

Information Transmission and Voting ^{*}

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Abstract

I analyze an individual's incentive to disclose hard information in the context of committee voting. A committee consists of three members, one left-leaning, one right-leaning, and the third ex ante unbiased. They decide on whether to pursue a left or a right policy by majority rule. One of them has private information about the merits of policies and can privately send verifiable messages to the others. If the informed member is unbiased, he withholds information to neutralize his opponents' votes when preferences are diverse. If the informed member is biased, then the others can better infer his information, knowing that any information favoring his agenda will be shared. In the latter case, because more information is effectively shared, higher social welfare results.

Keywords: committee decision-making, voting, verifiable disclosure, sender-receiver game.

JEL Codes: D71, D72, D83.

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1 Introduction

1.1 Motivation

In many economic and political interactions, collective decisions are made by small committees through voting. For instance, a jury needs to decide whether a defendant is guilty or innocent. A board of policy makers has to approve or reject a proposal. More often than not, committee members make their voting decisions under uncertainty. They may be uncertain about the underlying state. They may not know others' preferences precisely. One of them may know the state better than the others. In such situations, if the better-informed member can send costless, verifiable messages to the others, will he disclose his private information voluntarily? If yes, with whom will he share and to what extent?

As an example, consider a recruiting committee which has to pick one of two candidates. Committee members' preferences diverge in regard to what qualities suit the position. In addition, one member might know the candidates' quality better than others. As another example, consider a firm deciding which of the two projects to invest in. Different divisions have diverse interests in the projects, due to such factors as career concerns or effort aversion. One division might hold verifiable information about the viability of each project. Will the better-informed member or division disclose the information before votes are cast?

To address these questions, I study a setting in which a three-member committee is to select one of two policies—a left policy or a right one—by majority rule. Members are uncertain about a state variable that is payoff-relevant. Each member's payoff depends on the state, the winning policy, and his preference type. One member, whose identity is commonly known, might have private information (a signal) about the state and can share this information with the other two members in private. Communication takes the form of *verifiable disclosure*: the signal is costless to communicate and ex post verifiable. Hence, the informed member's strategic choice is whether to share his private information and, if so, with whom. Finally, committee members vote simultaneously.

Committee members have diverse interests, and do not know the others' exact preferences. To model conflicts of interests and preference uncertainty, I assume that committee members' preference types are drawn from uniform distributions with different supports, and I refer to the mean of a member's preference distribution as his ideology. Preference distributions are commonly known, yet each individual privately learns his preference type before he votes. This formulation is consistent with the observation that policy makers' ideologies are publicly known, but their stances on any particular issue are private information. A stylized committee consists of three members: Left, Central, and Right.

Ex ante, Left is biased toward the left-leaning policy and Right toward the right-leaning policy, while Central is unbiased.

1.2 A preview of results

I explore which member, if informed, is more willing to share hard information and what the welfare implications are. In particular, I compare the situation in which Central is informed with the situation in which a peripheral member is informed. The term “a peripheral member” refers to a member who, ex ante, is leaning toward one of the policies, so both Left and Right are peripheral members. I show that more information is effectively shared when it is a peripheral member who has the private information. This is because Central’s incentive to be pivotal deters him from sharing when the committee becomes sufficiently polarized. However, if a peripheral member is informed, then the others can better infer his information, knowing that any information favoring his agenda will be shared. My analysis leads to the somewhat surprising conclusion that social welfare—measured as the sum of all members’ expected payoffs—is higher when an ex ante biased member is informed than when an ex ante neutral member is informed.

To obtain some intuition, let me illustrate Central’s incentives to share or to hide information. Central has an incentive to share information with his opponents in order to coordinate their votes. With coordinated votes, when Central obtains information favoring one policy, that policy is more likely to be chosen by the majority rule. On the other hand, Central wants to neutralize the votes of his opponents in order to be pivotal. Being pivotal allows a member to adjust his vote according to his preference type, and is valuable in the setting in which members’ preference types are subject to idiosyncratic shocks. When conflicts of interests are sufficiently large, Central chooses not to share any information at all. His opponents always vote differently from each other, which makes Central a de facto dictator. This analysis prompts one intuitive conclusion: the more polarized the committee is, the less information Central will share.

It is worth emphasizing that this nondisclosure equilibrium arises even when the peripheral members are not partisans yet. They would vote informatively if Central’s information were public. However, Central’s withholding information in the voting context polarizes Left and Right even further, thereby placing Central in an advantageous position. To illustrate the intuition, take the perspective of Left. He conditions his analysis on his vote being pivotal, in which Right (most likely) votes for the right-leaning policy while Central votes for the left-leaning policy, making Left’s vote decisive. Left would then infer from Central’s vote that the hard information favors the left-leaning policy. Therefore Left is more likely to cast a vote in favor of the left-leaning policy. As a result, with Left and Right

further polarized, Central withholds information and dictates the voting outcome.

A peripheral member is, however, less likely to be pivotal. In addition, a peripheral member, if informed, is inclined to share information that promotes his *ex ante* preferred policy, enabling the uninformed members to better infer the signal that a peripheral member obtains. Since more information is transmitted within the committee, the voting outcome better reflects members' preferences, and thus, a higher welfare level results.

1.3 Related literature

This paper is closely related to the literature on communication and voting in committees, which focuses on the role of communication prior to voting in aggregating private information/preferences and in altering election outcomes. In most of this literature, communication is public and modeled as a simultaneous cheap-talk stage. Coughlan (2000) examines the situation in which members have common values¹ and shows that cheap-talk communication can restore *full-information equivalence*—the voting outcome is the same as if all private information were publicly known—under a unanimous voting rule. Coughlan (2000) and Austen-Smith and Feddersen (2006) show that this result relies on the assumption of common values. If conflicts of interests prevail, however, communication cannot guarantee full-information equivalence regardless of voting rules. In other words, no matter what the voting rules are, not all members vote informatively. Although this paper also focuses on the relationship between communication and voting in committees, it differs from the current literature in three fundamental ways.

Asymmetry. In the papers mentioned above, each committee member receives a private signal regarding the fundamentals. Yet, in many collective-decision organizations, information is unevenly distributed among members. Frequently, some committee members are better informed of the fundamentals. To model this *asymmetry* explicitly, I assume that only one member receives a noisy private signal while the other two do not. Gilligan and Krehbiel (1989) and Krishna and Morgan (2001) also explicitly model asymmetric information among committee members. Yet, they model collective decision-making as a sender-receiver game: two committee members have superior information and make proposals to the third member, who chooses a policy either from between the proposals or from a different pool. In my model, one informed member decides whether to share information with the other two before they vote. As it turns out, the better-informed member's incentives to share information are quite different in the voting context than in the context of these two papers.

¹If committee members' preferences over two alternatives are identical whenever they have the same information, they share common values.

Verifiability. The second way in which my model is different is that the signal is hard evidence. This corresponds to scenarios in which verifiable information is available and disclosure is discretionary. It is frequently observed that talk is not “cheap.” More often than not, communication is less about what messages to send and more about whether to speak out or not. In this regard, my paper adds new insights to the literature on sender-receiver games with hard evidence. As argued by Grossman (1981), Milgrom (1981), and Seidmann and Winter (1997), if a seller has more favorable information than buyers do about his product, he would always reveal it to buyers if the information is verifiable and revelation costless. This information revelation unravels and buyers infer from no disclosure on the seller’s side that he has the lowest quality. It follows that the seller will always disclose information about product quality and mandated disclosure is not necessary. In sender-receiver games with hard evidence, unraveling is a robust result. In the voting context, however, full revelation is not always an equilibrium outcome.

Private communication. Last but not least, I assume a *private communication* setting instead of public debate about alternatives. There are many ways in which communication might be organized. In particular, subgroups may conduct private meetings in which meaningful communication takes place. Private communication allows the informed member to share with one fellow member while keeping the other in the dark. Hence, I am able to examine with whom the informed member shares his private information. In this aspect, my paper is related to Farrell and Gibbons (1989) and Goltsman and Pavlov (2008). Both papers study a cheap-talk communication game with one sender and two receivers, and examine how the presence of one receiver affects the sender’s communication with the other receiver. In the context of voting, the informed member’s incentives to share with one uninformed member are naturally affected by the presence of the other uninformed member. I will show that the underlying forces in a voting game are different.

The paper is organized as follows: The model and the solution concept are introduced in Section 2. Section 3 illustrates the preliminary results. In Section 4 and 5, I present the equilibria for when an ex ante neutral member is informed and those for when an ex ante biased member is informed. I compare the two scenarios and perform a welfare analysis in Section 6. Section 7 concludes.

2 The setup

2.1 The model

Environment. A committee consists of three players, $\ell \in N = \{1, 2, 3\}$. They select a policy, $p \in \{L, R\}$, by majority rule. Let L and R be $-w < 0$ and $w > 0$, respectively. No restriction

is imposed on the size of w since the distance between two policies does not affect the analysis, as I show later. If policy p is selected, then the outcome is $p - s$, where s is an unknown state drawn uniformly from $[-1/2, 1/2]$. This unknown state represents players' uncertainty regarding the effect of the candidate policies.

For a fixed policy p , its outcome $p - s$ is also uniformly distributed. The supports of $L - s$ and $R - s$ are $[-w - 1/2, -w + 1/2]$ and $[w - 1/2, w + 1/2]$, which are symmetric with respect to the origin. Figure 1 shows the density functions of outcome $L - s$ and $R - s$, when w equals $1/5$.

