Modes of Persuasion Toward Unanimous Consent

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Features

- A sender promotes a project to a group of voters.
- Voters decide collectively on approval through unanimity rule.
- Voters vary in:
  - their payoff states, which are positively correlated,
  - their thresholds of doubt.
- Before states are realized, the sender commits to an information policy:
  - general policies: information conditioned on the entire state profile.
  - individual policies: information conditioned only on individual payoff state.

Motivating examples

Example 1:

- An industry representative persuades multiple regulators to approve a project.
- Each regulator is concerned about different yet correlated aspects of the project.
- An approval entails the endorsement of all regulators.
- The representative sets an institutionalized standard on the amount of information to be provided to each regulator.

Example 2:

- Within organizations, new ideas are born in the R&D department.
- These ideas are required to find broad support from other departments with varied interests.
- The R&D department designs tests to persuade other departments.

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- Which voters have better information? Will any voter learn her state perfectly?
- Which voters obtain positive payoffs? Which voters are made indifferent?
- What is the optimal policy if we move away from unanimity?

Related work

- Information design:
  - One agent: Rayo and Segal (2010), Kamenica and Gentzkow (2011).
  - Multiple agents: Bergemann and Morris (2016a, 2016b, 2017), Taneva (2016), Mathevet, Perego and Taneva (2016), Arieli and Babichenko (2016).
  - Voting game: Caillaud and Tirole (2007), Alonso and Câmara (2016), Schnakenberg (2015), Wang (2015), Chan, Gupta, Li and Wang (2016).
- Information aggregation/acquisition in voting:
  - Information aggregation: Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1997).
  - Information acquisition: Li (2001), Persico (2004), Gerardi and Yariv (2008), Gershkov and Szentes (2009).

Roadmap

1. Model
2. General persuasion
3. Individual persuasion

Model: Players, states and payoffs

- One Sender (he) and n voters $\{R_i\}_{i=1}^{n}$ (she).

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- Nature draws θ according to f. f is common knowledge.
- The realized θ is unobservable to all players.

Model: Players, states and payoffs

- We assume that f is exchangeable.

  For every θ and every permutation ρ of $(1, ..., n)$:

$$f(\theta_1, ..., \theta_n) = f\left(\theta_{\rho(1)}, ..., \theta_{\rho(n)}\right).$$

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- We assume that f is affiliated.

  For any θ, θ',

$$f(\theta \vee \theta') f(\theta \wedge \theta') \geqslant f(\theta) f(\theta').$$

$\theta \vee \theta'$ denote the component-wise maximum state profile.

$\theta \wedge \theta'$ denote the component-wise minimum state profile.

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If the project is rejected, all players receive a payoff of 0.

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- Environment: f, $\{\ell_i\}_{i=1}^n$.

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- We allow for any policy that is the limit of full-support policies.
- We solve for the Sender-optimal policy.

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Proposition:

Suppose voters' states are perfectly correlated. The unique optimal policy is

$$\pi(\hat{d}^a|H..H) = 1, \quad \pi(\hat{d}^a|L..L) = \frac{f(H..H)}{f(L..L)}\frac{1}{\ell_1}.$$

Only R_1's IC binds.

Imperfect correlation:

Under unanimity, a binding IC is equivalent to a zero payoff.

Imperfect correlation: Strictest voters' ICs bind

Proposition:

In any optimal policy, the IC constraints for a subgroup of the strictest voters bind, i.e. IC binds for $i \in \{1, ..., i'\}$ for some $i' \geqslant 1$.

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- Examine the dual of the linear programming problem.

The dual problem

- Sender's problem is:

$$\max_{\pi(\hat{d}^a|\theta)\geqslant 0} \sum_{\theta\in\{H,L\}^n} f(\theta)\pi(\hat{d}^a|\theta)$$

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$$\min_{\gamma_\theta\geqslant 0,\mu_i\geqslant 0} \sum_{\theta\in\{H,L\}^n} \gamma_\theta,$$

$$\text{s.t. } \gamma_\theta \geqslant f(\theta)\left(1 + \sum_{i:\theta_i=H} \mu_i - \sum_{i:\theta_i=L} \mu_i\ell_i\right), \ \forall\theta.$$

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- Examine the dual of the linear programming problem.
- Think of each IC as a "resource constraint."
- Granting surplus to a tough voter is more expensive than to a lenient one.

