is the assumption that the disclosure decision is made before the informed player learns his bias. I call this bias-independent sharing. This assumption corresponds to the situations in which committee members keep receiving idiosyncratic shocks which influence their preferences over the policies. These shocks might arrive after the communication stage. Consider again a legislative body picking one of two policies. Legislators keep gathering information about their constituents' preferences by having private meetings with party leaders/constituents before voting.

2.2 Strategies and solution concept

Strategy profile. A strategy profile consists of the informed player's disclosure strategy and all players' voting strategies.

Let $D = \{11, 10, 01, 00\}$ denote the space of disclosure actions. Action 11 refers to sharing the signal with both uninformed players, 10 sharing with j_1 but not j_2, etc. A mixed disclosure strategy, π_i, is a map from Θ to $\Delta(D)$, the set of probability distributions over D. If the informed player has not obtained a signal, he has no choice to make in the communication stage. Therefore, the set of histories after the communication stage is $M_i = \{\Theta \times D\} \cup \{\varnothing\}$, with m_i being a generic element. A mixed voting strategy of player i is a map $v_i : M_i \times B_i \to [0, 1]$. Here, $v_i(m_i, b_i)$ is the probability that player i votes for L, when the communication stage ends at node $m_i \in M_i$ and his bias is b_i.

At the end of the communication stage, an uninformed player $j \in \{j_1, j_2\}$ receives one of three possible messages from player i: signal θ_r, signal θ_l, or signal $\varnothing$ (which means that player i shares nothing with j). Let $M_j = \{\theta_r, \theta_l, \varnothing\}$ denote the set of j's information sets, with m_j being a generic element. Since j only partially observes i's disclosure action, M_j is a partition of M_i. The information set $m_j = \theta_r$ contains two nodes in M_i: either the informed player shares signal θ_r with the other uninformed player or he does not. Similarly, $m_j = \theta_l$ also contains two nodes in M_i. The information set $m_j = \varnothing$ contains all the nodes in M_i at which either player i withholds the signal from player j, or player i has not obtained a signal. A mixed voting strategy for player j is a map $v_j : M_j \times B_j \to [0,1]$, where $v_j(m_j, b_j)$ is the probability that player j votes for L given m_j and b_j. I let $\sigma = (\pi_i, v = (v_i, v_{j_1}, v_{j_2}))$ denote a mixed strategy profile.

Solution concept. The solution concept is a refinement of a weak perfect Bayesian equilibrium, which requires that beliefs, denoted by μ, be updated in accordance with Bayes' rule whenever possible and, in addition, satisfy a "no-signaling-what-you-don't-know" condition (Fudenberg and Tirole (1991)). This condition requires that a player's deviation does not signal information that he himself does not possess. In my model, if player j receives a signal off-path, then he could believe that player i's and j''s biases are perfectly correlated. The "no-signaling-what-you-don't-know" condition rules out cases like this. A full description of the "no-signaling-what-you-don't-know" condition is relegated to appendix 8.1.

Definition 1 (**Perfect Bayesian Equilibrium**)**.** An assessment (μ, σ) is a perfect Bayesian equilibrium if (i) the beliefs are updated in accordance with Bayes' rule whenever possible, and satisfy a "no-signaling-what-you-don't-know" condition (B-1, B-2, and B-3); (ii) the strategy profile σ is sequentially rational given μ; and (iii) no one uses a weakly dominated strategy in the voting stage: for every $\ell \in \{i, j_1, j_2\}$, v_ℓ is not weakly dominated.

3 Preliminaries

I begin with a single-player decision problem. Then, I proceed to characterize players' voting strategies and the informed player's disclosure strategy.

3.1 Single-player decision problem

Consider a single player ℓ who chooses $p \in \{L, R\}$ to maximize his expected payoff. The information structure and timeline of events are similar to the game introduced earlier. The expected payoff of player ℓ conditional on signal θ, bias b_ℓ, and policy p is given by:

$$\mathrm{E}[u_\ell \mid \theta, b_\ell, p] = \mathrm{E}[-(b_\ell - (p - s))^2 \mid \theta, b_\ell, p] = -(b_\ell + \mathrm{E}[s \mid \theta] - p)^2 - \mathrm{Var}[s \mid \theta],$$

where $\mathrm{E}[s \mid \theta]$ and $\mathrm{Var}[s \mid \theta]$ are the expected value and variance of s, given θ. Player ℓ's induced preferences over policies are symmetric and single-peaked at $b_\ell + \mathrm{E}[s \mid \theta]$. Therefore, player ℓ employs a threshold decision rule, choosing L if his bias b_ℓ is below $-\mathrm{E}[s \mid \theta]$ and R otherwise. The corresponding thresholds are as follows:

$$\{-\mathrm{E}[s \mid \theta=\theta_r], -\mathrm{E}[s \mid \theta=\theta_l], -\mathrm{E}[s \mid \theta=\varnothing]\} = \left\{-\frac{1}{6}, \frac{1}{6}, 0\right\}.$$

Signal θ_r leads to a lower threshold than signal θ_l and suggests that R is more likely to be better. The thresholds depend only on the realized signal, not on the player's bias. Nonetheless, for a fixed signal and hence the threshold, player 1's bias is more likely to fall below the threshold than player 2's bias, which in turn is more likely to fall below the threshold than player 3's. Therefore, player 1 is most likely to choose policy L, player 3 is least likely, and player 2 is in between.

3.2 Threshold voting strategies

Returning to the voting stage of the original game, I now demonstrate that player ℓ's voting strategy is also characterized by thresholds. The only time that a player can influence the policy choice is when his vote is pivotal, that is, when the votes of the other two players cancel each other out. Thus, player ℓ makes his voting decision conditional on not only his information set $h_\ell = (m_\ell, b_\ell)$ but also his vote being pivotal. Let piv denote the event that a vote is pivotal. The expected payoff to player ℓ conditional on information set h_ℓ, ℓ being pivotal, the strategy profile σ, and policy p is:

$$\begin{aligned}\mathrm{E}[u_\ell \mid h_\ell, piv, \sigma, p] &= \mathrm{E}[-(b_\ell - (p-s))^2 \mid h_\ell, piv, \sigma, p] \\ &= -(b_\ell + \mathrm{E}[s \mid h_\ell, piv, \sigma] - p)^2 - \mathrm{Var}[s \mid h_\ell, piv, \sigma] \\ &= -(b_\ell + \mathrm{E}[s \mid m_\ell, piv, \sigma] - p)^2 - \mathrm{Var}[s \mid m_\ell, piv, \sigma],\end{aligned}$$

where $\mathrm{E}[s \mid m_\ell, piv, \sigma]$ and $\mathrm{Var}[s \mid m_\ell, piv, \sigma]$ are the conditional expected value and variance of s. The last equality holds because player ℓ's bias is drawn independently of other random variables. Thus, player ℓ votes for L if his bias b_ℓ is below $-\mathrm{E}[s \mid m_\ell, piv, \sigma]$ and R if above. Lemma 1 characterizes the informed player's thresholds. Lemma 2 characterizes an uninformed player's thresholds when he receives a signal from the informed player.

Lemma 1 (Informed player's voting thresholds). *At information set $h_i = (m_i, b_i)$, the informed player votes for L if $b_i < -\mathrm{E}[s \mid m_i, piv, \sigma]$ and for R otherwise. The voting thresholds are as follows:*

$$-\mathrm{E}[s \mid m_i, piv, \sigma] = \begin{cases} -\frac{1}{6} & \text{if } m_i \in \{\theta_r\} \times D, \\ \frac{1}{6} & \text{if } m_i \in \{\theta_l\} \times D, \\ 0 & \text{if } m_i = \varnothing. \end{cases}$$

Lemma 2 (Uninformed player's voting thresholds). *At information set $h_j = (m_j, b_j)$, the uninformed player j votes for L if $b_j < -\mathrm{E}[s \mid m_j, piv, \sigma]$ and for R otherwise. The voting thresholds are as* *follows:*

$$-\mathrm{E}[s \mid m_j, piv, \sigma] = \begin{cases} -\frac{1}{6} & \textit{if } m_j = \theta_r, \\ \frac{1}{6} & \textit{if } m_j = \theta_l. \end{cases}$$

The proofs of Lemmas 1 and 2 are in appendix 8.2. The intuition lies in the fact that the signal exhausts all the relevant information regarding the state. Once a player observes the signal—either he is the informed player or he receives a signal from the informed player—he knows all the relevant information regarding the state. Being pivotal, in this case, does not give him more information. Similarly, the informed player uses zero as the threshold if no signal is obtained, since being pivotal gives him no information about the state in that case.

It follows that the only nontrivial thresholds are those used by uninformed players when they receive no signal: $-\mathrm{E}[s \mid m_{j_1} = \varnothing, piv, \sigma]$ and $-\mathrm{E}[s \mid m_{j_2} = \varnothing, piv, \sigma]$. Since not receiving a signal from the informed player is always on path, these thresholds can be calculated according to Bayes' rule. Here, I briefly illustrate how to calculate the thresholds (detailed formulae can be found in appendix 8.2.1). Let $\Pr[m_j = \varnothing, piv \mid s, \sigma]$ be the probability that (i) player j receives no signal and (ii) his vote is pivotal, conditional on state s. The density function of state s, conditional on j receiving no signal and being pivotal, is:

$$g(s \mid m_j = \varnothing, piv, \sigma) = \frac{\Pr[m_j = \varnothing, piv \mid s, \sigma] f(s)}{\int_{-\frac{1}{2}}^{\frac{1}{2}} \Pr[m_j = \varnothing, piv \mid s, \sigma] f(s) ds},$$

where $f(s)$ is the prior distribution. Therefore, player j's voting threshold, conditional on j receiving no signal and being pivotal, is:

$$-\mathrm{E}[s \mid m_j = \varnothing, piv, \sigma] = -\int_{-\frac{1}{2}}^{\frac{1}{2}} s g(s \mid m_j = \varnothing, piv, \sigma) ds.$$

When the informed player's disclosure strategy is fixed, an uninformed player's threshold, $-\mathrm{E}[s \mid m_j = \varnothing, piv, \sigma]$, depends only on the other players' voting strategies. Thus, $-\mathrm{E}[s \mid m_{j_1} = \varnothing, piv, \sigma]$ is a function of $-\mathrm{E}[s \mid m_{j_2} = \varnothing, piv, \sigma]$ and vice versa.[7] Given two equations in two unknowns, I can solve for these thresholds. When uninformed players observe no signal, they know that there is a positive probability that the informed player obtained no signal either. Therefore, the thresholds must be in the range $(-1/6, 1/6)$. From now on, I focus on strategy profiles that satisfy Lemmas 1 and 2.[8]

[7] Once we fix player i's disclosure strategy and restrict attention to strategy profiles satisfying Lemmas 1 and 2, the threshold $-\mathrm{E}[s \mid m_{j_1} = \varnothing, piv, \sigma]$ depends only on j_2's voting behavior when j_2 receives no signal.