Preferences. Player ℓ prefers policies that minimize the distance between his bias b_ℓ and outcome $p - s$. His realized payoff is given by the quadratic loss function:

$$u_\ell = -(b_\ell - (p - s))^2 = -(p - (b_\ell + s))^2.$$

The higher the state is or the higher the bias is, the more likely it is that player ℓ prefers R . Player ℓ 's bias b_ℓ is drawn uniformly from support B_ℓ , independently across players and of the state. The support B_ℓ is parameterized by a positive constant β , as follows:

$$(B_1, B_2, B_3) = \left(\left[-\frac{1}{2} - \beta, \frac{1}{2} - \beta \right], \left[-\frac{1}{2}, \frac{1}{2} \right], \left[-\frac{1}{2} + \beta, \frac{1}{2} + \beta \right] \right).$$

The support of player 2's bias distribution is symmetric around the origin. The supports of player 1's and player 3's bias distributions are symmetric around $-\beta$ and β , respectively. Here, β captures the ex ante misalignment of players' preferences (or how polarized the committee is). Let G_ℓ and g_ℓ designate the cdf and pdf of b_ℓ .

The bias distributions are common knowledge, but each player learns his bias privately before voting. Figure 2 shows the pdf of the bias distributions when β equals $3/10$. I focus on the parameter region $\beta \in (0, 2/3)$. It becomes clear later that the analysis is trivial for $\beta \geq 2/3$ since player 1 and 3 become partisans.

For a fixed policy p , player ℓ 's expected payoff, before s and b_ℓ are realized, is:

$$\mathbb{E}[u_\ell | p] = \mathbb{E}[-(b_\ell - (p - s))^2 | p] = -(p - (\mathbb{E}[b_\ell] + \mathbb{E}[s]))^2 - \text{Var}[s] - \text{Var}[b_\ell],$$

where $\mathbb{E}[s]$ and $\text{Var}[s]$ are the expected value and variance of s ; and $\mathbb{E}[b_\ell]$ and $\text{Var}[b_\ell]$ are the expected value and variance of b_ℓ . Ex ante, player ℓ 's induced preference over policies is single-peaked at $\mathbb{E}[b_\ell] + \mathbb{E}[s]$. In particular, player 2's induced preference over policies is single-peaked at 0, implying that ex ante he is indifferent between policy L and R . Since $\mathbb{E}[b_1] + \mathbb{E}[s] < 0$ and $\mathbb{E}[b_3] + \mathbb{E}[s] > 0$, ex ante player 1 strictly prefers L and player 3 strictly prefers R . For this reason, players 1, 2, and 3 are also called the left, central, and right players, respectively.

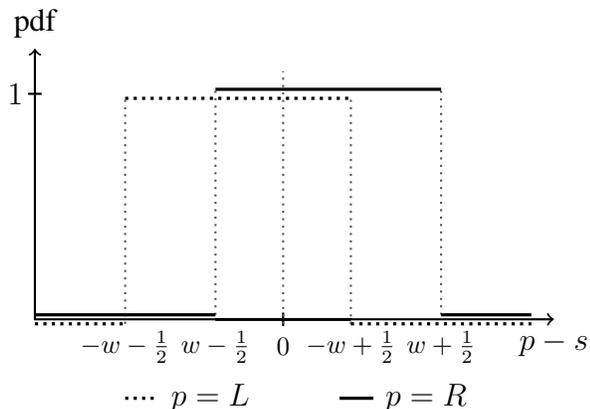


Figure 1: Distributions of outcome, $p - s$

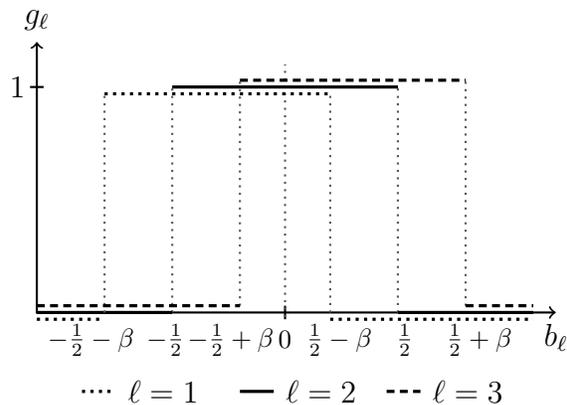


Figure 2: Distributions of bias, b_ℓ

Communication. Players do not know the state s . But one player, labeled i , has superior information and is called the *informed player*. With probability $1 - h \in (0, 1)$, player i obtains a binary signal about the state, $\theta \in \Theta = \{1, 0\}$. The signal takes value 1 with probability $1/2 + s$ and 0 with probability $1/2 - s$. With complementary probability h , he receives no signal at all, denoted by \emptyset . I am primarily interested in the limiting behavior and limiting payoffs as the probability of no signal becomes negligible, i.e., $h \rightarrow 0$.² Player i 's identity is commonly known.

The other two are the *uninformed players*, denoted by j_1 and j_2 . After observing the signal, the informed player can disclose the signal to his opponents. The signal is hard evidence and the communication is private, so player i chooses with whom to share the signal if he obtains one. An uninformed player observes the signal if player i shares with him, and observes nothing if player i does not, so an uninformed player does not observe whether player i has shared with the other uninformed player. If the informed player does not receive a signal, then he has no strategic choice to make. Therefore, if an uninformed player receives no signal from i , either player i chooses not to share with him or player i himself does not receive a signal. The uninformed players are not allowed to talk with each other.

Timeline. The game consists of a communication stage and a voting stage. In the communication stage, nature draws the state s and the signal θ , and reveals signal θ to player i with probability $1 - h$. If player i obtains a signal, then he decides whether to share it with j_1, j_2 . Otherwise, he has no strategic choice to make in this stage. In the voting stage, each player learns his bias privately and votes under majority rule. Then payoff is realized. I use ℓ to represent a generic player when discussing biases, payoffs, and the voting stage, while I use i to represent the informed player and j_1, j_2 to represent the

²Allowing h to be strictly positive simplifies the analysis compared with the case in which $h = 0$. Moreover, the limiting equilibrium-payoff set as h approaches zero is the same as the payoff set when $h = 0$.

uninformed players when discussing the communication stage.

Before proceeding, I want to emphasize three key features of the model. The first is the assumption that players are asymmetrically informed. This assumption lays the foundation for analyzing the better-informed player's incentives to disclose information. If players have the same information about the state, then communication prior to voting plays no role. The second feature is that players have heterogeneous ideologies, measured by the constant β . Introducing heterogeneous ideologies enriches the model in two ways. I can analyze with whom an informed player shares the information. In addition, I can compare the equilibria in which the central player is informed with the equilibria in which a peripheral player is informed. The third feature is the assumption that the disclosure decision is made *before* the informed player learns his bias. I call this *bias-independent* sharing. This assumption corresponds to the situations in which committee members keep receiving idiosyncratic shocks which influence their preferences over the policies. These shocks might arrive after the communication stage. Consider again the example of a firm facing an investment decision. The IT division might change its views toward the project "at the last minute" because it would lose several good programmers and its chance of being assigned the project would be largely reduced.

2.2 Strategies and solution concept

Strategy profile. A strategy profile consists of the informed player's disclosure strategy and all players' voting strategies.

Let $D = \{11, 10, 01, 00\}$ denote the space of disclosure actions. Action 11 refers to sharing the signal with both uninformed players, 10 sharing with j_1 but not j_2 , etc. A mixed disclosure strategy, π_i , is a map from Θ to $\Delta(D)$, the set of probability distributions over D . If the informed player has not obtained a signal, he has no choice to make in the communication stage, denoted as \emptyset . Therefore, the set of histories after the communication stage is $M_i = \{\Theta \times D\} \cup \{\emptyset\}$, with m_i being a generic element.

A mixed voting strategy of player i is a map $v_i : M_i \times B_i \rightarrow [0, 1]$. Here, $v_i(m_i, b_i)$ is the probability that player i votes for L , when the communication stage ends at node $m_i \in M_i$ and his bias is b_i .

At the end of the communication stage, an uninformed player $j \in \{j_1, j_2\}$ receives one of three possible messages from player i : signal 1, signal 0, or signal \emptyset (which means that player i shares nothing with j). Let $M_j = \{1, 0, \emptyset\}$ denote the set of j 's information sets, with m_j being a generic element. Since j only partially observes i 's disclosure action, M_j is a partition of M_i . The information set $m_j = 1$ contains two nodes in M_i : either the informed player shares signal 1 with the other uninformed player or he does not. Similarly, $m_j = 0$ also contains two nodes in M_i . The information set

$m_j = \emptyset$ contains all the nodes in M_i at which either player i hides the signal from player j , or player i has not obtained a signal. A mixed voting strategy for player j is a map $v_j : M_j \times B_j \rightarrow [0, 1]$, where $v_j(m_j, b_j)$ is the probability that player j votes for L given m_j and b_j .

I let $\sigma = (\pi_i, v = (v_i, v_{j_1}, v_{j_2}))$ denote a mixed strategy profile.

Solution concept. The solution concept is a refinement of a weak perfect Bayesian equilibrium, which requires that beliefs be updated in accordance with Bayes' rule whenever possible and, in addition, satisfy a “no-signaling-what-you-don't-know” condition (Fudenberg and Tirole, 1991). This condition requires that a player's deviation does not signal information that he himself does not possess. In my model, it requires that the informed player's deviation in the communication stage not affect beliefs about players' bias realizations. A precise description of the solution concept now follows. Readers who are more interested in equilibrium analysis and main results may skip this part and proceed to Section 3.