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$$\Pr(\theta_i = H | R_{-i} \text{ approve}) = \frac{\sum_{\theta \in \Theta_i^H} f(\theta) \prod_{j \neq i} \pi_j(\theta_j)}{\sum_{\theta \in \{H,L\}^n} f(\theta) \prod_{j \neq i} \pi_j(\theta_j)}.$$

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- R_i obeys an approval recommendation if:

$$\Pr(\theta_i = H | R_{-i} \text{ approve}) \pi_i(H) - \ell_i \Pr(\theta_i = L | R_{-i} \text{ approve}) \pi_i(L) \geqslant 0.$$

Individual persuasion

- R_i's approval IC is easily written as:

$$\Pr(\theta_i = H | R_{-i} \text{ approve}) \geqslant \frac{\ell_i \pi_i(L)}{\ell_i \pi_i(L) + \pi_i(H)}.$$

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- Sender chooses $(\pi_i(H), \pi_i(L))_{i=1}^n$ to maximize his payoff:

$$\sum_\theta f(\theta) \prod_i \pi_i(\theta_i),$$

subject to the approval ICs.

Optimal policy under perfect correlation

Proposition:

Suppose voters' states are perfectly correlated. Any optimal policy is of the form:

$$\pi_i(H) = 1 \text{ for all } i,$$

$$(\pi_1(L), ..., \pi_n(L)) \in [0,1]^n \text{ such that } \prod_i \pi_i(L) = \frac{f(H..H)}{f(L..L)} \frac{1}{\ell_1}.$$

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- The distribution over state profiles is:

$$f(HHH) = \frac{9}{50},\ f(HHL) = \frac{1}{20},\ f(HLL) = \frac{2}{25},\ f(LLL) = \frac{43}{100}.$$

- The thresholds of doubt are:

$$(\ell_1, \ell_2, \ell_3) = (41, 40, 39).$$

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- The optimal policy is:

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- R_3 rubber-stamps the others' approval decisions, obtaining a positive payoff.

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- R_1 learns her state perfectly. The only slack IC is R_1's IC.

Monotonicity of persuasion

Proposition:

There exists an optimal policy in which the approval probability in low state weakly decreases in the threshold of doubt:

$$\pi_i(L) \leqslant \pi_{i+1}(L) \text{ for all } i \in \{1, ..., n-1\}.$$

Moreover, in any optimal policy in which R_i's IC binds, $\pi_i(L) > \pi_j(L)$ for all $j \in \{1, 2, ..., i-1\}$.

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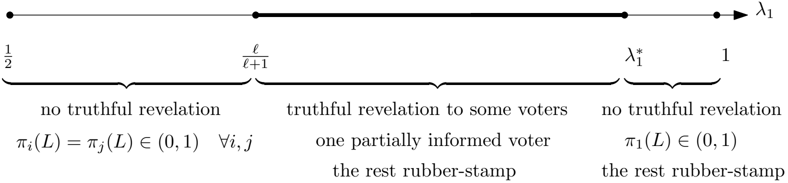
- More demanding voters' policies are more informative.
- Voters are divided into three subgroups:

perfectly informed	partially manipulated	rubber-stampers
$\pi_i(L) = 0$	$\pi_i(L) \in (0, 1)$	$\pi_i(L) = 1$

When do some voters learn their states perfectly?

- $\omega \in \{G, B\}$ such that $\Pr(\omega = G) = p_0$.
- $\Pr(H|G) = \Pr(L|B) = \lambda_1 \in [1/2, 1]$.
- $\ell_i = \ell$ for all i.