[8] I have assumed that each player ℓ's bias b_ℓ is uniformly distributed. Without this assumption, all I need to change is that

3.3 Disclosure strategy

3.3.1 Coordination motive and pivotal motive

To characterize the informed player's disclosure strategy, I first delineate two key motives to share or to withhold information. Consider a situation in which the central player is informed. After observing a signal favoring policy R, he would like to share the signal to persuade the other players to vote for R, because policy R delivers a higher expected payoff than L does, conditional on the signal. This is the *coordination motive*. With votes coordinated, when a signal favoring one policy is observed, that policy is more likely to be chosen by majority rule. However, despite a signal favoring R, the central player might find his bias sufficiently low in the voting stage, so that he strictly prefers policy L to be chosen. The central player wants to neutralize the other players' votes so that he is able to adjust his vote—the policy to be implemented if the other two vote differently—according to his bias realization. I call this the *pivotal motive*. With the presence of the pivotal motive, the informed player tends to withhold information.

3.3.2 Incentive constraints

Following this line of reasoning, I formalize the incentive constraints governing the informed player's disclosure behavior. From the perspective of player i, the votes of the uninformed players can be one of three cases:

$$\left(LL,\ RR,\ \underbrace{LR, RL}_{piv} \right),$$

where the underbrace gathers two cases that are equivalent. Given any signal $\theta \in \Theta$, any disclosure action $d \in D$, and voting strategies by the uninformed players v_{-i}, let $\mathbf{Pr}(\theta, d, v_{-i})$ be the probability vector that the uninformed players' votes are LL, RR, and piv:[9]

$$\mathbf{Pr}(\theta, d, v_{-i}) = \left(\Pr[LL \mid \theta, d, v_{-i}], \Pr[RR \mid \theta, d, v_{-i}], \Pr[piv \mid \theta, d, v_{-i}]\right).$$

player ℓ's probability of voting L is $G_\ell(-\mathrm{E}[s \mid m_\ell, piv, \sigma])$ where G_ℓ is the cdf of b_ℓ. From this aspect, the analysis and the main results can be generalized readily to a more general class of distributions, but the algebra will be more involved.

[9] I use boldface characters to represent vectors.

For any $\theta \in \Theta$, let $\mathbf{E}[u_i \mid \theta, v_i]$ be the expected payoff vector of player i given signal θ and the voting strategy v_i as in Lemma 1, when the other two votes are LL, RR, or piv:[10]

$$\mathbf{E}[u_i \mid \theta, v_i] = \left(\mathrm{E}[u_i \mid LL, \theta, v_i], \mathrm{E}[u_i \mid RR, \theta, v_i], \mathrm{E}[u_i \mid piv, \theta, v_i]\right).$$

The sum product of $\mathbf{Pr}(\theta, d, v_{-i})$ and $\mathbf{E}[u_i \mid \theta, v_i]$ is the expected payoff to player i given θ, d and v:

$$\mathrm{E}[u_i \mid \theta, d, v] = \mathbf{Pr}(\theta, d, v_{-i}) \cdot \mathbf{E}[u_i \mid \theta, v_i].$$

For any equilibrium profile (π_i, v), if d is played with positive probability after signal θ, then there is no other action with a higher payoff:

$$\mathrm{E}[u_i \mid \theta, d, v] \geqslant \mathrm{E}[u_i \mid \theta, d', v],\ \forall\, d' \in D. \tag{IC}$$

Any voting strategies characterized in section 3.2 and any disclosure strategy satisfying (IC) constitute an equilibrium.

4 Central player is informed

I first study the case in which the central player is informed. Due to symmetry between the signals, I focus on the case in which the central player obtains signal θ_r. The expected payoff vector if the other two' votes are (LL, RR, piv) is:

$$\mathbf{E}[u_2 \mid \theta = \theta_r, v_2] = \left(-w^2 - \frac{1}{3}w - \frac{1}{6}, -w^2 + \frac{1}{3}w - \frac{1}{6}, -w^2 + \frac{5}{9}w - \frac{1}{6}\right) = \left(0, \frac{2w}{3}, \frac{8w}{9}\right) + c\,\mathbf{e},$$

where c is a constant and $\mathbf{e}$ the vector $(1, 1, 1)$. It is easy to verify that the following holds:

$$\mathrm{E}[u_2 \mid piv, \theta = \theta_r, v_2] > \mathrm{E}[u_2 \mid RR, \theta = \theta_r, v_2] > \mathrm{E}[u_2 \mid LL, \theta = \theta_r, v_2].$$

Because the central player is ex ante unbiased, he prefers RR to LL since signal θ_r indicates that a higher state is more likely to occur. However, the central player most prefers being pivotal. The difference between $\mathrm{E}[u_2 \mid piv, \theta = \theta_r, v_2]$ and $\mathrm{E}[u_2 \mid RR, \theta = \theta_r, v_2]$ measures the option value of being pivotal.

Suppose the probabilities with which player 1 and player 3 vote for L are, respectively, p_1^L and p_3^L after signal θ_r and disclosure action d. Suppose that switching to disclosure decision d' increases p_1^L by $\Delta > 0$. The change in $\mathbf{Pr}(\theta = \theta_r, d, v_{-2})$ is:

$$\mathbf{Pr}(\theta = \theta_r, d', v_{-2}) - \mathbf{Pr}(\theta = \theta_r, d, v_{-2}) = \left(p_3^L, -(1 - p_3^L), 1 - 2p_3^L\right)\Delta.$$

[10] If the other votes are LL or RR, the informed player's vote does not affect his expected payoff. The expectation is taken over his bias and the state. If the other votes cancel each other out, the informed player's vote is the winning policy. The expectation is taken over i's bias, the state, and i's policy choice.

Accordingly, the change in the central player's expected payoff is $\mathrm{E}[u_2 \mid \theta = \theta_r, d', v] - \mathrm{E}[u_2 \mid \theta = \theta_r, d, v]$, which equals:

$$\underbrace{\Delta p_3^L 0 + (-\Delta)(1 - p_3^L)\frac{2w}{3}}_{coordination(-)} + \underbrace{\Delta(1 - 2p_3^L)\frac{8w}{9}}_{pivotal(-/+)} = \Delta\frac{2w}{9}(1 - 5p_3^L). \tag{1}$$

The first two terms measure the loss due to the failure to coordinate. Increasing the probability with which player 1 votes for L causes LL to occur more frequently and RR to occur less frequently, lowering the central player's expected payoff. The third term measures the loss or gain due to the change in the probability that the central player finds himself pivotal. When player 3 votes for R sufficiently often ($p_3^L < 1/5$), the gain from being pivotal outweighs the loss due to coordination failure. The central player prefers that player 1 votes for L more often.

Lemma 3 (Central player's disclosure strategy). *Given signal θ_r, player 2 prefers a lower p_1^L if $p_3^L > 1/5$ and a higher p_1^L if $p_3^L < 1/5$. Analogously, player 2 prefers a lower p_3^L if $p_1^L > 1/5$ and a higher p_3^L if $p_1^L < 1/5$.*

4.1 Equilibria

In this section, I show that, as β increases from 0 to $2/3$, the central player's disclosure behavior is characterized by three phases: full disclosure, partial disclosure, and no disclosure.

4.1.1 Full disclosure

When the misalignment of preferences is sufficiently small, neither player 1's nor player 3's voting behavior is sufficiently extreme. When $\beta < 2/15$, both p_1^L and p_3^L are at least $1/5$:

$$p_1^L \geqslant -\frac{1}{6} - \left(-\frac{1}{2} - \beta\right) > \frac{1}{5}, \quad p_3^L \geqslant -\frac{1}{6} - \left(-\frac{1}{2} + \beta\right) > \frac{1}{5}.$$

This is because the lowest threshold that an uninformed player ever employs is $-1/6$. According to Lemma 3, the central player prefers p_3^L and p_1^L to be as low as possible, so he shares signal θ_r to persuade the others to vote for R. Analogously, the central player fully discloses signal θ_l. The equilibrium voting outcome is the same as if the signal were public.

Proposition 1 (Full disclosure). *When $\beta < 2/15$, for any $h \in (0, 1)$ there exists a unique equilibrium. Player 2 fully discloses his private signal to both player 1 and player 3. The uninformed players, if receiving no signal, infer that the informed player obtains no signal and use 0 as the voting threshold.*

4.1.2 Partial disclosure

Regardless of the message that player 1 receives, he votes for L with probability greater than $1/5$ because

$$p_1^L \geqslant -\frac{1}{6} - \left(-\frac{1}{2} - \beta\right) = \frac{1}{3} + \beta > \frac{1}{5}.$$

According to Lemma 3, the central player prefers p_3^L to be as low as possible. If the central player shares signal θ_r with player 3, then player 3 uses $-1/6$ as his voting threshold after signal θ_r. Hence, p_3^L equals

$$p_3^L = \max\left\{-\frac{1}{6} - \left(-\frac{1}{2} + \beta\right), 0\right\} = \max\left\{\frac{1}{3} - \beta, 0\right\},$$

which is strictly smaller than $1/5$ when $\beta > 2/15$.[11] According to Lemma 3, the central player withholds signal θ_r from player 1. We summarize the discussion in the following lemma.

Lemma 4 (Central player withholds unfavorable signals). *When $\beta > 2/15$, for any $h \in (0,1)$ and in any equilibrium, (i) player 2 withholds θ_r from player 1 and θ_l from player 3; (ii) if θ_r realizes, then player 3 votes for L with probability* $\max\{1/3 - \beta, 0\}$*, and if θ_l realizes, then player 1 votes for R with probability* $\max\{1/3 - \beta, 0\}$.