At each information set, the player about to move must have a belief about which history in the information set has been reached. In particular, I need to specify (i) the informed player's belief before he takes a disclosure action, and (ii) all three players' beliefs before they vote. Since the informed player's information sets before he takes a disclosure action are singletons, I need to specify only the players' beliefs before they vote. The set of histories before the players vote is:

$$H = M_i \times B_i \times B_{j_1} \times B_{j_2}.$$

For each $\ell \in N$, the set of player ℓ 's information sets before he votes is $H_\ell = M_\ell \times B_\ell$, with h_ℓ being an element. Let $\mu_\ell(\tilde{m}_i, \tilde{b}_i, \tilde{b}_{j_1}, \tilde{b}_{j_2} \mid h_\ell)$ be the probability that player ℓ assigns to the event that \tilde{m}_i occurs in the communication stage and that the bias vector is weakly below $(\tilde{b}_i, \tilde{b}_{j_1}, \tilde{b}_{j_2})$, conditional on h_ℓ . The system of beliefs is denoted by $\mu = \{\{\mu_\ell(\cdot \mid h_\ell)\}_{h_\ell \in H_\ell}\}_{\ell \in N}$.

I first characterize the informed player's beliefs $\mu_i(\cdot \mid h_i)_{h_i \in H_i}$. Since player i 's unreached information sets can be reached only by his own deviations, his beliefs at those sets can be derived by Bayes' rule (assuming perfect recall). Given $h_i = (m_i, b_i)$, player i knows exactly what happened in the communication stage. Hence, his belief is given by

$$\mu_i(\tilde{m}_i, \tilde{b}_i, \tilde{b}_{j_1}, \tilde{b}_{j_2} \mid m_i, b_i) = \begin{cases} G_{j_1}(\tilde{b}_{j_1}) G_{j_2}(\tilde{b}_{j_2}) & \text{if } b_i \leq \tilde{b}_i \text{ and } \tilde{m}_i = m_i, \\ 0 & \text{otherwise.} \end{cases} \quad (\text{B-1})$$

Here, recall that $G_\ell(\cdot)$ is the cdf of b_ℓ . For an uninformed player j , if he sees a message m_j that is on path, he can calculate the probability that m_i occurs in the communication stage, conditional on

m_j , denoted by $\mu_j(m_i | m_j)$.³ Hence, $\mu_j(\tilde{m}_i, \tilde{b}_i, \tilde{b}_j, \tilde{b}_{j'} | m_j, b_j)$ is well-defined according to Bayes' rule. If player j receives an out-of-equilibrium message from player i , Bayes' rule does not impose any restriction on player j 's beliefs μ_j . For example, if player i never shares any signal with player j on path, then player j could believe that player i 's and j 's biases are perfectly correlated if he receives a signal from player i . However, those beliefs violate the “no-signaling-what-you-don't-know” condition since biases are drawn independently after the communication stage. Player j 's belief regarding player i 's disclosure behavior should be independent of player j 's belief regarding his opponents' bias realizations. Moreover, both beliefs should be independent of player j 's bias realization. This motivates the following restrictions imposed on the uninformed players' beliefs, where $u_j(\cdot | m_j)$ are extended to include those nodes m_j that are off path:

- (i) For $j \in \{j_1, j_2\}$, there exists a belief system $\{\mu_j(\cdot | m_j) : \mu_j(\cdot | m_j) \in \Delta(M_i)\}_{m_j \in M_j}$ such that $\mu_j(\cdot | m_j)$ assigns positive probability only to nodes contained in m_j . That is,

$$\sum_{m_i \in m_j} \mu_j(m_i | m_j) = 1, \quad \forall m_j \in M_j. \quad (\text{B-2})$$

The beliefs $\{\mu_j(\cdot | m_j)\}_{m_j \in M_j}$ are defined by Bayes' rule whenever possible.

- (ii) For $j \in \{j_1, j_2\}$ and $h_j = (m_j, b_j) \in H_j$, the beliefs at h_j are defined as follows:

$$\mu_j(\tilde{m}_i, \tilde{b}_i, \tilde{b}_j, \tilde{b}_{j'} | m_j, b_j) = \begin{cases} \mu_j(\tilde{m}_i | m_j) G_i(\tilde{b}_i) G_{j'}(\tilde{b}_{j'}) & \text{if } b_j \leq \tilde{b}_j \text{ and } \tilde{m}_i \in m_j, \\ 0 & \text{otherwise,} \end{cases} \quad (\text{B-3})$$

where $j' \in \{j_1, j_2\}$ and $j' \neq j$.

Definition 1 (Perfect Bayesian Equilibrium).

An assessment (μ, σ) is a perfect Bayesian equilibrium (PBE) if (i) the beliefs are updated in accordance with Bayes' rule whenever possible, and satisfy a “no-signaling-what-you-don't-know” condition (B-1, B-2, and B-3); (ii) the strategy profile σ is sequentially rational given μ ; and (iii) no one uses a weakly dominated strategy in the voting stage: for every $\ell \in \{i, j_1, j_2\}$, v_ℓ is not weakly dominated.

In Section 4 and 5, I characterize the set of all PBE when h is negligible, with a focus on the equilibria that give the informed player the highest expected payoff. I examine and compare the scenario in which the central player is informed with the scenario in which a peripheral player is informed.

³For example, if player i discloses signal 1 to player j with probability one and to player j' with probability α , then player j assigns probability α to the node at which j' also receives signal 1, and assigns probability $1 - \alpha$ to the node at which j' receives no signal, conditional on $m_j = 1$.

3 Preliminaries

I begin with a single-player decision problem. Then, I proceed to characterize players' voting strategies and the informed player's disclosure strategy.

3.1 Single-player decision problem

Consider a single player ℓ who chooses $p \in \{L, R\}$ to maximize his expected payoff. The information structure and timeline of events are similar to the game introduced earlier. Nature draws state s and a signal $\theta \in \Theta$. With probability $1 - h$, player ℓ is informed of signal θ . With complementary probability, he obtains no signal, denoted as $\theta = \emptyset$. Then player ℓ learns his bias b_ℓ and chooses a policy. The expected payoff of player ℓ conditional on signal θ , bias b_ℓ , and policy p is given by:

$$E[u_\ell \mid \theta, b_\ell, p] = E[-(b_\ell - (p - s))^2 \mid \theta, b_\ell, p] = -(b_\ell + E[s \mid \theta] - p)^2 - \text{Var}[s \mid \theta],$$

where $E[s \mid \theta]$ and $\text{Var}[s \mid \theta]$ are the expected value and variance of s , given θ . Player ℓ 's induced preferences over policies are symmetric and single-peaked at $b_\ell + E[s \mid \theta]$. Therefore, player ℓ employs a threshold decision rule, choosing L if his bias b_ℓ is below $-E[s \mid \theta]$ and R otherwise. The corresponding thresholds are as follows,

$$\{-E[s \mid \theta = 1], -E[s \mid \theta = 0], -E[s \mid \theta = \emptyset]\} = \left\{-\frac{1}{6}, \frac{1}{6}, 0\right\}.$$

Signal 1 leads to a lower threshold than signal 0 and suggests that R is more likely to be better. The thresholds depend only on the realized signal, not on the player's bias. Nonetheless, for a fixed signal and hence the threshold, player 1's bias is more likely to fall below the threshold than player 2's bias, which in turn is more likely to fall below the threshold than player 3's. Therefore, player 1 is most likely to choose policy L , player 3 is least likely, and player 2 is in between.

3.2 Threshold voting strategies

Returning to the voting stage of the original game, I now demonstrate that player ℓ 's voting strategy is also characterized by thresholds. The only time that a player can influence the policy choice is when his vote is pivotal, that is, when the votes of the other two players cancel each other out. Thus, player ℓ makes his voting decision conditional on not only his information set h_ℓ but also his vote being pivotal. Let piv denote the event that a vote is pivotal. The expected payoff to player ℓ conditional on

information set h_ℓ , ℓ being pivotal, the strategy profile σ , and policy p is

$$\begin{aligned} E[u_\ell \mid h_\ell, piv, \sigma, p] &= E[-(b_\ell - (p - s))^2 \mid h_\ell, piv, \sigma, p] \\ &= -(b_\ell + E[s \mid h_\ell, piv, \sigma] - p)^2 - \text{Var}[s \mid h_\ell, piv, \sigma] \\ &= -(b_\ell + E[s \mid m_\ell, piv, \sigma] - p)^2 - \text{Var}[s \mid m_\ell, piv, \sigma], \end{aligned}$$

where $E[s \mid m_\ell, piv, \sigma]$ and $\text{Var}[s \mid m_\ell, piv, \sigma]$ are the conditional expected value and variance of s . The last equality holds because player ℓ 's bias is drawn independently of other random variables. Thus, player ℓ votes for L if his bias b_ℓ is below $-E[s \mid m_\ell, piv, \sigma]$ and R if above. Lemma 1 characterizes the informed player's thresholds. Lemma 2 characterizes an uninformed player's thresholds when he receives a signal from the informed player.

Lemma 1 (Informed player's voting thresholds).