Concluding remarks

- We explore group persuasion in the context of unanimity rule, affiliated payoff states and heterogeneous thresholds of doubt.
- We compare two modes of persuasion: general and individual persuasion.
- General persuasion makes the strictest voters indifferent. Individual persuasion divides the group into perfectly-informed voters, partially-informed voters, and rubber-stampers.
- Under non-unanimous rules, general persuasion leads to certain approval, while individual persuasion does not.
- Future work:
  - Non-unanimous rules under individual persuasion.
  - Communication among voters.
  - Sequential persuasion.

Thank you!

Independent states under general persuasion

- Three voters' states are drawn independently.
- Each voter's state is H with probability $19/20$.
- The threshold profile is $\{\ell_1, \ell_2, \ell_3\} = \{41, 40, 20\}$.
- One optimal policy is

$$\pi(\hat{d}^a|\theta) = 1 \text{ for } \theta \in \{HHH, HHL\},$$

$$\pi(\hat{d}^a|LHH) = \frac{820}{1639}, \quad \pi(\hat{d}^a|HLH) = \frac{840}{1639},$$

$$\pi(\hat{d}^a|\theta) = 0 \text{ for } \theta \in \{HLL, LLH, LHL, LLL\}.$$

- R_1's and R_2's IC constraints bind. R_3's does not.

Back

$k < n$ votes: Independent general persuasion

- Each voter's recommendation is drawn independently conditional on the state profile.
- We can construct a certain approval policy which is the limit of a sequence of full-support policies.
- For state profiles with k high-state voters, these voters are recommended to approve. The low-state voters are recommended to reject.
- In all other state profiles, all voters are recommended to approve.
- When R_i receives an approval recommendation and conditions on being pivotal, she believes that her state is high.

$k < n$ votes: Certain approval under IGP

Proposition:

Under independent general persuasion, Sender's payoff is one.

- This strengthens the previous result by showing that Sender achieves a certain approval even when constrained to independent general persuasion.
- The voters impose no check on Sender if Sender is allowed to condition on the entire state profile.

Roadmap

1. Model
2. General persuasion
3. Individual persuasion
4. Extensions:
   - Public and sequential persuasion
   - Non-unanimous rule

Non-unanimous decision rule

- The project is approved if at least $k < n$ voters approve.

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- A trivial policy achieves certain approval:
  - Recommend that all voters approve all the time.
  - This policy relies crucially on the failure to be pivotal.
  - Each voter is exactly indifferent.

Non-unanimous decision rule

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- A trivial policy achieves certain approval:
  - Recommend that all voters approve all the time.
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  - Each voter is exactly indifferent.
- We only allow for any policy that is the limit of a sequence of full-support incentive-compatible policies.

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- Suppose that $n = 2$, $k = 1$.

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	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$
HH	ε^2	ε	ε	$1 - 2\varepsilon - \varepsilon^2$
HL	ε^2	ε^2	ε^2	$1 - 3\varepsilon^2$
LH	ε^2	ε^2	ε^2	$1 - 3\varepsilon^2$
LL	ε	ε^2	ε^2	$1 - \varepsilon - 2\varepsilon^2$

General policy for $n = 2, k = 1$

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- Sender recommends k to approve much more frequently when every voter's state is H than under any other state profile.

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- Sender recommends $k - 1$ to approve much more frequently when every voter's state is L than under any other state profile.

- For each state profile, the remaining probability is mainly allocated to the unanimous approval recommendation.

$k < n$ votes: Certain approval under general persuasion

Proposition:

Under general persuasion, Sender's payoff is one.

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Under general persuasion, Sender's payoff is one.

- The certain approval result does not rely on the failure to be pivotal or the voters being exactly indifferent.

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- Each voter approves more frequently under high state than under low state.

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- Any voter in this coalition does not become more optimistic about her state being H conditional on an approval recommendation and being pivotal.

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Proposition:

Under individual persuasion, Sender's payoff is strictly below one.

$k < n$ votes: No certain approval under individual persuasion

- Each voter approves more frequently under high state than under low state.

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- Each voter approves more frequently under high state than under low state.
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- Each voter approves more frequently under high state than under low state.
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Proposition:

Under individual persuasion, each voter's payoff is strictly higher than that under general persuasion.