Since player 3 doesn't vote for L very often, the central player conceals signal θ_r from player 1 so as to induce him to vote for L more often, maximizing the probability that the central player is pivotal. Meanwhile, the central player prefers player 3 to vote for R as often as possible. He can do so by sharing signal θ_r with player 3, which confirms player 3's ex ante bias for R. Because the central player discloses his signal to one side, I call this behavior *partial disclosure*.

Proposition 2 (Partial disclosure). *When $\beta > 2/15$, there exists an equilibrium such that player 2 shares signal θ_l only with player 1 and shares signal θ_r only with player 3. Player 1's and player 3's voting thresholds, if they see no signal, are:*

$$-\mathrm{E}[s \mid m_1 = \varnothing, piv, \sigma] = \begin{cases} -\frac{(4-3\beta)(1-h)}{6(3\beta(h-1)+5h+4)} & \text{if } \beta < \frac{1}{3}, \\ -\frac{1-h}{12h+6} & \text{if } \beta \geqslant \frac{1}{3}, \end{cases}$$

$$-\mathrm{E}[s \mid m_3 = \varnothing, piv, \sigma] = \mathrm{E}[s \mid m_1 = \varnothing, piv, \sigma].$$

When $2/15 < \beta < (h+8)/18$, this is the unique equilibrium.

[11] When $\beta = 2/15$, multiple equilibria exist. The central player shares θ_r with player 3 and θ_l with player 1. He is indifferent between sharing θ_r with player 1 or not, and between sharing θ_l with player 3 or not. We show in Proposition 7 in appendix that the central player's most preferred equilibrium is full disclosure.

4.1.3 No disclosure

Lastly, I show that for a large parameter region, there exists an equilibrium in which the central player shares no information.

I first illustrate how "conditional on being pivotal" makes an uninformed player more likely to vote for his ex ante preferred policy, when the central player never shares information with the peripheral players. Take player 1 as an example. When he receives no signal, he reasons that, conditional on being pivotal, it is more likely that player 3 votes for R and the central player votes for L. For this reason, player 1 puts more weight on the event of signal θ_l being received by the central player, and is more likely to vote for L. To what extent player 1 exaggerates the probability that the central player votes for L depends on how likely player 3 votes for R. If player 3 always votes for R, player 1 puts probability one on the event that the central player votes for L. This reasoning, along with a moderate bias toward L, induces player 1 to vote for L with certainty. The same logic applies to player 3. The central player takes advantage of this reasoning by not sharing with the uninformed players. He dictates the voting outcome and receives the highest possible payoff.

Proposition 3 (No disclosure). *When $\beta \geqslant (h+8)/18$, there exists an equilibrium such that player 2 always conceals his signal. Player 1's and player 3's voting thresholds, if they see no signal, are:*

$$-\mathrm{E}[s \mid m_1 = \varnothing, piv, \sigma] = \frac{1-h}{18}, \text{ and } -\mathrm{E}[s \mid m_3 = \varnothing, piv, \sigma] = \frac{h-1}{18}.$$

This is player 2's most preferred equilibrium.

4.1.4 Summary

The following corollary summarizes the central player's most preferred equilibrium for any $\beta \in (0, 2/3)$ and $h \in (0,1)$. This follows directly from Propositions 1 to 3.

Corollary 1 (Central player's most preferred equilibrium). *Player 2's most preferred equilibrium is full disclosure for $\beta \leqslant 2/15$, no disclosure for $\beta \geqslant (h+8)/18$, and partial disclosure (as characterized in Proposition 2) for $2/15 < \beta < (h+8)/18$.*

Figure 3 depicts the equilibrium that gives the central player the highest expected payoff. When players' preferences are not too dissimilar, the central player fully discloses his information with the other two. As preference conflicts grow, the coordination motive gradually gives way to the pivotal motive. The central player begins to withhold information and eventually stops sharing. When $\beta \geqslant (h+8)/18$, there might be multiple equilibria. The central player strictly prefers the no disclosure equilibrium which gives him the highest possible payoff.

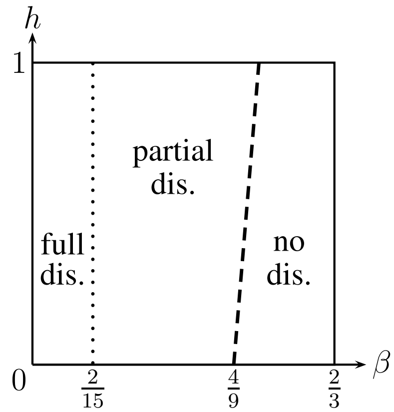

Figure 3: Player 2's most preferred equilibrium

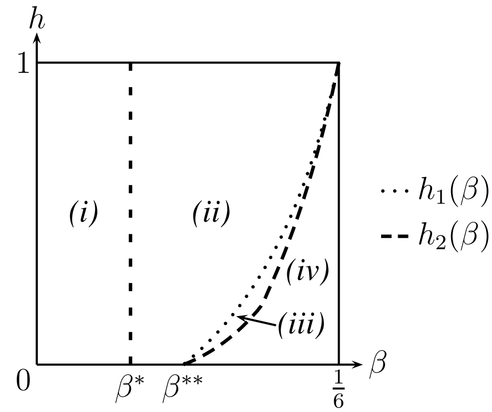

Figure 4: Equilibrium when player 1's is informed

The multiplicity of equilibria when $\beta \geqslant (h+8)/18$ arises due to the multiplicity of self-fulfilling beliefs. Take the limiting case $h \to 0$ as an example. When player 1 expects the central player to share signal θ_l with him and optimally uses the threshold $-1/6$ if he receives no signal, it is optimal for the central player to share signal θ_l with player 1. If player 1 expects the central player to stay quiet and player 3 to vote for R with certainty, then he optimally uses the threshold $1/18$ (as characterized in Proposition 3) if he receives no signal. In this case, it is optimal for the central player not to share any information.

5 A peripheral player is informed

5.1 The unraveling motive and equilibria

I now turn to the case in which a peripheral player, for example, the left player, is informed.[12] Unlike the central player, the left player's disclosure behavior is mainly subject to the *unraveling motive*: he is inclined to share signal θ_l—the signal favorable to policy L—and conceal signal θ_r, due to his ex ante bias toward policy L. The unraveling motive increases as the left player becomes more biased toward L. It overshadows both the coordination and the pivotal motives when β is sufficiently large. I name this motive the unraveling motive, because the induced disclosure behavior enables the uninformed players to better infer the signal obtained by the left player.

When the left player observes signal θ_l, the expected payoff vector if the other two' votes are

[12]Due to symmetry, the situation in which the right player is informed is similar.

(LL, RR, piv) is:

$$\mathbf{E}[u_1 \mid \theta = \theta_l, v_1] = \begin{cases} \left(\frac{2}{3} + 4\beta, 0, \frac{2}{9}(2+3\beta)^2\right) w + c'\ \mathbf{e} & \beta \in [0, \frac{1}{3}], \\ \left(\frac{2}{3} + 4\beta, 0, \frac{2}{3} + 4\beta\right) w + c'\ \mathbf{e} & \beta \in (\frac{1}{3}, \frac{2}{3}], \end{cases}$$

where c' is a constant. The option value of being pivotal, measured as the difference between $\mathrm{E}[u_1 \mid piv, \theta = \theta_l, v_1]$ and $\mathrm{E}[u_1 \mid LL, \theta = \theta_l, v_1]$, decreases in β. When β is greater than $1/3$, the option value of being pivotal reduces to zero because the left player always votes for L after signal θ_l. The following lemma shows that the left player always shares signal θ_l with the other two players.

Lemma 5. *Player 1 always shares signal θ_l with uninformed players.*

When the left player observes signal θ_r, the expected payoff vector if the other two' votes are (LL, RR, piv) is:

$$\mathbf{E}[u_1 \mid \theta = \theta_r, v_1] = \left(-\frac{2}{3} + 4\beta, 0, \frac{2}{9}(1+3\beta)^2\right) w + c''\ \mathbf{e},$$

where c'' is a constant. When $\beta < 1/6$, the left player prefers RR to LL and has incentives to coordinate votes. However, the coordination motive diminishes as β increases. When $\beta > 1/6$, the left player prefers LL to RR in spite of signal θ_r. The pivotal motive is always present in the sense that the left player strictly prefers being pivotal to LL or RR. Yet, when $\beta > 1/6$, the option value of being pivotal, measured as the difference between $\mathrm{E}[u_1 \mid piv, \theta = \theta_r, v_1]$ and $\mathrm{E}[u_1 \mid LL, \theta_1, v_1]$, decreases in β. The next result shows that for $\beta \geqslant 1/6$, there is essentially a unique equilibrium in which the left player conceals signal θ_r.

Proposition 4. *When $\beta \geqslant 1/6$, there exists an equilibrium in which player 1 conceals θ_r. There exists an increasing function $\hat{\beta}(h) : [0,1] \to [1/3, 1/2]$ such that (i) this is the unique equilibrium for any $1/6 \leqslant \beta \leqslant \hat{\beta}(h)$, and (ii) all equilibria are outcome equivalent to this equilibrium for any $\beta > \hat{\beta}(h)$.*

Based on Lemma 5 and Proposition 4, I need to specify the equilibrium disclosure strategy only when the left player receives signal θ_r and β is less than $1/6$. Suppose the probabilities with which player 2 and 3 vote for L are p_2^L and p_3^L. The expected payoff of the left player is:

$$\mathrm{E}[u_1 \mid \theta = \theta_r, d, v] = p_2^L p_3^L \left(4\beta - \frac{2}{3}\right) w + \frac{2}{9}(3\beta+1)^2 \left(p_2^L + p_3^L - 2p_2^L p_3^L\right) w + c''.$$

the derivatives of $\mathrm{E}[u_1 \mid \theta = \theta_r, d, v]$ with respect to p_2^L and p_3^L are:

$$\begin{aligned} \frac{\partial \mathrm{E}[u_1 \mid \theta = \theta_r, d, v]}{\partial p_2^L} &= c''' \left(\frac{(3\beta+1)^2}{18\beta^2 - 6\beta + 5} - p_3^L\right), \\ \frac{\partial \mathrm{E}[u_1 \mid \theta = \theta_r, d, v]}{\partial p_3^L} &= c''' \left(\frac{(3\beta+1)^2}{18\beta^2 - 6\beta + 5} - p_2^L\right), \end{aligned}$$

where c''' is a constant and $(3\beta+1)^2/(18\beta^2-6\beta+5)$ increases in β for $\beta \in (0, 1/6)$. When β is sufficiently small, both derivatives are negative, so the left player wants both p_2^L and p_3^L to be as small as possible. As a result, he shares signal θ_r with both player 2 and player 3. As β increases, the left player prefers a higher p_2^L and a lower p_3^L and hence shares signal θ_r with player 3 but not with player 2. When β is sufficiently large, both derivatives are positive. The left player conceals signal θ_r from both uninformed players. Proposition 5 summarizes the equilibrium disclosure strategy as β ranges from 0 to $1/6$. I also show that this is the unique equilibrium when β is small enough or $h < 4/13$.