At information set $h_i = (m_i, b_i)$, the informed player votes for L if $b_i < -E[s \mid m_i, piv, \sigma]$ and for R otherwise. The voting thresholds are as follows:

$$-E[s \mid m_i, piv, \sigma] = \begin{cases} -\frac{1}{6} & \text{if } m_i \in \{1\} \times D, \\ \frac{1}{6} & \text{if } m_i \in \{0\} \times D, \\ 0 & \text{if } m_i = \emptyset. \end{cases}$$

Lemma 2 (Uninformed player's voting thresholds).

At information set $h_j = (m_j, b_j)$, the uninformed player j votes for L if $b_j < -E[s \mid m_j, piv, \sigma]$ and for R otherwise. The voting thresholds are as follows:

$$-E[s \mid m_j, piv, \sigma] = \begin{cases} -\frac{1}{6} & \text{if } m_j = 1, \\ \frac{1}{6} & \text{if } m_j = 0. \end{cases}$$

The proofs of Lemma 1 and 2 are in Appendix 8.1. The intuition lies in the fact that the signal exhausts all the relevant information regarding the state. Once a player observes the signal—either he is the informed player or he receives a signal from the informed player—he knows all the relevant information regarding the state. Being pivotal, in this case, does not give him more information. Similarly, the informed player uses zero as the threshold if no signal is obtained, since being pivotal gives him no information about the state in that case. From now on, I focus on strategy profiles that satisfy Lemma 1 and 2.

It follows that the only nontrivial thresholds are those used by uninformed players when they receive no signal: $-E[s \mid m_{j_1} = \emptyset, piv, \sigma]$ and $-E[s \mid m_{j_2} = \emptyset, piv, \sigma]$. Since not receiving a signal from the

informed player is always on path, these thresholds can be calculated according to Bayes' rule. Here, I briefly illustrate how to calculate the thresholds (detailed formulae can be found in Appendix 8.2). Let $\Pr[m_j = \emptyset, piv \mid s, \sigma]$ be the probability that (i) player j receives no signal and (ii) his vote is pivotal, conditional on state s . The density function of state s , conditional on j receiving no signal and being pivotal, is

$$g(s \mid m_j = \emptyset, piv, \sigma) = \frac{\Pr[m_j = \emptyset, piv \mid s, \sigma]f(s)}{\int_{-\frac{1}{2}}^{\frac{1}{2}} \Pr[m_j = \emptyset, piv \mid s, \sigma]f(s)ds},$$

where $f(s)$ is the prior distribution. Therefore, player j 's voting threshold, conditional on j receiving no signal and being pivotal, is

$$-E[s \mid m_j = \emptyset, piv, \sigma] = -\int_{-\frac{1}{2}}^{\frac{1}{2}} sg(s \mid m_j = \emptyset, piv, \sigma)ds. \quad (1)$$

When the informed player's disclosure strategy is fixed, an uninformed player's threshold, $-E[s \mid m_j = \emptyset, piv, \sigma]$, depends only on his opponents' voting strategies. Thus, $-E[s \mid m_{j_1} = \emptyset, piv, \sigma]$ is a function of $-E[s \mid m_{j_2} = \emptyset, piv, \sigma]$ and vice versa.⁴ Given two equations in two unknowns, I can solve for these thresholds. When uninformed players observe no signal, they know that there is a positive probability that the informed player obtained no signal either. Therefore, the thresholds must be in the range $(-1/6, 1/6)$.

3.3 Disclosure strategy

3.3.1 Coordination motive and pivotal motive

To characterize the informed player's disclosure strategy, I first delineate two key motives to share or to withhold information. Consider a situation in which the central player is informed. After observing a signal favoring policy R , he would like to share the signal to persuade his opponents to vote for R , because policy R delivers a higher expected payoff than L does, conditional on the signal. This is the *coordination motive*. With votes coordinated, when a signal favoring one policy is observed, that policy is more likely to be chosen by majority rule. However, despite a signal favoring R , the central player might find his bias sufficiently low in the voting stage, so that he strictly prefers policy L to be chosen. The central player wants to neutralize his opponents' votes so that he is able to adjust his vote—the policy to be implemented if the other two vote differently—according to his bias realization. I call this the *pivotal motive*. With the presence of the pivotal motive, the informed player tends to withhold

⁴Once we fix player i 's disclosure strategy and restrict attention to strategy profiles satisfying Lemma 1 and Lemma 2, the threshold $-E[s \mid m_{j_1} = \emptyset, piv, \sigma]$ depends only on j_2 's voting behavior when j_2 receives no signal.

information.

3.3.2 Incentive constraints

Following this line of reasoning, I formalize the incentive constraints governing the informed player's disclosure behavior. From the perspective of player i , the votes of the uninformed players can be one of three cases,

$$\left(LL, RR, \underbrace{LR, RL}_{piv} \right),$$

where the underbrace gathers two cases that are equivalent. Given any signal $\theta \in \Theta$, any disclosure action $d \in D$, and voting strategies by the uninformed players v_{-i} , let $\mathbf{Pr}(\theta, d, v_{-i})$ be the probability vector that the uninformed players' votes are LL , RR , and piv .⁵

$$\mathbf{Pr}(\theta, d, v_{-i}) = (\Pr[LL \mid \theta, d, v_{-i}], \Pr[RR \mid \theta, d, v_{-i}], \Pr[piv \mid \theta, d, v_{-i}]).$$

For any $\theta \in \Theta$, let $\mathbf{E}[u_i \mid \theta, v_i]$ be the expected payoff vector of player i given signal θ and the voting strategy v_i as in Lemma 1, when the other two votes are LL , RR , or piv .⁶

$$\mathbf{E}[u_i \mid \theta, v_i] = (\mathbf{E}[u_i \mid LL, \theta, v_i], \mathbf{E}[u_i \mid RR, \theta, v_i], \mathbf{E}[u_i \mid piv, \theta, v_i]).$$

The sum product of $\mathbf{Pr}(\theta, d, v_{-i})$ and $\mathbf{E}[u_i \mid \theta, v_i]$ is the expected payoff to player i given θ , d and v :

$$\mathbf{E}[u_i \mid \theta, d, v] = \mathbf{Pr}(\theta, d, v_{-i}) \cdot \mathbf{E}[u_i \mid \theta, v_i].$$

For any equilibrium profile (π_i, v) , if d is played with positive probability after signal θ , then there is no other action with a higher payoff:

$$\mathbf{E}[u_i \mid \theta, d, v] \geq \mathbf{E}[u_i \mid \theta, d', v], \quad \forall d' \in D. \tag{IC}$$

Any voting strategies characterized in Subsection 3.2 and any disclosure strategy satisfying IC constitute a PBE.

4 Central player is informed

I first study the case in which the central player is informed. Due to symmetry between the signals, I focus on the case in which the central player obtains signal 1. The expected payoff vector if his

⁵I use boldface characters to represent vectors.

⁶If the other votes are LL or RR , the informed player's vote does not affect his expected payoff. The expectation is taken over his bias and the state. If the other votes cancel each other out, the informed player's vote is the winning policy. The expectation is taken over i 's bias, the state, and i 's policy choice.

opponents' votes are (LL, RR, piv) is

$$\mathbf{E}[u_2 \mid \theta = 1, v_2] = \left(-\omega^2 - \frac{1}{3}\omega - \frac{1}{6}, -\omega^2 + \frac{1}{3}\omega - \frac{1}{6}, -\omega^2 + \frac{5}{9}\omega - \frac{1}{6} \right) = \left(0, \frac{2\omega}{3}, \frac{8\omega}{9} \right) + c \mathbf{e},$$

where c is a constant and \mathbf{e} the vector $(1, 1, 1)$. It is easy to verify that the following holds:

$$\mathbf{E}[u_2 \mid piv, \theta = 1, v_2] > \mathbf{E}[u_2 \mid RR, \theta = 1, v_2] > \mathbf{E}[u_2 \mid LL, \theta = 1, v_2].$$

Because the central player is ex ante unbiased, he prefers RR to LL since signal 1 indicates that a higher state is more likely to occur. However, the central player most prefers being pivotal. The difference between $\mathbf{E}[u_2 \mid piv, \theta = 1, v_2]$ and $\mathbf{E}[u_2 \mid RR, \theta = 1, v_2]$ measures the option value of being pivotal.

Suppose the probabilities with which player 1 and player 3 vote for L are, respectively, p_1^L and p_3^L after signal 1 and disclosure action d . Suppose that switching to disclosure decision d' increases p_1^L by $\Delta > 0$. The change in $\Pr(\theta = 1, d, v_{-2})$ is

$$\Pr(\theta = 1, d', v_{-2}) - \Pr(\theta = 1, d, v_{-2}) = (p_3^L, -(1 - p_3^L), 1 - 2p_3^L) \Delta.$$

Accordingly, the change in the central player's expected payoff is $\mathbf{E}[u_2 \mid \theta = 1, d', v] - \mathbf{E}[u_2 \mid \theta = 1, d, v]$ which equals:

$$\underbrace{\Delta p_3^L 0 + (-\Delta)(1 - p_3^L) \frac{2\omega}{3}}_{\text{coordination}(-)} + \underbrace{\Delta(1 - 2p_3^L) \frac{8\omega}{9}}_{\text{pivotal}(-/+)} = \Delta \frac{2\omega}{9} (1 - 5p_3^L). \quad (2)$$

The first two terms measure the loss due to the failure to coordinate. Increasing the probability with which player 1 votes for L causes LL to occur more frequently and RR to occur less frequently, lowering the central player's expected payoff. The third term measures the loss or gain due to the change in the probability that the central player finds himself pivotal. When player 3 votes for R sufficiently often ($p_3^L < 1/5$), the gain from being pivotal outweighs the loss due to coordination failure. The central player prefers that player 1 votes for L more often.