Proposition 5. *There exists $0 < \beta^* < \beta^{**} < 1/6$, and two increasing functions $h_1(\beta) > h_2(\beta)$: $[\beta^{**}, 1/6] \to [0, 1]$ such that player 1*

(i) shares θ_r with player 2 and 3 for $\beta < \beta^*$;

(ii) shares θ_r only with player 3 for $\beta^* < \beta \leqslant h_1^{-1}(h)$;

(iii) shares θ_r with player 3 with an interior probability for $h_1^{-1}(h) < \beta < h_2^{-1}(h)$;

(iv) conceals θ_r for $\beta \geqslant h_2^{-1}(h)$.

This is the unique equilibrium for any $\beta \in (0, \beta^*) \cup (\beta^*, \beta^{**})$ or $h < 4/13$.[13]

Figure 4 displays the disclosure behavior if the left player obtains signal θ_r. From the left to the right, the four regions correspond to the four cases discussed in Proposition 5. The left player becomes less likely to share θ_r as β grows. Full-information equivalence occurs if and only if β is sufficiently small.

6 Incentives and welfare

The central player's incentives to disclose information are quite different from those of a peripheral player. When the central player is informed, the coordination motive calls for more disclosure while the pivotal motive calls for less. As β increases, the coordination motive gives way to the pivotal motive (see Table 1). When the left player is informed, the unraveling motive overshadows the coordination and pivotal motives. The left player always discloses signal θ_l, enabling the uninformed players to partially infer the informed player's signal.[14]

Following Jackson and Tan (2013), I take the sum of the players' expected payoffs as the social welfare. Our next result shows that when $\beta \geqslant (h+8)/18$, there exists a decreasing function $\bar{h}(\beta)$ such

[13]When $\beta = \beta^*$, there are multiple equilibria. The left player shares θ_r with player 3 and is indifferent between sharing

Informed player	Coordination motive	Pivotal motive	Unraveling motive
Central player	$\Downarrow$	$\Uparrow$	not available
Left player	$\Downarrow$	$\Downarrow$	$\Uparrow$

Table 1: Incentives governing disclosure decisions as β increases

that the social welfare is strictly lower when the central player is informed than when the left player is informed for $h < \bar{h}(\beta)$. In this parameter region $\beta \geqslant (h+8)/18$, the central player conceals both signals, whereas the left player shares signal θ_l. An informed central player leads to a lower social welfare for two reasons. First, he shares less information than the left player, so uninformed players' votes do not respond to the information. Second, when the central player is informed but withholds information, he further polarizes the uninformed players, who become partisans and always vote for their ex-ante preferred policy. This leads to less preference aggregation than when the left player is informed.[15]

Proposition 6. *When $\beta \geqslant (h+8)/18$, there exists a decreasing function $\bar{h}(\beta) : \left[\bar{\beta}, 2/3\right] \to [0,1]$ such that the social welfare is strictly lower when the central player is informed than when a peripheral player is informed for any (β, h) with $h < \bar{h}(\beta)$.*[16]

When $\beta \geqslant (h+8)/18$, the central player, if informed, is effectively dictating the policy choice. It is easy to show that the social welfare would be higher if the central player were the dictator than if the left player were the dictator. This further shows that the informed central player leads to a lower social welfare than the informed left player does, due to the fact that he shares less information and polarizes the uninformed players.

As a corollary to Proposition 6, we compare the two scenarios as h goes to zero. If the left player is informed, the others can perfectly infer his private information due to the fact that he always shares θ_l. The social welfare level is the same as if the signal were public. If the central player is informed, the social welfare is the same as if the signal were public if and only if $\beta \leqslant 4/9$. If $\beta > 4/9$, the social

θ_r with player 2 or not.

[14] I can measure the amount of information transmitted by an informed player by the probability that uninformed players vote mistakenly. In the case where $\beta \geqslant 4/9$ and $h \to 0$, the probability that a peripheral player would vote against his ideology, if he observed the signal, would be $1/3 - \beta/2$. When the central player is informed, peripheral voters always vote in accordance with his ideology. When the left player is informed, all information is effectively shared, so this mistake is avoided. This contributes to a higher welfare level.

[15] When $\beta \to 2/3$, both player 1 and player 3 become partisans, so the central player's withholding behavior no longer matters. An informed central player hence does better than an informed left player for all $h \in (0,1)$ when $\beta = 2/3$. This explains why an informed central player does better for $h > \bar{h}(\beta)$.

[16] The value of $\bar{\beta}$ is approximately .54.

welfare level is strictly lower than that if the left player is informed, as shown in Figure 5. Hence, if the informed player almost surely has information, the social welfare is always higher if a peripheral player is informed than if a central player is informed.

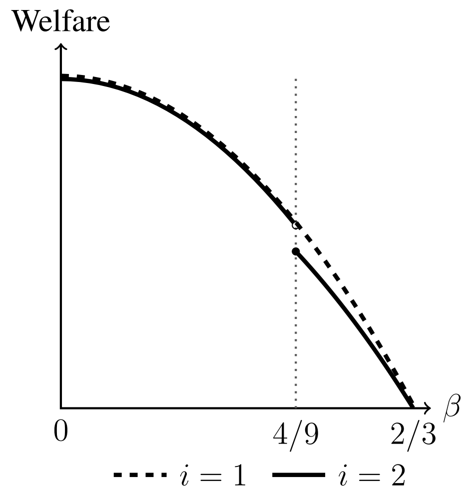

Figure 5: Welfare comparison as $h \to 0$

7 Concluding remarks

This paper aims to understand verifiable disclosure in the environment of committee decision making. My work shows that there is a crucial distinction between polarization and bias. When the central member is informed, he discloses his signal in order to coordinate other committee members' voters only when the committee members' preferences are relative homogeneous. However, when the committee is sufficiently polarized, the central member stays silent in order to stay pivotal. In contrast, if a biased member possesses private information, more information is effectively shared, leading to a higher welfare level. I now conclude by revisiting the main features of the model and discussing future research directions.

Cheap talk vs hard evidence. I previously concluded that a committee with an informed peripheral member leads to collectively desirable outcomes. However, I do not expect this result to hold where communication is modeled as cheap talk. In such a case, a peripheral member's incentives to promote his agenda impair his credibility when he is perceived to be biased. Even if the peripheral member is informed and shares his unverifiable information, it will be disregarded due to its questionable credibility. This effectively nullifies his effort to share, leading to a suboptimal social welfare. Note that the central member is also plagued by the credibility problem due to the incentives he has to neutralize the others' votes and stay pivotal. However, when the committee is not too polarized, an informed central member can and does transmit information. My conjecture is that, with unverifiable information, an informed central member is better than an informed peripheral member.

Simple majority vs unanimity voting rules. I assumed that the policy is selected by a simple majority rule. This is a natural assumption given the symmetry across the candidate policies and the voters. Here, I briefly discuss how the disclosure behavior changes if a policy needs unanimous approval. Without loss of generality, I assume that policy L needs unanimous approval and policy R needs only one vote. In this case, an informed player is pivotal if and only if both uninformed players vote for L. As a result, an informed player would like LL to occur more often, and does so by sharing the signal that promotes policy L. From this aspect, a more demanding voting rule for one policy encourages the informed player to share information that favors this policy. As the parameters vary, the comparison between the majority and unanimity rules in terms of information revelation and welfare can go either way.

Bias-independent vs bias-dependent sharing. I focused on settings where an informed player chooses his disclosure strategy before learning his bias realization. This applies to settings where players keep gathering information about their ideal policies. There are also settings in which an informed player chooses his disclosure strategy after learning his bias. This will change the disclosing behavior when β is small: an informed player knows for sure whether he likes policy R or L, and will share any signal that promotes his preferred policy.

However, when β is moderate (i.e., $\beta \geqslant (h+8)/18$), our equilibrium characterization and main welfare result (i.e., Propositions 3, 4 and 6) still hold. If the central player is informed, his most preferred equilibrium is the no-disclosure one as in Proposition 3. The central player gets his highest possible payoff in this equilibrium, so no disclosure is his best response even under bias-dependent sharing. If the left player is informed, he always shares θ_l, and shares θ_r only if his bias is high. We show in Proposition 8 in appendix that the resulting equilibrium is outcome-equivalent to that in Proposition 4. Therefore, under bias-dependent sharing, the informed central player shares less information and polarizes the uninformed players, leading to a lower social welfare than the informed left player.

Public vs private communication. Requiring communication to be public does not affect the basic intuitions and results when the committee is moderately polarized. A peripheral member, if informed, publicly announces the signal that favors his agenda. In contrast, an informed central member stays quiet in public debate when the committee is sufficiently polarized. Similarly, the results for a moderately polarized committee are robust if the communication is private but uninformed players are allowed to talk to each other.

Nonetheless, the possibility of private communication does improve the informed member's welfare in certain circumstances. For example, when the central member is informed and conflicts of interests are moderate, he tailors his information to uninformed members' ex ante biases, sharing a signal favorable to the left-leaning policy with the left member and a signal favorable to the right-leaning policy with the right player. In this case, private communication allows the central player to maximize the benefit of coordinating votes on the one hand and staying pivotal as often as possible on the other. I leave a complete comparison between public and private communication modes for future research.

8 Appendix

8.1 The "no-signaling-what-you-don't-know" condition

At each information set, the player about to move must have a belief about which history in the information set has been reached. In particular, I need to specify (i) the informed player's belief before he takes a disclosure action, and (ii) all three players' beliefs before they vote. Since the informed player's information sets before he takes a disclosure action are singletons, I need to specify only the players' beliefs before they vote. The set of histories before the players vote is:

$$H = M_i \times B_i \times B_{j_1} \times B_{j_2}.$$

For each $\ell \in N$, the set of player ℓ's information sets before he votes is $H_\ell = M_\ell \times B_\ell$, with h_ℓ being an element. Let $\mu_\ell\left(\tilde{m}_i, \tilde{b}_i, \tilde{b}_{j_1}, \tilde{b}_{j_2} \mid h_\ell\right)$ be the probability that player ℓ assigns to the event that $\tilde{m}_i$ occurs in the communication stage and that the bias vector is weakly below $\left(\tilde{b}_i, \tilde{b}_{j_1}, \tilde{b}_{j_2}\right)$, conditional on h_ℓ. The system of beliefs is denoted by $\mu = \{\{\mu_\ell(\cdot \mid h_\ell)\}_{h_\ell \in H_\ell}\}_{\ell \in N}$.

I first characterize the informed player's beliefs $\mu_i(\cdot \mid h_i)_{h_i \in H_i}$. Since player i's unreached information sets can be reached only by his own deviations, his beliefs at those sets can be derived by Bayes' rule (assuming perfect recall). Given $h_i = (m_i, b_i)$, player i knows exactly what happened in the communication stage. Hence, his belief is given by

$$\mu_i\left(\tilde{m}_i, \tilde{b}_i, \tilde{b}_{j_1}, \tilde{b}_{j_2} \mid m_i, b_i\right) = \begin{cases} G_{j_1}\left(\tilde{b}_{j_1}\right) G_{j_2}\left(\tilde{b}_{j_2}\right) & \text{if } b_i \leqslant \tilde{b}_i \text{ and } \tilde{m}_i = m_i, \\ 0 & \text{otherwise.} \end{cases} \tag{B-1}$$

Here, recall that $G_\ell(\cdot)$ is the cdf of b_ℓ. For an uninformed player j, if he sees a message m_j that is on path, he can calculate the probability that m_i occurs in the communication stage, conditional on

m_j, denoted by $\mu_j\left(m_i \mid m_j\right)$.[17] Hence, $\mu_j\left(\tilde{m}_i, \tilde{b}_i, \tilde{b}_j, \tilde{b}_{j'} \mid m_j, b_j\right)$ is well-defined according to Bayes' rule. If player j receives an out-of-equilibrium message from player i, Bayes' rule does not impose any restriction on player j's beliefs μ_j. For example, if player i never shares any signal with player j on path, then player j could believe that player i's and j''s biases are perfectly correlated if he receives a signal from player i. However, those beliefs violate the "no-signaling-what-you-don't-know" condition since biases are drawn independently after the communication stage. Player j's belief regarding player i's disclosure behavior should be independent of player j's belief regarding the other players' bias realizations. Moreover, both beliefs should be independent of player j's bias realization. This motivates the following restrictions imposed on the uninformed players' beliefs, where $u_j(\cdot \mid m_j)$ are extended to include those nodes m_j that are off path:

(i) For $j \in \{j_1, j_2\}$, there exists a belief system $\{\mu_j(\cdot \mid m_j) : \mu_j(\cdot \mid m_j) \in \Delta(M_i)\}_{m_j \in M_j}$ such that $\mu_j(\cdot \mid m_j)$ assigns positive probability only to nodes contained in m_j. That is,

$$\sum_{m_i \in m_j} \mu_j(m_i \mid m_j) = 1, \; \forall m_j \in M_j. \tag{B-2}$$

The beliefs $\{\mu_j(\cdot \mid m_j)\}_{m_j \in M_j}$ are defined by Bayes' rule whenever possible.

(ii) For $j \in \{j_1, j_2\}$ and $h_j = (m_j, b_j) \in H_j$, the beliefs at h_j are defined as follows:

$$\mu_j\left(\tilde{m}_i, \tilde{b}_i, \tilde{b}_j, \tilde{b}_{j'} \mid m_j, b_j\right) = \begin{cases} \mu_j\left(\tilde{m}_i \mid m_j\right) G_i\left(\tilde{b}_i\right) G_{j'}\left(\tilde{b}_{j'}\right) & \text{if } b_j \leqslant \tilde{b}_j \text{ and } \tilde{m}_i \in m_j, \\ 0 & \text{otherwise,} \end{cases} \tag{B-3}$$

where $j' \in \{j_1, j_2\}$ and $j' \neq j$.

8.2 Proofs for section 3

Proof of Lemmas 1 and 2. Throughout this proof, the dependence on σ is dropped when no confusion arises. It is shown in section 3.2 that players employ threshold voting strategy. Given the information structure, the highest (resp. lowest) threshold a voter ever employs is $1/6$ (resp. $-1/6$). Let $p_\ell(m_\ell) = \Pr[b_\ell < -\mathrm{E}[s \mid m_\ell, piv]]$ denote the probability that player ℓ votes for L given m_ℓ where the expectation is taken over ℓ's bias. Conditional on s, the probability that $m_i = \varnothing$ is $\Pr[m_i = \varnothing \mid s] = h$. The probability that i is pivotal conditional on $m_i = \varnothing$ and s is:

$$\Pr[piv \mid m_i = \varnothing, s] = p_{j_1}(\varnothing)(1 - p_{j_2}(\varnothing)) + p_{j_2}(\varnothing)(1 - p_{j_1}(\varnothing)).$$

[17] For example, if player i discloses signal θ_r to player j with probability one and to player j' with probability α, then player j assigns probability α to the node at which j' also receives signal θ_r, and assigns probability $1 - \alpha$ to the node at which j' receives no signal, conditional on $m_j = \theta_r$.

The probability that i receives no signal and is pivotal conditional on state s is:

$$\Pr[m_i = \varnothing, piv \mid s] = h\left(p_{j_1}(\varnothing)(1 - p_{j_2}(\varnothing)) + p_{j_1}(\varnothing)(1 - p_{j_1}(\varnothing))\right),$$

which is independent of s. The probability distribution over state conditional on $m_i = \varnothing$ and being pivotal is given by

$$g(s \mid m_i = \varnothing, piv) = \frac{\Pr[m_i = \varnothing, piv \mid s] f(s)}{\int_{-\frac{1}{2}}^{\frac{1}{2}} \Pr[m_i = \varnothing, piv \mid s] f(s) ds} = f(s) = 1.$$

This implies that $-\mathrm{E}[s \mid m_i = \varnothing, piv] = 0$. When $m_i \in \{\theta_r\} \times D$, player i knows the thresholds utilized by the uninformed players. The probability of m_i and i being pivotal conditional on s, $\Pr[m_i, piv \mid s]$, is proportional to $1/2 + s$. Therefore, I have

$$g(s \mid m_i, piv) = \frac{1}{2} + s, \quad -\mathrm{E}[s \mid m_i, piv] = -\frac{1}{6}.$$

Analogously, when an uninformed player j receives signal θ_r, the probability of m_j and j being pivotal conditional on s, $\Pr[m_j = \theta_r, piv \mid s]$, is proportional to $1/2 + s$. Therefore, j uses $-1/6$ as voting threshold as well.[18] Similarly, the threshold that player i (resp. player j) employs after $m_i \in \{\theta_l\} \times D$ (resp. $m_j = \theta_l$) is $1/6$. ■

8.2.1 Voting thresholds of the uninformed when $m_j = \varnothing$

Here, I show how to calculate $-\mathrm{E}[s \mid m_{j_1} = \varnothing, piv, \sigma]$ while restricting attention to strategy profile σ satisfying Lemmas 1 and 2. When j_1 receives no signal, either player i receives no signal or player i chooses not to disclose his signal to j_1. Let θ be the signal that player i obtains. I have:

$$\begin{aligned}
\Pr[m_{j_1} = \varnothing, piv \mid s, \sigma] &= \sum_{\theta \in \{\theta_r, \theta_l, \varnothing\}} \Pr[m_{j_1} = \varnothing, piv, \theta \mid s, \sigma] \\
&= \sum_{\theta \in \{\theta_r, \theta_l, \varnothing\}} \Pr[\theta \mid s, \sigma] \Pr[m_{j_1} = \varnothing, piv \mid \theta, s, \sigma] \\
&= \sum_{\theta \in \{\theta_r, \theta_l, \varnothing\}} \Pr[\theta \mid s] \Pr[m_{j_1} = \varnothing, piv \mid \theta, \sigma].
\end{aligned}$$

According to how the signal is drawn, I have

$$\Pr[\theta = \theta_r \mid s] = (1 - h)\left(\frac{1}{2} + s\right), \quad \Pr[\theta = \theta_l \mid s] = (1 - h)\left(\frac{1}{2} - s\right), \quad \Pr[\theta = \varnothing \mid s] = h.$$

[18] The argument applies also when an uninformed player j receives signal θ_r off path. This is because the informed player can not fake the signal.

Next, I show how to calculate $\Pr[m_{j_1} = \varnothing, piv \mid \theta, \sigma]$. To illustrate, take $\theta = \varnothing$ as an example. I have

$$\Pr[m_{j_1} = \varnothing, piv \mid \theta = \varnothing, \sigma] = p_i(\varnothing)(1 - p_{j_2}(\varnothing)) + p_{j_2}(\varnothing)(1 - p_i(\varnothing)).$$

I want to show that this probability is strictly positive when $\beta \in [0, 2/3)$. For this probability to be zero, it must be the case that both i and j_2 vote for the same policy with probability one. For any pair (i, j_2), this is not possible. Therefore, the probability $\Pr[m_{j_1} = \varnothing, piv \mid s, \sigma]$ is strictly positive for any $s \in [-1/2, 1/2]$. The density function of state s conditional on j_1 receiving no signal and being pivotal is:

$$g(s \mid m_{j_1} = \varnothing, piv, \sigma) = \frac{\Pr[m_{j_1} = \varnothing, piv \mid s, \sigma] f(s)}{\int_{-\frac{1}{2}}^{\frac{1}{2}} \Pr[m_{j_1} = \varnothing, piv \mid s, \sigma] f(s) ds}.$$

Therefore, the expectation of state variable s conditional on j_1 receiving no signal and being pivotal is:

$$-\mathrm{E}[s \mid m_{j_1} = \varnothing, piv, \sigma] = -\int_{-\frac{1}{2}}^{\frac{1}{2}} s g(s \mid m_j = \varnothing, piv, \sigma) ds.$$

The probability $\Pr[m_j = \varnothing, piv \mid s, \sigma]$ is a sum product of $\Pr[m_j = \varnothing \mid s, \sigma]$ and $\Pr[piv \mid m_j = \varnothing, s, \sigma]$. Regardless of the state, there is a strictly positive probability that player i receives no signal and hence shares no signal with player j. Therefore, $\Pr[m_j = \varnothing \mid s, \sigma]$ is well-defined and strictly positive for any s. Once I restrict attention to those strategy profiles satisfying Lemmas 1 and 2, the probability $\Pr[piv \mid m_j = \varnothing, s, \sigma]$ is strictly positive for any s.