Lemma 3 (Central player's disclosure strategy).

Given signal 1, the central player prefers a lower p_1^L if $p_3^L > 1/5$ and a higher p_1^L if $p_3^L < 1/5$. Analogously, the central player prefers a lower p_3^L if $p_1^L > 1/5$ and a higher p_3^L if $p_1^L < 1/5$.

4.1 Equilibria

To characterize the equilibria, I need to consider only the situation in which $\beta \in (0, 2/3)$, because the left (right) player becomes partisan and always votes for L (R) when β exceeds $2/3$. As β increases, the central player's disclosure behavior is characterized by three phases: full disclosure, partial disclosure, and no disclosure.

4.1.1 Full disclosure

When the misalignment of preferences is sufficiently small, neither player 1's nor player 3's voting behavior is sufficiently extreme. When $\beta < 2/15$, both p_1^L and p_3^L are at least $1/5$:

$$p_1^L \geq -\frac{1}{6} - \left(-\frac{1}{2} - \beta\right) > \frac{1}{5}, \quad p_3^L \geq -\frac{1}{6} - \left(-\frac{1}{2} + \beta\right) > \frac{1}{5}.$$

(This is because the lowest threshold that an uninformed player ever employs is $-1/6$.) According to Lemma 3, the central player prefers p_3^L and p_1^L to be as low as possible, so he shares signal 1 to persuade them to vote for R . Analogously, the central player fully discloses signal 0. The equilibrium voting outcome is the same as if the signal were public.

Proposition 1 (Full disclosure).

When $\beta < 2/15$, there exists a unique equilibrium. Player 2 fully discloses his private signal to both player 1 and player 3. The uninformed players, if receiving no signal, infer that the informed player obtains no signal and use 0 as the voting threshold.

4.1.2 Partial disclosure

Regardless of the message that player 1 receives, he votes for L with probability greater than $1/5$ because

$$p_1^L \geq -\frac{1}{6} - \left(-\frac{1}{2} - \beta\right) = \frac{1}{3} + \beta > \frac{1}{5}.$$

According to Lemma 3, player 2 prefers a lower p_3^L given that $p_1^L > 1/5$. If player 2 shares signal 1 with player 3, then player 3 uses $-1/6$ as his voting threshold after signal 1. Hence, p_3^L equals

$$p_3^L = \max \left\{ -\frac{1}{6} - \left(-\frac{1}{2} + \beta\right), 0 \right\} = \max \left\{ \frac{1}{3} - \beta, 0 \right\},$$

which is smaller than $1/5$ when $\beta \geq 2/15$.⁷ According to Lemma 3, player 2 withholds signal 1 from the left player.

By sharing signal 1 with the right player, which confirms the right player's ex ante bias for R , the central player induces him to vote for R with high probability. Meanwhile, the central player conceals signal 1 from the left player so as to induce him to vote for L , maximizing the probability of being pivotal. Analogously, the central player discloses signal 0 to the left player but not to the right player. Because the central player discloses his signal to one side, I call this behavior *partial disclosure*.

⁷When $\beta = 2/15$, there are multiple equilibria since the central player is indifferent between sharing signal 1 with player 1 and not sharing it. When $\beta > 2/15$, the central player strictly prefers to withhold signal 1 from player 1.

Proposition 2 (Partial disclosure).

When $\frac{2}{15} \leq \beta \leq \beta^*(h)$, there exists an equilibrium such that player 2 shares signal 0 only with player 1 and shares signal 1 only with player 3. If the uninformed players see no signal, the thresholds are:

$$-E[s \mid m_1 = \emptyset, piv, \sigma] = \begin{cases} -\frac{(4-3\beta)(1-h)}{6(3\beta(h-1)+5h+4)} & \text{if } \beta < \frac{1}{3}, \\ -\frac{1-h}{12h+6} & \text{if } \beta \geq \frac{1}{3}, \end{cases}$$

$$-E[s \mid m_3 = \emptyset, piv, \sigma] = E[s \mid m_1 = \emptyset, piv, \sigma].$$

When $\frac{2}{15} < \beta < \beta^{**}(h)$, this is the unique equilibrium.⁸

4.1.3 No disclosure

Lastly, I show that for a large parameter region, there exists an equilibrium in which player 2 shares no information.

I first illustrate how “conditional on being pivotal” makes an uninformed player more likely to vote for his ex ante preferred policy, when the central player never shares information with the peripheral players. Take the left player as an example. When he receives no signal, he reasons that, conditional on being pivotal, it is more likely that the right player votes for R and the central player votes for L . For this reason, the left player puts more weight on the event of signal 0 being received by the central player and is more likely to vote for L . To what extent the left player exaggerates the probability that the central player votes for L depends on how partisan the right player is. If the right player always votes for R , the left player puts probability one on the event that the central player votes for L . This reasoning, along with a relatively large bias toward L , induces the left player to vote for L with certainty. The same logic applies to the right player. The central player takes advantage of this reasoning by not sharing with uninformed players. He dictates the voting outcome and receives the highest possible payoff.

Proposition 3 (No disclosure).

When $\beta \geq \beta^{**}(h)$, there exists an equilibrium such that the central player always conceals his signal. If the uninformed players see no signal, the thresholds are

$$-E[s \mid m_1 = \emptyset, piv, \sigma] = \frac{1-h}{18}, \text{ and } -E[s \mid m_3 = \emptyset, piv, \sigma] = \frac{h-1}{18}.$$

4.1.4 Summary

Figure 3 depicts the equilibria when the central player is informed as the probability of no signal becomes negligible, i.e., $h \rightarrow 0$. The x axis variable is β , ranging from 0 to $2/3$.

⁸The boundary conditions are $\beta^*(h) = \frac{5h+4}{6(2h+1)}$ and $\beta^{**}(h) = \frac{h+8}{18}$. It is easily verified that $\beta^{**}(h) < \beta^*(h)$ for any $h \in (0, 1)$.

The multiplicity of equilibria between $\beta^{**}(0)$ and $\beta^*(0)$ arises due to the multiplicity of self-fulfilling beliefs: when the left player expects the central player to share signal 0 with him and optimally uses the threshold $-1/6$ if he receives no signal, it is better for the central player to share signal 0 with the left player. If the left player expects the central player to stay quiet and the right player to vote for R with certainty, then he optimally uses the threshold $1/18$ (as shown in Proposition 3) if he receives no signal. In this case, it is better for the central player not to share any information.

Figure 4 depicts the equilibrium that gives the informed player the highest expected payoff. When players' preferences are not too dissimilar, the central player fully discloses his information with his opponents. As preference conflicts grow, the coordination motive gradually gives way to the pivotal motive. The central player begins to withhold information from his opponents and eventually stops sharing. When multiple equilibria exist, the central player strictly prefers the no disclosure equilibrium which gives him the highest possible expected payoff.

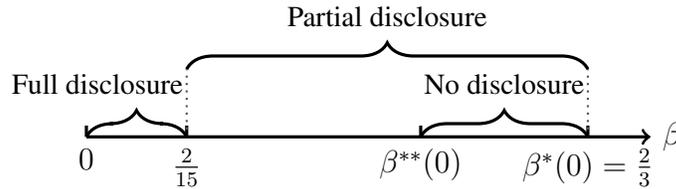


Figure 3: Equilibria when central player is informed

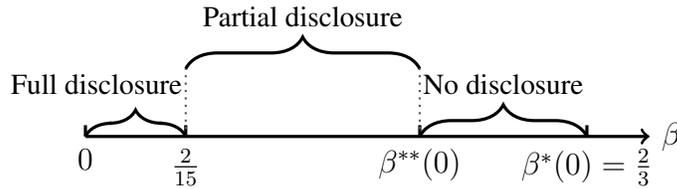


Figure 4: Equilibrium most preferred by central player

5 A peripheral player is informed

5.1 The unraveling motive

I now turn to the case in which a peripheral player, for example, the left player, is informed.⁹ Unlike the central player, the left player's disclosure behavior is mainly subject to the *unraveling motive*: he

⁹Due to symmetry, the situation in which the right player is informed is similar.

is inclined to share signal 0—the signal favorable to policy L —and conceal signal 1, due to his ex ante bias toward policy L . The unraveling motive increases as the left player becomes more biased toward L . It dominates both the coordination and the pivotal motives when β is sufficiently large. I name this motive the unraveling motive, because the induced disclosure behavior enables the uninformed players to better infer the signal obtained by the left player.

When the left player observes signal 0, the expected payoff vector if his opponents' votes are (LL, RR, piv) is

$$\mathbf{E}[u_1 \mid \theta = 0, v_1] = \begin{cases} \left(\frac{2}{3} + 4\beta, 0, \frac{2}{9}(2 + 3\beta)^2\right) + c' \mathbf{e} & \beta \in [0, \frac{1}{3}], \\ \left(\frac{2}{3} + 4\beta, 0, \frac{2}{3} + 4\beta\right) + c' \mathbf{e} & \beta \in (\frac{1}{3}, \frac{2}{3}], \end{cases}$$

where c' is a constant. The option value of being pivotal, measured as the difference between $\mathbf{E}[u_1 \mid piv, \theta = 0, v_1]$ and $\mathbf{E}[u_1 \mid LL, \theta = 0, v_1]$, decreases in β . When β is greater than $1/3$, the option value of being pivotal reduces to zero because the left player always votes for L after signal 0. The following lemma shows that the left player always shares signal 0 with the other two players.