8.3 Proofs for section 4

To simplify notation, let $t_1, t_3 \in (-1/6, 1/6)$ denote the voting thresholds employed by player 1 and 3 if they observe no signal. Due to symmetry, I focus on the disclosure behavior after the central player observes signal θ_r. Let p_1^L and p_3^L denote probabilities with which player 1 and 3 vote for L in equilibrium.

Proof of Proposition 1. When $\beta < 2/15$, an uninformed player votes for policy L with probability at least $1/5$ regardless of the messages he receives. Because the lowest threshold an uninformed player ever employs is $-1/6$, both p_1^L and p_3^L are bounded from below by $1/5$,

$$p_1^L \geqslant -\frac{1}{6} - \left(-\frac{1}{2} - \beta\right) > \frac{1}{5}, \quad p_3^L \geqslant -\frac{1}{6} - \left(-\frac{1}{2} + \beta\right) > \frac{1}{5}.$$

According to Lemma 3, player 2 would like p_3^L and p_1^L to be as low as possible. The unique optimal disclosure strategy is to share signal θ_r with both player 1 and 3. Analogously, player 2 fully disclose signal θ_l to the uninformed players. ■

Proof of Proposition 2. For $\beta > \frac{2}{15}$, we know from Lemma 4 that player 2 withholds θ_r from player 1 (and withholds θ_l from player 3). Moreover, after signal θ_r, player 2 would like p_3^L to be as small as possible. If player 2 shares θ_r with player 3, then player 3 will use the voting threshold $-\frac{1}{6}$. If player 2 doesn't share θ_r with player 3, then player 3 will use the voting threshold t_3, which is strictly above $-\frac{1}{6}$. Therefore, sharing θ_r with player 3 is always a best response (although it might not be the unique best response, as we later show in Proposition 3). Given player 2's disclosure strategy, we calculate player 1's and 3's voting thresholds t_1, t_3 after signal $\varnothing$, which are given in Proposition 2. This shows that the profile in Proposition 2 is an equilibrium.

We next argue that this partial disclosure equilibrium is the unique equilibrium for $\frac{2}{15} < \beta < \frac{8+h}{18}$. First, if $\beta < \frac{1}{3}$, then according to Lemma 4 the probability that player 3 votes for L, if θ_r realizes, is $\max\left\{\frac{1}{3} - \beta, 0\right\} > 0$. Therefore, player 2's unique best response is to share θ_r with player 3 (and, similarly, to share θ_l with player 1). We thus focus on the region that $\beta \geqslant \frac{1}{3}$. We let q_1 and q_3 be the probabilities with which player 2 shares θ_l with player 1 and shares θ_r with player 3, respectively. We want to argue that if $\beta < \frac{8+h}{18}$, then $q_1 = q_3 = 1$ is the unique equilibrium. Suppose not. Suppose that $q_3 < 1$. This implies that not sharing θ_r with player 3 is among player 2's best responses. Then, it must be true that $t_3 \leqslant -\frac{1}{2} + \beta$. We now discuss two cases, $q_1 < 1$ and $q_1 = 1$, separately.

(i) If $q_1 < 1$, then it must be true that $t_1 \geqslant \frac{1}{2} - \beta$. Given this condition, the voting thresholds if player 1 and 3 see no signal are:

$$t_1 = \frac{(h-1)(2q_1 - 1)}{12(h-1)q_1 + 18}, \quad \text{and} \quad t_3 = \frac{(h-1)(2q_3 - 1)}{12(h-1)q_3 + 18}.$$

They satisfy the conditions $t_3 \leqslant -\frac{1}{2} + \beta$ and $t_1 \geqslant \frac{1}{2} - \beta$ only if $\beta \geqslant \frac{8+h}{18}$.

(ii) If $q_1 = 1$, then $t_1 = \frac{h-1}{6+12h}$. Substituting t_1 into the formula for t_3, we can solve for t_3 explicitly. It is easy to verify that t_3 increases in q_3 so the condition $t_3 \leqslant -\frac{1}{2} + \beta$ is the easiest to satisfy when $q_3 = 0$. Substituting $q_3 = 0$ into t_3, we show that $t_3 \leqslant -\frac{1}{2} + \beta$ is true only if $\beta \geqslant \frac{8+h}{18}$.

This completes the proof that $q_1 = q_3 = 1$ is the unique equilibrium when $\frac{2}{15} < \beta < \frac{8+h}{18}$. ■

Proposition 7. *When $\beta = \frac{2}{15}$, player 2 shares θ_r with player 3 and θ_l with player 1. He is indifferent between sharing θ_r with player 1 or not, and is indifferent between sharing θ_l with player 3 or not. Player 2's most preferred equilibrium is the full disclosure one.*

Proof of Proposition 7. When $\beta = \frac{2}{15}$, after θ_r realizes, the probability that player 1 votes for L (i.e., p_1^L) is strictly higher than $\frac{1}{5}$. Hence, player 2 prefers p_3^L to be as small as possible. Player 2 hence shares

θ_r with player 3, so p_3^L equals $\frac{1}{5}$. This implies that player 2 is indifferent between sharing θ_r with player 1 or not. We let q_1 be the probability with which player 2 shares θ_r with player 1. Similarly, player 2 shares θ_l with player 1 and is indifferent between sharing θ_l with player 3 or not. We let q_3 be the probability that player 2 shares θ_l with player 3. Given this sharing strategy, player 1's and player 3's voting thresholds if they see no signal are:

$$t_1 = \frac{(h-1)(q_1-1)}{h(6q_1+9)-6q_1+6}, \quad \text{and} \quad t_3 = \frac{(h-1)(q_3-1)}{h(6q_3+9)-6q_3+6}.$$

Player 2's equilibrium payoff is:

$$\frac{1}{180}\left(w\left(68-\frac{3}{5}h(300t_1t_3-40t_1+40t_3+33)\right)-180w^2-30\right),$$

which is maximized when q_1 and q_3 both equal one. ■

Proof of Proposition 3. We have argued in Lemma 4 that player 2 would like p_3^L to be as small as possible if θ_r has realized. If player 2 doesn't share θ_r with player 3, then it must be true that player 3 votes for L with zero probability (i.e., $t_3 \leqslant -\frac{1}{2}+\beta$). Similarly, if player 2 doesn't share θ_l with player 1, it must be true that $t_1 \geqslant \frac{1}{2}-\beta$. Given this no-disclosure strategy and the conditions $t_3, -t_1 \leqslant -\frac{1}{2}+\beta$, player 1's and player 3's voting thresholds after no signal are $t_3 = -t_1 = \frac{h-1}{18}$. These satisfy the conditions $t_3, -t_1 \leqslant -\frac{1}{2}+\beta$ if and only if $\beta \geqslant \frac{8+h}{18}$. ■

8.4 Proofs for sections 5 and 6

Proof of Lemma 5. Suppose that $\beta < 1/3$. If the probabilities with which player 2 and 3 vote for L are p_2^L and p_3^L, respectively, the expected payoff to player 1 is:

$$\mathrm{E}[u_1 \mid \theta=\theta_l, d, v] = p_2^L p_3^L\left(4\beta+\frac{2}{3}\right)w + \frac{2}{9}(3\beta+2)^2\left(p_2^L+p_3^L-2p_2^Lp_3^L\right)w + c'.$$

Since the highest possible threshold that player 2 or player 3 employs is $1/6$, both p_2^L and p_3^L are weakly smaller than $2/3$. The derivatives of $\mathrm{E}[u_1 \mid \theta=\theta_l, d, v]$ with respect to p_2^L and p_3^L are strictly positive:

$$\frac{\partial \mathrm{E}[u_1 \mid \theta=\theta_l, d, v]}{\partial p_2^L} = \frac{2}{9}\left((3\beta+2)^2 - p_3^L(6\beta(3\beta+1)+5)\right)w > 0,$$
$$\frac{\partial \mathrm{E}[u_1 \mid \theta=\theta_l, d, v]}{\partial p_3^L} = \frac{2}{9}\left((3\beta+2)^2 - p_2^L(6\beta(3\beta+1)+5)\right)w > 0.$$

Therefore, player 1 prefers p_2^L, p_3^L to be as large as possible. He fully discloses signal θ_l, so player 2 and 3 both use $1/6$ as their threshold after signal θ_l. If β lies in $[1/3, 2/3]$, player 1 fully discloses signal θ_l so as to decrease the probability that RR occurs. ■

Proof of Proposition 4. Suppose that if θ_r realizes the probabilities with which player 2 and 3 vote for L are p_2^L and p_3^L. The expected payoff of player 1 is:

$$\mathrm{E}[u_1 \mid \theta = \theta_r, d, v] = p_2^L p_3^L \left(4\beta - \frac{2}{3}\right) w + \frac{2}{9}(3\beta+1)^2 \left(p_2^L + p_3^L - 2p_2^L p_3^L\right) w + c''.$$

The derivatives of $\mathrm{E}[u_1 \mid \theta = \theta_r, d, v]$ with respect to p_2^L and p_3^L are:

$$\frac{\partial \mathrm{E}[u_1 \mid \theta = \theta_r, d, v]}{\partial p_2^L} = \frac{2}{9}\left(1 - 2p_3^L\right)(3\beta+1)^2 w + p_3^L \left(4\beta - \frac{2}{3}\right) w, \tag{2}$$

$$\frac{\partial \mathrm{E}[u_1 \mid \theta = \theta_r, d, v]}{\partial p_3^L} = \frac{2}{9}\left(1 - 2p_2^L\right)(3\beta+1)^2 w + p_2^L \left(4\beta - \frac{2}{3}\right) w. \tag{3}$$

If an uninformed player receives no signal, his voting threshold is in the range $(-1/6, 0]$, because signal θ_l is always shared as shown in Lemma 5. If an uninformed player observes θ_r, his voting threshold is $-1/6$. This implies that $p_2^L \leqslant 1/2$ and $p_3^L < 1/2$, for any $\beta \geqslant 1/6$. Hence, for any $\beta \geqslant 1/6$, $\frac{\partial \mathrm{E}[u_1|\theta=\theta_r,d,v]}{\partial p_2^L} > 0$, so player 1 doesn't share θ_r with player 2. This further implies that $p_2^L < 1/2$, so $\frac{\partial \mathrm{E}[u_1|\theta=\theta_r,d,v]}{\partial p_3^L} > 0$. As a result, player 1 would like p_3^L to be as large as possible.

We let $t_2, t_3 \in (-1/6, 0]$ be player 2's player 3's voting thresholds if they see no signal. Let q_3 be the probability with which player 1 shares θ_r with player 3. If $t_3 \leqslant -\frac{1}{2} + \beta$, then player 1 is indifferent between sharing θ_r with player 3 or not. If $t_3 > -\frac{1}{2} + \beta$, then player 1 strictly prefers not to share θ_r. As a result, $q_3 = 0$ is always a best response by player 1. Moreover, if $\beta \leqslant \frac{1}{3}$, then $-\frac{1}{2} + \beta \leqslant -\frac{1}{6} < t_3$, so $q_3 = 0$ is player 1's unique best response. Hence, we only need to focus on the region $\beta > \frac{1}{3}$. We next examine when it is possible that some $q_3 > 0$ is a best response by player 1.

Suppose that player 1 chooses some $q_3 > 0$. It must be true that $t_3 \leqslant -\frac{1}{2} + \beta$. Then we can solve t_2, t_3 as a function of q_3. For $\beta \geqslant 1/2$, we have:

$$\begin{aligned} t_2 &= \frac{(3\beta+1)(h-1)}{6(-3\beta(h-1)+5h+1)}, \\ t_3 &= -\frac{(h-1)(q_3-1)(2(6\beta-1)t_2-3)}{12t_2(6\beta(h-1)(q_3-1)-h(q_3-7)+q_3-1)-18((h-1)q_3+h+1)}. \end{aligned} \tag{4}$$

The value t_3 increases in q_3 so the inequality $t_3 \leqslant -\frac{1}{2}+\beta$ is easiest to satisfy when $q_3 \to 0$. Substituting $q_3 = 0$ into t_3, we obtain that $t_3 \leqslant -\frac{1}{2} + \beta$ holds for any $\beta \geqslant 1/2$ and $0 < h < 1$. For $\beta < 1/2$, we have:

$$\begin{aligned} t_2 &= \frac{(3\beta+1)(h-1)}{6(3\beta(h+1)+2h+1)}, \\ t_3 &= -\frac{(h-1)(q_3-1)(2(6\beta-1)t_2-3)}{12t_2(6\beta((h-1)q_3+h+1)+h(-q_3)+h+q_3-1)-18((h-1)q_3+h+1)}. \end{aligned} \tag{5}$$

The value t_3 increases in q_3 so the inequality $t_3 \leqslant -\frac{1}{2}+\beta$ is easiest to satisfy when $q_3 \to 0$. Substituting

$q_3 = 0$ into t_3, we obtain that $t_3 \leqslant -\frac{1}{2} + \beta$ is equivalent to:

$$h \leqslant \frac{81\beta^2 + 9\sqrt{36\beta^6 - 12\beta^5 - 11\beta^4 + 8\beta^3 + 3\beta^2 + 1} - 3\beta - 19}{54\beta^3 - 90\beta^2 - 12\beta + 35}.$$

The right-hand side strictly increases in β. It equals 0 when $\beta = \frac{1}{3}$, and equals 1 when $\beta = \frac{1}{2}$. Let $\hat{\beta}(h)$ be the inverse function of the right-hand side function. Hence, it is possible to have some $q_3 > 0$ as a best response by player 1 if and only if $\beta > \hat{\beta}(h)$.

We have argued that for $\beta > \hat{\beta}(h)$, it is possible to have some $q_3 > 0$ in equilibrium other than $q_3 = 0$. However, the value t_2 is independent of q_3 and the condition $t_3 \leqslant -\frac{1}{2} + \beta$ implies that, no matter what q_3 is, player 3 must vote for R for sure if no signal or signal θ_r realizes. This implies that, when multiple equilibria exist, they are all outcome equivalent to the equilibrium in which player 1 doesn't share θ_r with player 3. ■

Proof of Proposition 5. The signs of $\frac{\partial \mathrm{E}[u_1|\theta=\theta_l,d,v]}{\partial p_2^L}$ and $\frac{\partial \mathrm{E}[u_1|\theta=\theta_l,d,v]}{\partial p_3^L}$ are the same as those of $A - p_3^L$ and $A - p_2^L$, respectively, where $A := \frac{(3\beta+1)^2}{18\beta^2-6\beta+5}$ is strictly positive and increasing for $\beta \in [0, 1/6)$. Since the voting threshold by player 3 or 2 is at least $-1/6$, it is always true that $p_3^L \geqslant \frac{1}{3} - \beta$ and that $p_2^L \geqslant \frac{1}{3}$. We let β^* be the root of $A = \frac{1}{3} - \beta$, and β^{**} the root of $A = \frac{1}{3}$. We have $(\beta^*, \beta^{**}) \approx (0.05, 0.08)$. For any $\beta < \beta^*$, it holds that $A - p_3^L < 0$, so player 1 shares θ_r with player 2. We let β. For any $\beta < \beta^{**}$, it holds that $A - p_2^L < 0$, so player 1 shares θ_r with player 3. We are ready to state the equilibrium for any $\beta < \beta^{**}$.

(i) For any $\beta < \beta^*$, there is a unique equilibrium in which player 1 shares θ_r with players 2 and 3.

(ii) For any $\beta = \beta^*$, player 1 shares θ_r with player 3, so $p_3^L = \frac{1}{3} - \beta$. As a result, $\frac{\partial \mathrm{E}[u_1|\theta=\theta_l,d,v]}{\partial p_2^L} = 0$ so player 1 is indifferent between sharing θ_r with player 2 or not.

(iii) For any $\beta \in (\beta^*, \beta^{**})$, there is a unique equilibrium. Player 1 shares θ_r with player 3 so $p_3^L = \frac{1}{3} - \beta$. As a result, $\frac{\partial \mathrm{E}[u_1|\theta=\theta_l,d,v]}{\partial p_2^L} = 0$ so player 1 does not share θ_r with player 2.

From now on, we focus on the parameter region in which $\beta \in \left[\beta^{**}, \frac{1}{6}\right)$. We let q_2, q_3 be the probabilities with which player 1 shares signal θ_r with player 2 and 3. We let t_2, t_3 be the voting thresholds by player 2 and 3 if they see no signal. We first argue that there exists an equilibrium in which $q_2 = 0$, so it must be true that $\frac{\partial \mathrm{E}[u_1|\theta=\theta_l,d,v]}{\partial p_2^L} \geqslant 0$.

(i) If $q_3 = 1$ in the equilibrium, then t_2 and t_3 are given by:

$$t_2 = \frac{(9\beta^2 + 2)(h - 1)}{3(18\beta^2(h + 1) + 5h + 4)} \quad \text{and} \quad t_3 = 0.$$

It is a best response to choose $q_3 = 1$ if and only if $\frac{\partial \mathrm{E}[u_1|\theta=\theta_l,d,v]}{\partial p_3^L} \leqslant 0$. The equilibrium conditions $\frac{\partial \mathrm{E}[u_1|\theta=\theta_l,d,v]}{\partial p_2^L} \geqslant 0$ and $\frac{\partial \mathrm{E}[u_1|\theta=\theta_l,d,v]}{\partial p_3^L} \leqslant 0$ are satisfied if and only if:

$$h \geqslant h_1(\beta) := \frac{4\left(9\beta^2+2\right)\left(3\beta(3\beta+8)-2\right)}{6\beta(18(\beta-3)\beta(3\beta-1)-49)+65}.$$

(ii) If $q_3 \in (0,1)$ in the equilibrium, then $\frac{\partial \mathrm{E}[u_1|\theta=\theta_l,d,v]}{\partial p_3^L} = 0$ must hold. This implies that:

$$t_2 = \frac{3(6\beta-1)}{2(6\beta(3\beta-1)+5)}.$$

Substituting this value into the formula for t_2 and t_3, we can solve t_3, q_3 as a function of h, β. We are left with the constraints that $\frac{\partial \mathrm{E}[u_1|\theta=\theta_l,d,v]}{\partial p_2^L} \geqslant 0$ and that $q_3 \in (0,1)$. They are satisfied if and only if:

$$h_2(\beta) < h < h_1(\beta),$$

where $h_2(\beta)$ solves the equation that $q_3 = 0$.

(iii) If $q_3 = 0$ in the equilibrium, then $\frac{\partial \mathrm{E}[u_1|\theta=\theta_l,d,v]}{\partial p_3^L} \geqslant 0$ must hold. We can solve t_2 and t_3 as a function of h, β. Substituting t_2, t_3 into the equilibrium conditions $\frac{\partial \mathrm{E}[u_1|\theta=\theta_l,d,v]}{\partial p_2^L} \geqslant 0$ and $\frac{\partial \mathrm{E}[u_1|\theta=\theta_l,d,v]}{\partial p_3^L} \geqslant 0$, we show that they are satisfied if and only if:

$$h \leqslant h_2(\beta).$$

We next examine the equilibrium in which $q_2 = 1$. This implies that $p_2^L = 1/3$.

(i) For any $\beta > \beta^{**}$, we have $A - 1/3 > 0$. This implies that $q_3 = 0$ for any $\beta > \beta^{**}$. Then t_2 and t_3 are given by:

$$t_2 = 0 \quad \text{and} \quad t_3 = -\frac{(3\beta+4)(h-1)}{6(3\beta(h-1)-5h-4)}.$$

Substituting t_2, t_3 into the equilibrium conditions $\frac{\partial \mathrm{E}[u_1|\theta=\theta_l,d,v]}{\partial p_2^L} \leqslant 0 \leqslant \frac{\partial \mathrm{E}[u_1|\theta=\theta_l,d,v]}{\partial p_3^L}$, we obtain that these conditions are satisfied if and only if:

$$h \geqslant \frac{2(3\beta+4)(3\beta(3\beta(6\beta-1)+13)-2)}{(6\beta-1)(6\beta(3\beta(3\beta-5)+11)-65)}.$$

The right-hand side increases in β for $\beta \in [\beta^{**}, 1/6)$, and equals $\frac{4}{13}$ when $\beta = \beta^{**}$. Therefore, it is possible to have $q_2 = 1$ and $q_3 = 0$ in an equilibrium only if $h \geqslant \frac{4}{13}$.