Lemma 4.

The left player always discloses signal 0 to his opponents.

Proof of Lemma 4. Suppose that β is below $1/3$. If the probabilities with which player 2 and 3 vote for L are p_2^L and p_3^L , respectively, the expected payoff to player 1 is

$$\mathbf{E}[u_1 \mid \theta = 0, d, v] = p_2^L p_3^L \left(4\beta + \frac{2}{3}\right) + \frac{2}{9} (3\beta + 2)^2 (p_2^L + p_3^L - 2p_2^L p_3^L) + 3c'.$$

Since the highest possible threshold that player 2 or player 3 employs is $1/6$, both p_2^L and p_3^L are weakly smaller than $2/3$. The derivatives of $\mathbf{E}[u_1 \mid \theta = 0, d, v]$ with respect to p_2^L and p_3^L are strictly positive:

$$\begin{aligned} \frac{\partial \mathbf{E}[u_1 \mid \theta = 0, d, v]}{\partial p_2^L} &= \frac{2}{9} \left((3\beta + 2)^2 - p_3^L (6\beta(3\beta + 1) + 5) \right) > 0, \\ \frac{\partial \mathbf{E}[u_1 \mid \theta = 0, d, v]}{\partial p_3^L} &= \frac{2}{9} \left((3\beta + 2)^2 - p_2^L (6\beta(3\beta + 1) + 5) \right) > 0. \end{aligned}$$

Therefore, player 1 prefers p_2^L, p_3^L to be as large as possible. He fully discloses signal 0, so player 2 and 3 both use $1/6$ as their threshold after signal 0. If β lies in $[1/3, 2/3]$, player 1 fully discloses signal 0 so as to decrease the probability that RR occurs. \square

When the left player observes signal 1, the expected payoff vector if his opponents' votes are (LL, RR, piv) is

$$\mathbf{E}[u_1 \mid \theta = 1, v_1] = \left(-\frac{2}{3} + 4\beta, 0, \frac{2}{9}(1 + 3\beta)^2\right) + c'' \mathbf{e},$$

where c'' is a constant. When $\beta < 1/6$, the left player prefers RR to LL and has incentives to coordinate votes. However, the coordination motive diminishes as β increases. When $\beta > 1/6$, the left player prefers LL to RR in spite of signal 1. The pivotal motive is always present in the sense that the left player strictly prefers being pivotal to LL or RR . Yet, when $\beta > 1/6$, the option value of being pivotal, measured as the difference between $E[u_1 | piv, \theta = 1, v_1]$ and $E[u_1 | LL, \theta_1, v_1]$, decreases in β .

Lemma 5.

The left player conceals signal 1 from uninformed players when $\beta \geq 1/6$.

Proof of Lemma 5. Suppose the probabilities with which player 2 and 3 vote for L are p_2^L and p_3^L . The expected payoff of player 1 is

$$E[u_1 | \theta = 1, d, v] = p_2^L p_3^L \left(4\beta - \frac{2}{3} \right) + \frac{2}{9} (3\beta + 1)^2 (p_2^L + p_3^L - 2p_2^L p_3^L) + 3c''.$$

When an uninformed player receives no signal, he infers that either the left player is concealing signal 1 or no signal is being transmitted by nature (because signal 0 is always shared as shown in Lemma 4). The corresponding voting threshold is a weighted average of $-1/6$ and 0. Therefore, the threshold that an uninformed player uses is weakly below 0 after signal 1. This implies that both p_2^L and p_3^L are weakly smaller than $1/2$. When $4\beta - 2/3 > 0$, the derivatives of $E[u_1 | \theta = 1, d, v]$ with respect to p_2^L and p_3^L are strictly positive:

$$\frac{\partial E[u_1 | \theta = 1, d, v]}{\partial p_2^L} = \frac{2}{9} (1 - 2p_3^L) (3\beta + 1)^2 + p_3^L \left(4\beta - \frac{2}{3} \right) > 0, \tag{3}$$

$$\frac{\partial E[u_1 | \theta = 1, d, v]}{\partial p_3^L} = \frac{2}{9} (1 - 2p_2^L) (3\beta + 1)^2 + p_2^L \left(4\beta - \frac{2}{3} \right) > 0. \tag{4}$$

Player 1 conceals signal 1 to increase the probability that player 2 and 3 vote for L . When $\beta = 1/6$, p_3^L is smaller than $1/2 - 1/6$. The derivative of $E[u_1 | \theta = 1, d, v]$ with respect to p_2^L is strictly positive. Therefore, player 1 does not share signal 1 with player 2. This means that $p_2^L < 1/2$ and the derivative of $E[u_1 | \theta = 1, d, v]$ with respect to player 3 is also strictly positive. Player 1 conceals signal 1 from player 3. □

5.2 Equilibria

Based on Lemmas 4 and 5, I need to specify the disclosure strategy only when the left player receives signal 1 and β is less than $1/6$. Suppose the probabilities with which player 2 and 3 vote for L are, respectively, p_2^L and p_3^L after signal 1. I argue that as h approaches 0, p_2^L and p_3^L approach $1/3$ and $1/3 - \beta$, respectively. This is obviously the case if player 1 chooses to share signal 1 with players 2 and

3. Even if player 1 chooses not to share signal 1, an uninformed player infers that player 1 has obtained signal 1 since signal 0 is always shared. In this case, an uninformed player's threshold, if he observes no signal, approaches $-1/6$ as h becomes negligible. Once I substitute $(p_2^L, p_3^L) = (1/3, 1/3 - \beta)$ into equations 3 and 4, the signs of the two derivatives pin down the sharing strategy by player 1. The disclosure strategy is characterized by two cutoffs $(\beta', \beta'') \approx (0.05, 0.08)$.¹⁰ When $\beta \leq \beta'$, both derivatives, $\frac{\partial E[u_1|\theta=1,d,v]}{\partial p_2^L}$ and $\frac{\partial E[u_1|\theta=1,d,v]}{\partial p_3^L}$, are negative. Player 1 shares signal 1 with both player 2 and player 3. When $\beta \in (\beta', \beta'')$, player 1 prefers a higher p_2^L and a lower p_3^L and hence shares signal 1 with player 3 but not with player 2. When $\beta \geq \beta''$, both derivatives are positive. Player 1 conceals signal 1 from both his opponents. Figure 5 summarizes the equilibrium disclosure strategy as β ranges from 0 to $2/3$. The left player always discloses signal 0. If he obtains signal 1, the left player shares it only when β is sufficiently small.

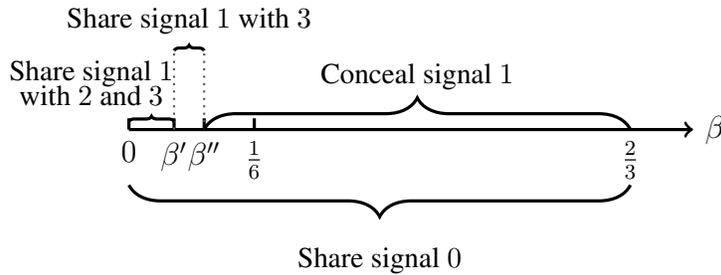


Figure 5: Equilibria when left player is informed

6 Incentives and welfare

The central player's incentives to disclose information are quite different from those of a peripheral player. When the central player is informed, the coordination motive calls for more disclosure while the pivotal motive calls for less. As β increases, the coordination motive gives way to the pivotal motive (see Table 1). When the left player is informed, the unraveling motive overshadows the coordination and pivotal motives. The left player always discloses signal 0, enabling the uninformed players to perfectly infer the informed player's signal when h is negligible.

Following convention, I take the sum of the players' expected payoffs as the social welfare. When the left player is informed, the equilibrium voting outcome is the same as if the signal were public. Hence, both the welfare level as well as individuals' expected payoffs is the same as if the signal were public.

¹⁰The cutoffs β' and β'' solve $\frac{\partial E[u_1|\theta=1,d,v]}{\partial p_2^L} = 0$ and $\frac{\partial E[u_1|\theta=1,d,v]}{\partial p_3^L} = 0$, respectively.

Informed player	Coordination motive	Pivotal motive	Unraveling motive
Central player	↓	↑	not available
Left player	↓	↓	↑

Table 1: Incentives governing disclosure decisions as β increases

The central player shares his private signal only when β is small. When $\beta < 4/9$, the central player either fully discloses his signal or partially reveals it. In both cases, the uninformed players can perfectly infer the central player’s signal when h is negligible. Hence, the equilibrium outcome is the same as if the signal were public. When $\beta \geq 4/9$, the central player shares no information and dictates the voting outcome. The voting outcome fails to reflect the uninformed players’ preferences, lowering the social welfare level.

Proposition 4.

When $\beta \in [\frac{4}{9}, \frac{2}{3})$, the social welfare is strictly lower when the central player is informed than when a peripheral player is informed.

Figure 6 shows how social welfare changes as β increases from 0 to $2/3$. The dashed line corresponds to the case in which the left player is informed. The solid line shows the welfare level when the central player is informed. Starting from the discontinuity point, $\beta = 4/9$, the central player withholds information. This entails a welfare loss. The central player achieves the highest possible expected payoff since he dictates the voting outcome. However, his gain comes at the expense of the uninformed players who receive no information and whose preferences are not reflected in the voting outcome at all.

7 Concluding remarks

This paper is a first step toward understanding committee decision making and verifiable disclosure. My work shows that there is a crucial distinction between polarization and bias. A polarized committee discourages its central member from disclosing information. In contrast, if a biased member possesses private information, full revelation results. Notably, this conclusion contradicts the conventional wisdom. When the US Congress establishes bipartisan special committees, its practice has sometimes been to have an equal number of Democrats and Republicans, plus some respected and experienced independents who were expected to lend some “neutrality” to the committee. However, my analysis shows that this is actually socially suboptimal, as this arrangement discourages central members from disclosing vital information in the case of a polarized committee.

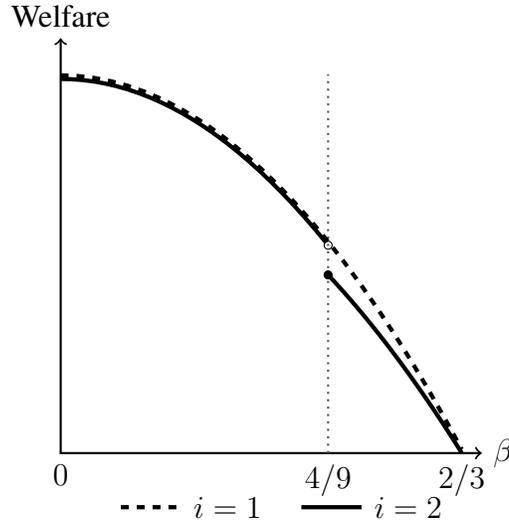


Figure 6: Welfare comparison

I will conclude by revisiting the main features of the model and discussing future research directions.

Cheap talk and hard evidence. I previously concluded that a committee with an informed peripheral member leads to collectively desirable outcomes. However, I do not expect this result to hold where communication is modeled as cheap talk. In such a case, a peripheral member's incentives to promote his agenda impair his credibility when he is perceived to be biased. Even if the peripheral member is informed and shares his unverifiable information, it will be disregarded due to its questionable credibility. This effectively nullifies his effort to share, leading to a suboptimal social welfare. Note that the central member is also plagued by the credibility problem due to the incentives he has to neutralize his opponents' votes and stay pivotal. However, when the committee is not too polarized, an informed central member can and does transmit information. My conjecture is that, with unverifiable information, an informed central member is better than an informed peripheral member.

Public and private communication. Requiring communication to be public does not affect the basic intuitions and results when $h \rightarrow 0$. A peripheral member, if informed, publicly announces the signal that favors his agenda and effectively shares more information. In contrast, an informed central member stays quiet in public debate when the committee is sufficiently polarized. Similarly, the results for $h \rightarrow 0$ are robust if the communication is private but uninformed players are allowed to talk to each other.

Nonetheless, the possibility of private communication does improve the informed member's welfare

in certain circumstances when h is not negligible. For example, when the central member is informed and conflicts of interests are moderate, he tailors his information to uninformed members' ex ante biases, sharing a signal favorable to the left-leaning policy with the left member and a signal favorable to the right-leaning policy with the right player. In this case, private communication allows the central player to maximize the benefit of coordinating votes on the one hand and staying pivotal as often as possible on the other. I leave a complete comparison between public and private communication modes for future research.

8 Appendix

8.1 Proof of Lemma 1 and 2

Throughout this subsection, the dependence on σ is dropped when no confusion arises. It is shown in Subsection 3.2 that players employ threshold voting strategy. Given the information structure, the highest (resp. lowest) threshold a voter ever employs is $1/6$ (resp. $-1/6$). Let $t_\ell(m_\ell) = \Pr[b_\ell < -E[s | m_\ell, piv]]$ denote the probability that player ℓ votes for L given m_ℓ where the expectation is taken over ℓ 's bias. Conditional on s , the probability that $m_i = \emptyset$ is $\Pr[m_i = \emptyset | s] = h$. The probability that i is pivotal conditional on $m_i = \emptyset$ and s is

$$\Pr[piv | m_i = \emptyset, s] = t_{j_1}(\emptyset)(1 - t_{j_2}(\emptyset)) + t_{j_2}(\emptyset)(1 - t_{j_1}(\emptyset)).$$

The probability that i receives no signal and is pivotal conditional on state s is

$$\Pr[m_i = \emptyset, piv | s] = h(t_{j_1}(\emptyset)(1 - t_{j_2}(\emptyset)) + t_{j_2}(\emptyset)(1 - t_{j_1}(\emptyset))),$$

which is independent of s . The probability distribution over state conditional on $m_i = \emptyset$ and being pivotal is given by

$$g(s | m_i = \emptyset, piv) = \frac{\Pr[m_i = \emptyset, piv | s]f(s)}{\int_{-\frac{1}{2}}^{\frac{1}{2}} \Pr[m_i = \emptyset, piv | s]f(s)ds} = f(s) = 1.$$

This implies that $-E[s | m_i = \emptyset, piv] = 0$. When $m_i \in \{1\} \times D$, player i knows the thresholds utilized by the uninformed players. The probability of m_i and i being pivotal conditional on s , $\Pr[m_i, piv | s]$, is proportional to $1/2 + s$. Therefore, I have

$$g(s | m_i, piv) = \frac{1}{2} + s, \quad -E[s | m_i, piv] = -\frac{1}{6}.$$

Analogously, when an uninformed player j receives signal 1, the probability of m_j and j being pivotal conditional on s , $\Pr[m_j = 1, piv | s]$, is proportional to $1/2 + s$. Therefore, j uses $-1/6$ as voting

threshold as well.¹¹ Similarly, the threshold that player i (resp. player j) employs after $m_i \in \{0\} \times D$ (resp. $m_j = 0$) is $1/6$.

8.2 Voting thresholds of the uninformed When $m_j = \emptyset$

Here, I show how to calculate $-\mathbb{E}[s \mid m_{j_1} = \emptyset, piv, \sigma]$ while restricting attention to strategy profile σ satisfying Lemma 1 and Lemma 2. When j_1 receives no signal, either player i receives no signal as well or player i chooses not to disclose his signal to j_1 . Let θ be the signal that player i obtains. I have

$$\begin{aligned} \Pr[m_{j_1} = \emptyset, piv \mid s, \sigma] &= \sum_{\theta \in \{1, 0, \emptyset\}} \Pr[m_{j_1} = \emptyset, piv, \theta \mid s, \sigma] \\ &= \sum_{\theta \in \{1, 0, \emptyset\}} \Pr[\theta \mid s, \sigma] \Pr[m_{j_1} = \emptyset, piv \mid \theta, s, \sigma] \\ &= \sum_{\theta \in \{1, 0, \emptyset\}} \Pr[\theta \mid s] \Pr[m_{j_1} = \emptyset, piv \mid \theta, \sigma]. \end{aligned}$$

According to how the signal is drawn, I have

$$\Pr[\theta = 1 \mid s] = (1 - h)\left(\frac{1}{2} + s\right), \quad \Pr[\theta = 0 \mid s] = (1 - h)\left(\frac{1}{2} - s\right), \quad \Pr[\theta = \emptyset \mid s] = h.$$

Next, I show how to calculate $\Pr[m_{j_1} = \emptyset, piv \mid \theta, \sigma]$. To illustrate, take $\theta = \emptyset$ as an example. I have

$$\Pr[m_{j_1} = \emptyset, piv \mid \theta = \emptyset, \sigma] = t_i(\emptyset)(1 - t_{j_2}(\emptyset)) + t_{j_2}(\emptyset)(1 - t_i(\emptyset)).$$

I want to show that this probability is strictly positive when $\beta \in [0, 2/3)$. For this probability to be zero, it must be the case that both i and j_2 vote for the same policy with probability one. For any pair (i, j_2) , this is not possible. Therefore, the probability $\Pr[m_{j_1} = \emptyset, piv \mid s, \sigma]$ is strictly positive for any $s \in [-1/2, 1/2]$. The density function of state s conditional on j_1 receiving no signal and being pivotal is

$$g(s \mid m_{j_1} = \emptyset, piv, \sigma) = \frac{\Pr[m_{j_1} = \emptyset, piv \mid s, \sigma] f(s)}{\int_{-1/2}^{1/2} \Pr[m_{j_1} = \emptyset, piv \mid s, \sigma] f(s) ds}.$$

Therefore, the expectation of state variable s conditional on j_1 receiving no signal and being pivotal is

$$-\mathbb{E}[s \mid m_{j_1} = \emptyset, piv, \sigma] = - \int_{-1/2}^{1/2} s g(s \mid m_j = \emptyset, piv, \sigma) ds.$$

The probability $\Pr[m_j = \emptyset, piv \mid s, \sigma]$ is a sum product of $\Pr[m_j = \emptyset \mid s, \sigma]$ and $\Pr[piv \mid m_j =$

¹¹If an uninformed player j receives signal 1 off the equilibrium path, since the informed player can not fake the signal, player j assigns probability one on the event that player i shares signal 1 with j and assigns probability $\mu_j(m_i \mid m_j)$ to the event that m_i occurs conditional on $m_j = 1$.

$\emptyset, s, \sigma]$. Regardless of the state, there is a strictly positive probability that player i receives no signal and hence shares no signal with player j . Therefore, $\Pr[m_j = \emptyset \mid s, \sigma]$ is well-defined and strictly positive for any s . Once I restrict attention to those strategy profiles satisfying Lemma 1 and Lemma 2, the probability $\Pr[piv \mid m_j = \emptyset, s, \sigma]$ is strictly positive for any s .