(ii) For $\beta = \beta^{**}$, the argument in the previous step shows that it is possible to have an equilibrium with $q_3 = 0$ for any $h \geqslant \frac{4}{13}$. It is not possible to have $q_3 = 1$ in an equilibrium since p_3^L would be

$1/3-\beta$ and $\frac{\partial \mathrm{E}[u_1|\theta=\theta_l,d,v]}{\partial p_2^L}$ would be strictly negative. It is possible to have $q_3 \in (0,1)$ if and only if:

$$h\left(\left(15-8\sqrt{2}\right)q_3+26\sqrt{2}-39\right)+4\left(2\sqrt{2}-3\right)(q_3-1)\leq 0.$$

This condition holds for some $q_3 \in (0,1)$ only if $h \geqslant \frac{4}{13}$.

To sum, it is possible to have $q_2 = 1$ in an equilibrium only if $h \geqslant \frac{4}{13}$.

Lastly, we examine the equilibrium in which $q_2 \in (0,1)$. First, it is not possible to have $q_3 = 1$. Suppose not. Then $p_3^L = \frac{1}{3} - \beta$. For any $\beta \geqslant \beta^{**}$, $\frac{\partial \mathrm{E}[u_1|\theta=\theta_l,d,v]}{\partial p_2^L} > 0$ so q_2 should be zero. This leads to a contradiction. Therefore, either $q_3 = 0$ or $q_3 \in (0,1)$.

(i) Suppose that $q_3 = 0$ in equilibrium. Given that $q_2 \in (0,1)$, $\frac{\partial \mathrm{E}[u_1|\theta=\theta_l,d,v]}{\partial p_2^L} = 0$ must hold. This implies that:

$$t_3 = \beta + \frac{3(6\beta-1)}{2(6\beta(3\beta-1)+5)}.$$

Substituting this value into the formula for t_2 and t_3, we can solve t_2, q_2 as a function of h, β. We are left with the constraints that $\frac{\partial \mathrm{E}[u_1|\theta=\theta_l,d,v]}{\partial p_3^L} \geqslant 0$ and that $q_2 \in (0,1)$. They are satisfied if and only if

$$\frac{2(3\beta-2)(3\beta+1)(\beta(\beta(6\beta(6\beta+7)+1)+39)-2)}{2\beta(\beta(3\beta(9\beta(2\beta(6\beta-13)+5)-74)-146)+138)-37} \geqslant h$$
$$> \frac{2(3\beta+4)(3\beta(3\beta(6\beta-1)+13)-2)}{(6\beta-1)(6\beta(3\beta(3\beta-5)+11)-65)}.$$

The left-hand side follows from the constraint that $\frac{\partial \mathrm{E}[u_1|\theta=\theta_l,d,v]}{\partial p_3^L} \geqslant 0$, and the right-hand side follows from the constraint that $q_2 < 1$.

(ii) Suppose that $q_3 \in (0,1)$ in equilibrium. We let $(a_1, a_2, a_3, 1-a_1-a_2-a_3)$ denote player 1's disclosure strategy, where a_1, a_2, a_3 correspond to the probability to share with both players 2 and 3, that to share only with player 2, and that to share only with player 3, respectively. Then, $q_2 = a_1 + a_2$ and $q_3 = a_1 + a_3$. There are two cases. First, player 1 chooses whether to share with player 2 and 3 independently, so $a_1 = q_2 q_3, a_2 = q_2(1-q_3), a_3 = (1-q_2)q_3$. In the second case, this condition does hold.

We now consider the first case. The conditions $\frac{\partial \mathrm{E}[u_1|\theta=\theta_l,d,v]}{\partial p_2^L} = \frac{\partial \mathrm{E}[u_1|\theta=\theta_l,d,v]}{\partial p_3^L} = 0$ imply that:

$$t_2 = \frac{6\beta(-3\beta q_2+q_2-9)-5q_2+9}{6(6\beta(3\beta-1)+5)(q_2-1)}, \quad \text{and} \quad t_3 = \frac{2-3\beta(3\beta(6\beta-1)+13)}{3(6\beta(3\beta-1)+5)(q_3-1)} - \frac{1}{6}.$$

Substituting these values into the formula for t_2 and t_3, we can solve q_2, q_3 as a function of h, β. The conditions $0 < q_2, q_3 < 1$ are satisfied if and only if:

$$h > \frac{2(3\beta-2)(3\beta+1)(\beta(\beta(6\beta(6\beta+7)+1)+39)-2)}{2\beta(\beta(3\beta(9\beta(2\beta(6\beta-13)+5)-74)-146)+138)-37}. \tag{6}$$

The right-hand side increases in β and equals $\frac{4}{13}$ when $\beta = \beta^{**}$. For the second case, the same condition 6 is required for such an equilibrium to exist.

To sum, it is possible to have $q_2 \in (0,1)$ in equilibrium only if $h > \frac{4}{13}$. This shows that for any $\beta \in [\beta^{**}, 1/6)$ and $h < \frac{4}{13}$, there is an unique equilibrium in which $q_2 = 0$. ■

Proof of Proposition 6. We first consider the case in which player 2 is informed. For $\beta \geqslant \frac{h+8}{18}$, player 2's most preferred equilibrium is the no disclosure one characterized in Proposition 3. Player 1 and 3 vote for L and R, respectively, with probability one if they see no signal. This is as if player 2 were choosing the policy by himself. The social welfare equals:

$$\frac{1}{18}(w(-5h-54w+14)-9)-2\beta^2. \tag{7}$$

We next consider the case in which player 1 is informed. When $\beta \geqslant \frac{h+8}{18}$, there is essentially a unique equilibrium characterized in Proposition 4 in which player 1 withholds θ_r and shares θ_l. When $\beta \geqslant 1/2$, player 2's and 3's voting thresholds if they see no signal are given by (4) with q_3 being zero. When $\beta < 1/2$, their voting thresholds if they see no signal are given by (5) with q_3 being zero. Substituting these voting thresholds into the social welfare, we obtain that for $\beta \geqslant 1/2$ the social welfare is:

$$a_1 t_2^2 + a_2 t_2 + a_3,$$

where t_2 is given by (4) and

$$\begin{aligned}
a_1 =& \frac{1}{3} w(3\beta(h-1)-5h-1), \\
a_2 =& \frac{1}{9}(3\beta+1)^2(h-1)w, \\
a_3 =& \frac{1}{108}\left(-54\left(4\beta^2+1\right)+w(9\beta(10\beta+1)(h-1)-61h+115)-324w^2\right).
\end{aligned}$$

For $\beta < 1/2$ the social welfare is:

$$b_1 t_2^2 + b_2 t_2 + b_3,$$

where t_2 is given by (5) and

$$b_1 = -\frac{1}{3}w(3\beta(h+1)+2h+1),$$
$$b_2 = -\frac{1}{18}w(6\beta(3\beta(h+1)-2h+2)-11h+2),$$
$$b_3 = \frac{1}{108}\left(-18\beta^2((h+5)w+12)+9\beta(7h-1)w+w(-61h-324w+115)-54\right).$$

It is easy to verify that for any $\beta < 1/2$, the social welfare when player 1 is informed is strictly higher than (7). For any $\beta \geqslant 1/2$, it is strictly higher than (7) if and only if

$$h < \bar{h}(\beta) = \frac{4(3\beta-2)(3\beta+1)(3\beta+2)}{(3\beta-7)(6\beta(6\beta+1)-11)}.$$

■

Proposition 8 (Bias-dependent sharing). *Suppose that $\beta \geqslant (h+8)/18$ and an informed player chooses his disclosure strategy after learning his bias. If player 2 is informed, his most preferred equilibrium is the no-disclosure one as in Proposition 3. If player 1 is informed, the equilibrium is outcome-equivalent to that in Proposition 4. Hence, Proposition 6 still holds.*

Proof of Proposition 8. We first argue that the strategy profile in Proposition 3 is an equilibrium even if player 2 learns his bias before sharing. First, player 2 gets the highest possible payoff, so he is best responding. Second, given player 2's withholding information, uninformed players are best responding, as shown in the proof of Proposition 3.

We next characterize the equilibrium when player 1 is informed. We let $t_2, t_3 \in (-1/6, 1/6)$ be player 2's and 3's voting thresholds if they see no signal. If player 1 sees θ_l, he prefers L for all bias realization. Therefore, he shares θ_l for sure. If player 1 sees θ_r, he prefers R if and only if $b_1 > -\frac{1}{6}$, so he shares θ_r if and only if $b_1 > -\frac{1}{6}$. Given this sharing behavior, we can solve for t_2, t_3. For $\beta \geqslant \frac{1}{2}$, we have:

$$t_2 = \frac{(3\beta+1)(h-1)}{6(-3\beta(h-1)+5h+1)},$$
$$t_3 = -\frac{(3\beta+1)(h-1)(3\beta(h-1)-8h-1)}{6(3\beta(h-1)-17h-1)(3\beta(h-1)-2h-1)}. \tag{8}$$

For $\beta < \frac{1}{2}$, we have:

$$t_2 = \frac{(3\beta+1)(h-1)}{6(3\beta(h+1)+2h+1)},$$
$$t_3 = -\frac{(2\beta+1)(h-1)(6\beta(2h+1)+7h+2)}{6\left(12\beta^2(h-1)(4h+1)-2\beta(h(h+30)+5)-17h(h+1)-2\right)}. \tag{9}$$

Note that the t_2 in (8) and (9) is the same as that in (4) and (5), respectively. Moreover, the values of t_3 in (4), (5), (8), and (9) are smaller than $-\frac{1}{2} + \beta$.

Lastly, we argue that the equilibrium is outcome-equivalent to that in Proposition 4. If θ_l realizes, it is shared and all three players use $\frac{1}{6}$ as their voting threshold. This is the same as before. If $\varnothing$ realizes, player 3 votes for R for sure, and player 1's and 2's voting thresholds are 0 and t_2, which are the same as before. If θ_r realizes, when player 1 votes for R, the policy will be R since player 3 votes for R for sure. When player 1 votes for L and hence withholds θ_r, then player 2 is the pivotal player and his voting threshold t_2 is the same as before. Therefore, this equilibrium is outcome equivalent to the one in Proposition 4. ■

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