8.3 Proof of Proposition 1-3

To simplify notation, let $t_1, t_3 \in (-1/6, 1/6)$ denote the voting thresholds employed by player 1 and 3 if they observe no signal. Let player 1 and 3 be the uninformed player j_1 and j_2 respectively. The disclosure actions have the following interpretation,

$$\left\{ \begin{array}{l} (1, 0, 0, 0) : \text{ share with 1 and 3,} \\ (0, 1, 0, 0) : \text{ share with 1,} \\ (0, 0, 1, 0) : \text{ share with 3,} \\ (0, 0, 0, 1) : \text{ share with neither 1 or 3.} \end{array} \right.$$

Due to symmetry, I focus on the disclosure behavior after the central player observes signal 1. Let p_1^L and p_3^L denote probabilities with which player 1 and 3 vote for L in equilibrium.

Proof of Proposition 1. When $\beta < 2/15$, an uninformed player votes for policy L with probability at least $1/5$ regardless of the messages he receives. Because the lowest threshold an uninformed player ever employs is $-1/6$, both p_1^L and p_3^L are bounded from below by $1/5$,

$$p_1^L \geq -\frac{1}{6} - \left(-\frac{1}{2} - \beta\right) > \frac{1}{5}, \quad p_3^L \geq -\frac{1}{6} - \left(-\frac{1}{2} + \beta\right) > \frac{1}{5}.$$

According to Lemma 3, player 2 would like p_3^L and p_1^L to be as low as possible. The unique optimal disclosure strategy is to share signal 1 with both player 1 and 3. Analogously, player 1 fully disclose signal 0 to the uninformed players. \square

Proof of Proposition 2. The equilibrium described in Proposition 2 corresponds to the situation where $\beta > 2/15$, $t_1 < 1/2 - \beta$, and $t_3 > -1/2 + \beta$. When $\beta > 2/15$, the probability with which player 1 votes for L is strictly larger than $1/5$ because

$$p_1^L \geq -\frac{1}{6} - \left(-\frac{1}{2} - \beta\right) > \frac{1}{5}.$$

Therefore, player 2 prefers p_3^L to be as low as possible. If $t_3 > -1/2 + \beta$, sharing signal 1 with player 3 leads to a strictly lower p_3^L than withholding signal 1 does. Hence, player 2 shares signal 1 with player

3. This implies that the threshold of player 3 after signal 1 is $-1/6$ and

$$p_3^L = -\frac{1}{6} - \left(-\frac{1}{2} + \beta\right) = \frac{1}{3} - \beta < \frac{1}{5}.$$

Player 2 prefers p_1^L to be as high as possible and withholds signal 1 from the left player. Analogously, the central player shares signal 0 with the left player while withholding it from the right player. Given the disclosure strategy, the thresholds if the uninformed players receive no message are

$$t_1 = -E[s \mid m_1 = \emptyset, piv, \sigma] = \begin{cases} -\frac{(4-3\beta)(1-h)}{6(3\beta(h-1)+5h+4)} & \text{if } \beta < \frac{1}{3}, \\ -\frac{1-h}{12h+6} & \text{if } \beta \geq \frac{1}{3}, \end{cases}$$

$$t_3 = -E[s \mid m_3 = \emptyset, piv, \sigma] = E[s \mid m_1 = \emptyset, piv, \sigma].$$

The condition that $t_1 < 1/2 - \beta$ and $t_3 > -1/2 + \beta$ requires that $\beta < \frac{5h+4}{12h+6}$.

Note that when $\beta = \frac{2}{15}$, the central player is indifferent between sharing signal 1 with player 1 or not. For any $q, q' \in [0, 1]$, the following disclosure strategy and voting thresholds constitute an equilibrium

$$\pi_2(1) = (q, 0, 1 - q, 0), \quad \pi_2(0) = (q', 1 - q', 0, 0),$$

$$t_1 = \frac{5h}{6(2(h-1)q + 3h + 2)} - \frac{1}{6}, \quad t_3 = \frac{1}{6} - \frac{5h}{6(2(h-1)q' + 3h + 2)}.$$

□

Proof of Proposition 3. The equilibrium described in Proposition 3 corresponds to the situation where $\beta > 2/15$, $t_1 \geq 1/2 - \beta$, and $t_3 \leq -1/2 + \beta$. Player 1 (player 3) always votes for L (R) if he receives no signal from the central player. In the equilibrium, it must be the case that the central player is always pivotal. Otherwise, the central player could choose not to share any information with his opponents who would vote differently. The probability with which player 1 votes for L is strictly larger than $1/5$ because

$$p_1^L \geq -\frac{1}{6} - \left(-\frac{1}{2} - \beta\right) > \frac{1}{5}.$$

Therefore, player 2 prefers p_3^L to be as low as possible. Since $-1/6 < t_3 \leq -1/2 + \beta$, player 3 votes for R with probability one whether player 2 shares signal 1 with him or not. Hence, p_3^L equals zero and player 2 prefers p_1^L to be as high as possible. Because player 1 uses $-1/6$ as voting threshold if he observes signal 1 and $t_1 > -1/6$ as threshold if he receives no signal, player 2 conceals signal 1 from the left player. The optimal disclosure rule after signal 1 is

$$\pi_2(1) = (0, 0, q, 1 - q), \tag{5}$$

where $q \in [0, 1]$. Analogously, the optimal disclosure rule after signal 0 is

$$\pi_2(0) = (0, q', 0, 1 - q'). \quad (6)$$

The central player only needs to make sure to withhold 0—a signal favorable to L —from the right player. Given the disclosure strategy, I can calculate the uninformed players' voting thresholds

$$t_1 = -E[s \mid m_1 = \emptyset, piv, \sigma] = \frac{1}{6} \left(-\frac{h+2}{3-2(1-h)q'} + 1 \right), \quad (7)$$

$$t_3 = -E[s \mid m_3 = \emptyset, piv, \sigma] = \frac{1}{6} \left(\frac{h+2}{3-2(1-h)q} - 1 \right). \quad (8)$$

The last step is to check that $t_1 \geq 1/2 - \beta$ and $t_3 \leq -1/2 + \beta$ does hold. This condition is easier to satisfy when q and q' both equal zero, because t_1 decreases in q' and t_3 increases in q . By setting q and q' to be zero, I have

$$\frac{1}{6} \left(-\frac{h+2}{3} + 1 \right) \geq \frac{1}{2} - \beta, \quad \frac{1}{6} \left(\frac{h+2}{3} - 1 \right) \leq -\frac{1}{2} + \beta.$$

Both inequalities are equivalent to $\beta \geq (8+h)/18$. For any pair (β, h) satisfying this condition, the following disclosure strategy and the associated thresholds form an equilibrium

$$\pi_2(1) = (0, 0, 0, 1), \quad \pi_2(0) = (0, 0, 0, 1).$$

Mixed disclosure strategies characterized by 5 and 6 are also allowed as long as q and q' are bounded from above by $\frac{18\beta-h-8}{4(3\beta-1)(1-h)}$. \square

The last group of equilibrium corresponds to the case where $\beta > 2/15$, $t_1 < 1/2 - \beta$, and $t_3 \leq -1/2 + \beta$.¹² Player 3 always votes for R if he receives no signal while player 1's vote is sensitive to his bias realization when he receives no signal. I first characterize player 2's disclosure strategy when he obtains signal 1. Because the probability with which player 1 votes for L is strictly larger than $1/5$, player 2 prefers p_3^L to be as low as possible. Since $-1/6 < t_3 \leq -1/2 + \beta$, player 3 votes for R with probability one whether he receives signal 1 or not. Hence, p_3^L equals zero and player 2 prefers p_1^L to be as large as possible. Because player 1 uses $-1/6$ as voting threshold if he observes signal 1 and $t_1 > -1/6$ as threshold if he receives no signal, player 2 conceals signal 1 from player 1. The optimal disclosure rule after signal 1 is

$$\pi_2(1) = (0, 0, q, 1 - q), \quad (9)$$

where $q \in [0, 1]$. Next, consider player 2's disclosure strategy when he observes signal 0. First, note that the probability with which player 3 votes for R is strictly larger than $1/5$. Because the highest

¹²The case where $\beta > 2/15$, $t_1 \geq 1/2 - \beta$, and $t_3 > -1/2 + \beta$ is omitted here due to symmetry.

voting threshold a player ever employs is $1/6$, player 3 votes for R with probability at least

$$\frac{1}{2} + \beta - \frac{1}{6} > \frac{1}{5}.$$

Therefore, player 2 prefers the left player to vote for policy L with higher probability. Since $t_1 < 1/2 - \beta$, player 2 strictly prefers to share signal 0 with player 1 who then votes for L with probability one. Given that, player 2 prefers player 3 to vote for R with higher probability. Hence, he withholds signal 0 from player 3. The optimal disclosure strategy after signal 0 is

$$\pi_2(1) = (0, 1, 0, 0). \quad (10)$$

The last step is to check that $t_1 < 1/2 - \beta$ and $t_3 \leq -1/2 + \beta$ does hold. This requires that the pair (β, h) satisfies the following inequality

$$\beta < \frac{5h + 4}{6(2h + 1)}, \text{ and } \beta \geq \frac{9h^2 + 33h - \sqrt{h(h(1473 - h(95h + 118)) + 1352) + 304} + 12}{12(h - 1)(2h + 1)}. \quad (11)$$

When the above condition holds, the condition $\beta \geq (8 + h)/18$ also holds. I can find an equilibrium such that the central player is always pivotal. Therefore, this subgroup of equilibria does not improve the informed player's welfare.

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